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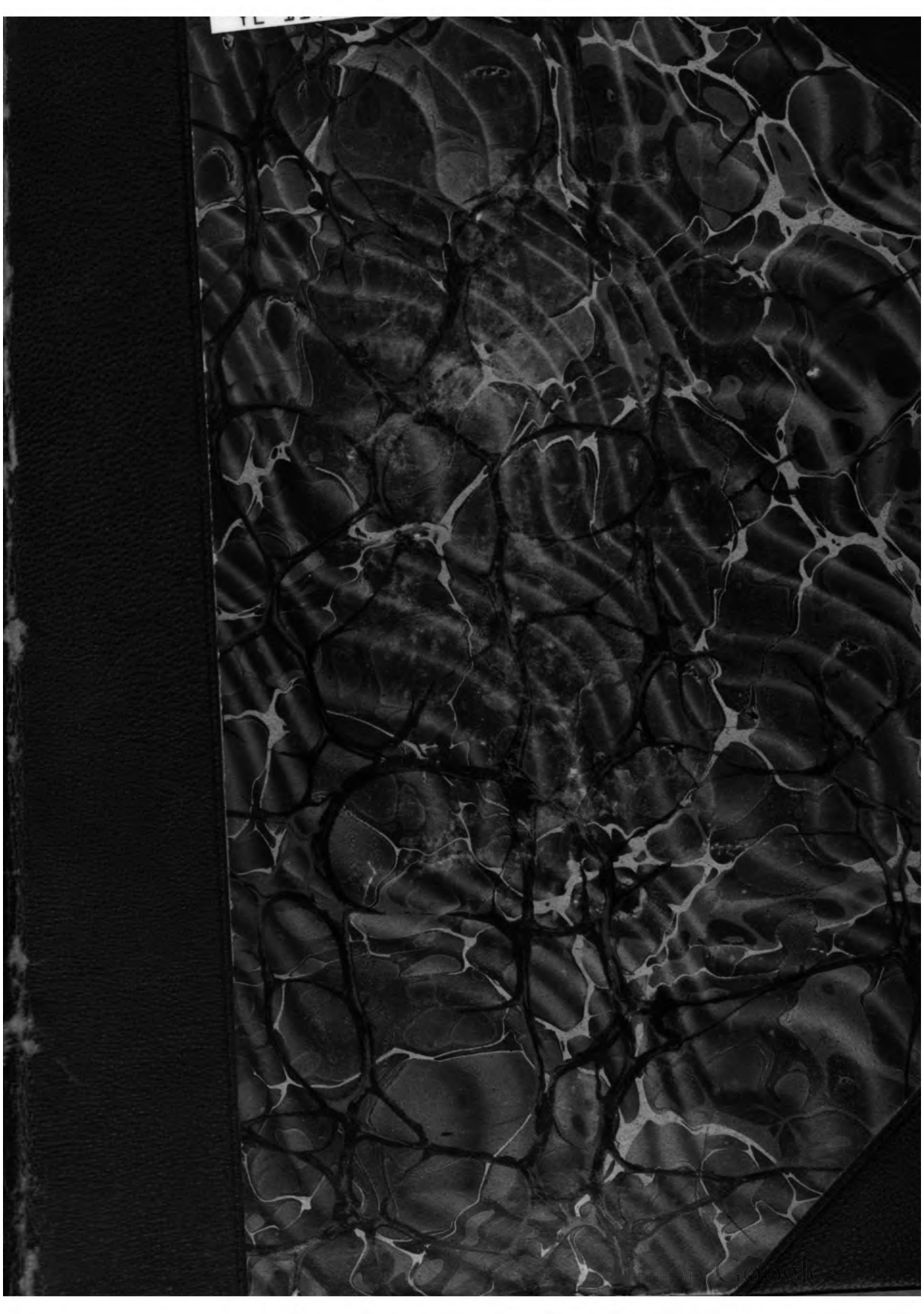
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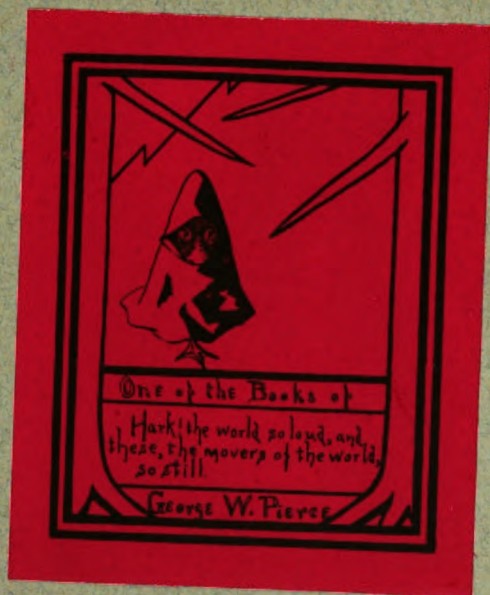
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THE
PHYSICAL REVIEW

*A JOURNAL OF EXPERIMENTAL AND
THEORETICAL PHYSICS*

CONDUCTED WITH THE CO-OPERATION
OF THE

AMERICAN PHYSICAL SOCIETY

BY

EDWARD L. NICHOLS, ERNEST MERRITT,
AND FREDERICK BEDELL

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THE PHYSICAL REVIEW.

THE MAGNETIC SUSCEPTIBILITY OF WATER.

BY HERMAN D. STEARNS.

THE magnetic susceptibility of water at a given temperature, such as 20° C., is regarded as one of the constants of nature, but there is no general agreement as to its numerical value. Table I. contains the results published by a number of careful observers. In writing the susceptibility (K) no correction has been made for the susceptibility of air. The temperature coefficient of (K) is of the order of .002, hence reduction to a common temperature could not bring the results into agreement.

TABLE I.
Determinations of (K).

Date.	Observer.	Temperature.	$K \cdot 10^6$.
1885	Quincke ¹	20° C.	— .84
1888	DuBois ¹	15	— .86
1892	Henrichsen ²		— .75
1895	Curie ³	18	— .79
1896	Townsend ⁴	10	— .77
1898 } 1901 }	Königsberger ⁵	22	— .80
1899	Jäger and Meyer ⁶	18	— .66

¹ Wied. Ann., Vol. 35, p. 137, 1888.

² Wied. Ann., Vol. 45, p. 38, 1892.

³ Journal de Physique, p. 206, 1895.

⁴ Proc. Roy. Soc. London, 60, p. 186, 1896-7.

⁵ Wied. Ann., Vol. 66, p. 698, 1898; Drude's Ann., Vol. 6, p. 506, 1901.

⁶ Wied. Ann., Vol. 67, p. 712, 1899; Drude's Ann., Vol. 6, p. 870, 1901.

With the assistance of Mr. Angus L. Cavanagh, a student of physics in Stanford University, I have made a new determination.

We have used that general method in which a right cylinder of water of cross-section (q) is set perpendicular to a magnetic field whose strength at one end of the cylinder is equal to (H) and at the other end is negligible. (K) is given by the equation,

$$K = -\frac{2g\rho}{H^2q},$$

where (ρ) is the numerical value of the magnetic repulsion in grams on the cylinder in the direction of its axis. The value of (g) in this locality is 980.

In determining (ρ) we used a balance as Jäger¹ and Meyer did in their third determination. A glass tube was suspended from one end of a non-magnetic balance. The lower end was sealed abruptly and reached the center of the air space between the parallel pole-pieces of a large electromagnet. In this position the change in the weight of the tube due to exciting the magnet was determined by several successive weighings. The tube was then filled with water to a height at which the field strength was negligible and the weighings were repeated. Table II. contains the record of the weighings and the computed value of (ρ) for the four different tubes that were used. The changes of weight indicated by (ρ') and (ρ'') in Table II. were made by the use of a single rider whose mass was determined by each of us separately by comparison with a standard mass. Our results differed by .05 per cent.

The cross-section (q) of the cylinder of water contained in each tube was determined by the usual method of calibrating tubes by means of mercury. The height of the mercury column was read by each of two micrometer microscopes furnished by the Geneva Society, one of them belonging to the "comparateur" furnished by this society. The mercury was weighed on an ordinary analytical

produce no appreciable effect, and below 1.5 cm. no lack of uniformity in the field was found. The values found for (q) are given in Table II.

The water used was distilled from rain water caught in a glass vessel as it fell from a tile roof after copious showers had cleaned the roof. The water was condensed in tubes Nos. 1, 2 and 3 directly from the still. That in tube No. 4 had been in another glass vessel for some months. Experiment showed, however, that even water from the University water system gave the same values for (K) as the nearly pure water used.

TABLE II.

Values of (p) , (q) and $\left(\frac{p}{q}\right)$.

Tube No. 1. $(q) = 1.282$ sq. cm.		
Temperature.	Current of Electromagnet.	Repulsion.
Tube empty.		
(t)	(c)	(p')
21	12	-.0039
23	12	-.0038
23	12	-.0038
24	12	-.0039
24	12	-.0038
24	12	-.00385
22	12	-.0039
24	12	-.0039
Mean 23	12	-.00385
Tube filled with water.		
(t)	(c)	(p'')
22	12	.0290
22	12	.0290
22	12	.0290
24	12	.0291
Mean 23	12	.02902

Water alone (calculated).

TABLE II.—*Continued.*

Tube No. 2. (q) = 1.279.		
Tube empty.		
(t)	(c)	(p')
25	12	-.0038
22	12	-.0039
Mean 23	12	-.00385
Tube filled with water.		
(t)	(c)	(p'')
24	12	.0291
22	12	.0291
22	12	.0290
20	12	.0290
Mean 22	12	.02905
Water alone (calculated).		
(t)	(c)	(p) = ($p'' - p'$)
23	12	.0329
Tube No. 3. (q) = 1.333.		
Tube empty.		
(t)	(c)	(p')
25	12	-.0068
	12	-.0066
Mean 25	12	-.0067
Tube filled with water.		
(t)	(c)	(p'')
23	12	.0275
23	12	.0275
23	12	.0274
Mean 23	12	.02747
Water alone (calculated).		
(t)	(c)	(p) = ($p'' - p'$)
23	12	.03417

TABLE II.—*Continued.*Tube No. 4. (q) = .469.

Tube empty.

(t)	(c)	(ρ')
21	11.7	-.0011
21	11.6	-.0012
24	11.3	-.00115
22	12	-.0012
Mean 22	11.7	-.00116

Tube filled with water.

(t)	(c)	(ρ'')
20	12	.0109
20	12	.0109
22	11.6	.0108
Mean 21	11.9	.01087

Water alone (calculated).

(t)	(c)	(ρ) = ($\rho'' - \rho'$)
21	12	.01205

Values of (p/q).

No. of Tube.	Temperature.	(p/q)
1	23	256.4
2	23	257.2
3	23	256.3
4	21	256.9
Mean	23	256.7

The close agreement of the values of (p/q) would seem to show that the mean value cannot be in error by more than .5 per cent.

In determining the field strength (H) three test coils were used. Coil No. 1 was mounted so that it could be turned through 180° in the field (H). Coils No. 2 and 3 were mounted on handles so that they could be drawn out of the same field. No. 2 and No. 3 were of nearly the same diameter and were wound with nine turns of wire each. Coil No. 1 was of smaller diameter and was wound with six turns of wire. Its effective area was almost exactly the same as that of the other two, its actual area being doubled on account of its turning through 180° . An astatic mirror galvanom-

eter of long period placed at a sufficient distance from the electromagnet was connected in series with the three coils, and the galvanometer deflections due to operating in the field (H) with the coils were observed. After continued practice a comparison of the three coils was made and the results are given in Table III.

TABLE III.

Comparison of the test coils in the field (H).

No. of Coil.	Area (A).	Deflection (a).	Current.	a/A
3	16.84	212	11.8	1.259
1	16.92	213	11.8	1.259
2	16.88	212	11.7	1.256
3		212	11.7	
1		213	11.7	
2		212	11.7	
3		212	11.7	
1		213	11.7	
2		212	11.7	

This agreement being satisfactory coils No. 1 and No. 3 were selected for further use.

As a standard instrument a current inductor of the pattern described on page 224 of Henderson's Practical Electricity and Magnetism was used. The formula for the number of lines of force cut by the test coil of such a current inductor when the current through its solenoid is broken is

$$N_1 = \frac{4\pi n n_1 C A}{10} \left(1 - \frac{1}{2} \frac{r^2}{l^2} \right).$$

Measurements of the constants of this instrument gave, for the value of (N_1),

$$N_1 = 210 \cdot 10^3 C.$$

(C) was measured by Weston direct-reading portable ammeter No. 10364 whose indications are certified to by the company as correct to .25 per cent.

To test the accuracy of the measured constant of the current inductor, this inductor was connected in series with an earth induc-

tor and the galvanometer, and the deflections of the two inductors were compared. The result is given in Table IV.

TABLE IV.

Comparison of earth inductor and current inductor.

Inductor Operated.	Deflection.	Current in Solenoid of Current Inductor.
Current	237	.717
Earth	235	
Current	236.5	.715
Earth	234.5	
Current	234	.710
Earth	234.5	
Current	234.5	.712
Earth	235	
Current	234.5	.712
Earth	235	
Mean { Current	235.3	.713
Earth	234.8	
Calculated { Current	234.8	.712
Earth	234.8	

The formula for the earth indicator is

$$N_2 = 2A'H',$$

where (H') is the horizontal component of the earth's magnetic field.

The mean of four determinations of (H') by means of a Kew magnetometer set up where the earth inductor was used gave

$$(H') = .2482.$$

Hence

$$N_2 = 2.301 \cdot 10^3 \cdot .248 = .149300.$$

For N' , using for (C) the value .712 taken from Table IV., we have

$$N' = 210 \cdot 10^3 \cdot .712 = 149500.$$

This good agreement seemed to justify the use of the current inductor as an absolute standard.

To determine (H) by comparison with the field of the current inductor, coil No. 1 was connected in series with the test coil of the inductor and with the galvanometer, and the deflections due to rotating coil No. 1 in the field (H) and to breaking the current through the solenoid of the current inductor were observed. Coil No. 1 was then replaced by coil No. 3 and two more series were made on different days. The results are given in Table V.

TABLE V.

Value of (H).

Instrument Operated.		Deflection.	Current in Solenoid.	Current in Electro-magnet.
First series.				
	Current inductor.	233	.669	
	Coil No. 1.	233		12
	Current inductor.	233	.668	
	Coil No. 1.	234		12
	Current inductor.	234	.670	
	Coil No. 1.	233		12
	Current inductor.	234	.670	
	Coil No. 1.	234		12
Mean	{ Current inductor.	233.5	.669	
	{ Coil No. 1.	233.5		12

Area of coil No. 1, 16.92.

$$H = \frac{210 \cdot 10^9 \cdot .669}{16.92} = 8300.$$

Second series.

Current inductor.		222	.666	
		220	.666	
		220	.664	
		220	.663	
		219	.663	
Coil No. 3.		219		11.7
		219		11.7
Mean	{ Current inductor.	220	.665	
	{ Coil No. 3.	219		11.7
Calculated	{ Current	220	.665	
	{ inductor.	220		12

TABLE V.—*Continued.*

Third series.			
Current inductor.	222	.678	
	220	.668	
	220	.668	
	223	.679	
Coil No. 3.	220		12
Current inductor.	219	.666	
Coil No. 3	219		12
Current inductor.	219	.665	
Coil No. 3.	219		12
Mean	Current inductor.	220.5	.6706
	Coil No. 3.	219.3	
Calculated	Current inductor.	219.3	.667
	Coil No. 3.	219.3	
Mean current in solenoid in second and third series		.666	
Area of coil No. 3		16.84	

$$H = \frac{210 \cdot 10^3 \cdot .666}{16.84} = 8300.$$

In view of the agreement between the current inductor and the earth inductor and also among the test coils, a greater error than 1 per cent. in the value of (H) seems very improbable.

Careful explorations of the magnetic field showed that the residual field and the field at the top of the water cylinder were negligible. Hence it is correct to use as the formula for (K)

$$K = -\frac{2g\rho}{H^2q}.$$

The first four lines of Table VI. contain the values for K calculated from the values of (ρ/q) in Table II. and from the value of (H) in Table V.

The poles of the electromagnet were moved closer together and another determination of (K) was made by the same method and with the same apparatus. This value appears in the fifth line of Table VI.

The poles were brought back to their original position and additional determinations were made, the field strength being varied

by changes in the exciting current of the electromagnet. The results are given in the last three lines of Table VI.

TABLE VI.

Values of (K).

Temperature.	(H)	($K \cdot 10^6$)
23	8300	-.730
23	8300	-.732
23	8300	-.730
21	8300	-.731
22	9700	-.734
22	8150	-.736
22	8030	-.735
21	7470	-.738
Mean 22		-.733

Admitting a possible error of 1 per cent. in the measurements of (H) and of .5 per cent. in the measurements of $\left(\frac{p}{q}\right)$ the maximum possible error in the result would be 2.5 per cent., and it would seem certain that the true value of (K) lies between the values $-.715 \cdot 10^{-6}$ and $-.750 \cdot 10^{-6}$ at the temperature of 22°C .

Opposed to this conclusion stand the results of all the observers quoted in Table I. except that of Henrichsen. The situation is well expressed by Jäger and Meyer,¹ who say, "Eine ausreichende Erklärung für die starken Abweichungen der Resultate der verschiedenen Beobachter steht sonach noch aus."

PHYSICAL LABORATORY,

STANFORD UNIVERSITY, CAL., September, 1902.

¹ Drude's Ann., Vol. 6, p. 870, 1901

THERMODYNAMIC FORMULÆ FOR ISOTROPIC
SOLIDS SUBJECT TO TENSION IN
ONE DIRECTION.

BY J. R. BENTON.

THE purposes of this note are (1) to establish some thermodynamic formulæ for isotropic solids, when acted upon by no external forces except a uniformly distributed tension in one direction; and (2) to show that in consequence of these formulæ the coefficient of expansion and the specific heat of most isotropic substances must increase with increasing tension, the change in specific heat being however very small.

Consider a wire or rod of uniform cross section, subjected to tension. Let us exclude all actions upon it from outside, except the following: (1) the tension may be changed (inside the elastic limit); and (2) heat may be added (or abstracted). Let us assume that no internal changes take place in the wire, except as accompanying one of the above-named actions from outside; and that then the internal change is determined by the action from outside. This assumption can be realized by metal wires. Under these limitations the system will have but two degrees of freedom.

Let us assume further that the wire has been reduced to the "normal elastic state" by repeated heating and cooling under varying tension; the effects of elastic hysteresis can then be neglected. When all these conditions are fulfilled, experiments show (1) that all processes which our assumptions admit, are reversible, or at least can be made as nearly so as we please, by making them proceed sufficiently slowly; and (2) that the state of the system can be completely defined by two independent variables. These variables may be chosen in various ways; we shall find it most convenient to use the absolute temperature (T) in centigrade degrees, and the tension (P) in dynes per square centimeter.

Suppose T and P to increase; we know from experiment that the corresponding increase in the length l is given by

$$dl = l_0(adT + bdP), \quad (1)$$

where l_0 is the length at 0° C. and no tension, a is the coefficient of linear thermal expansion, and b is the reciprocal of Young's modulus. a and b are not constant, but are to be considered as functions of T and P .¹

If heat is added to the wire, we have

$$dQ = M(c_p dT + \beta dP), \quad (2)$$

where Q is the quantity of heat in gram calories, M is the mass of the wire, c_p the specific heat at constant tension, and β is a coefficient to which no name has been given, but which might be called the latent heat of change of tension. c_p and β are to be considered as functions of T and P .¹

For the work W , in ergs, done upon the wire, we have

$$dW = qPdL = ql_0(PadT + PbdP), \quad (3)$$

where q is the cross-section. In this expression we neglect the very small amount of work done by (or against) atmospheric pressure as the wire contracts (or expands) laterally.

For the internal energy E , in ergs, we have, since mechanical work and heat are the only kinds of energy that enter or leave the system,

$$dE = dW + JdQ = (ql_0Pa + MJc_p)dT + (ql_0Pb + MJ\beta)dP, \quad (4)$$

where J is the mechanical equivalent of heat, in ergs per gram-calorie.

Since we consider only reversible processes, an infinitesimal change in the entropy (H) is given by the equation $dH = dQ/T$. We have accordingly

$$dH = \frac{Mc_p}{T}dT + \frac{M\beta}{T}dP. \quad (5)$$

Passing in review the quantities under our consideration, we may classify them into (1) known constants, l_0 , q , M , and J ; (2) quantities of independent physical significance, T , P , l , E , H , W and Q ; and (3) coefficients — a , b , c_p , and β — defined in terms of the derivatives of the quantities of class (2). It is our problem to establish relations between these coefficients and their derivatives with respect to T and P .

Equations (1)–(5), involving the coefficients a , b , c_p , β , and the known constants, give the differentials of l , W , Q , E , H , in terms of the differentials of the independent variables T and P . dW and dQ are not perfect differentials; but we know by experiment that dl is a perfect differential, under the conditions we have assumed. dE and dH are perfect differentials, by the first and second fundamental principles of thermodynamics.

We have, then, from (1), (4) and (5)

$$\left(\frac{\partial b}{\partial T}\right)_P = \left(\frac{\partial a}{\partial P}\right)_T \quad (6)$$

$$\left(\frac{\partial [ql_0 Pb + MJ\beta]}{\partial T}\right)_P = \left(\frac{\partial [ql_0 Pa + MJc_p]}{\partial P}\right)_T \quad (7)$$

$$\left(\frac{\partial [M\beta/T]}{\partial T}\right)_P = \left(\frac{\partial [Mc_p/T]}{\partial P}\right)_T, \quad (8)$$

where $\left(\frac{\partial}{\partial T}\right)_P$ means the derivative with respect to T at constant tension, and $\left(\frac{\partial}{\partial P}\right)_T$ means the derivative with respect to P at constant temperature.

Carrying out the differentiations in (7) and (8); and bearing in mind the relation (6), we have

$$MJ \left(\frac{\partial \beta}{\partial T}\right)_P = ql_0 a + MJ \left(\frac{\partial c_p}{\partial P}\right)_T \quad (9)$$

Substituting in (9) the value of $\left(\frac{\partial \beta}{\partial T}\right)_P$ from (10) we get

$$\beta = \frac{q'_0}{MJ} \cdot aT = \frac{1}{\rho_0 J} \cdot aT, \quad (11)$$

where ρ_0 is the density at 0°C. and no tension.

Remembering that b is the reciprocal of Young's modulus, we can put (6) in the form

$$\left(\frac{\partial Y}{\partial T}\right)_P = -Y^2 \left(\frac{\partial a}{\partial P}\right)_T, \quad (12)$$

where Y is Young's modulus.

By substituting in (10) the value of β from (11) we obtain

$$\left(\frac{\partial a}{\partial T}\right)_P = \frac{\rho_0 J}{T} \left(\frac{\partial c_P}{\partial P}\right)_T. \quad (13)$$

The last three equations are the ones to which it is desired to call attention.

Equation (11) gives an expression for the latent heat of increase of tension in terms of the coefficient of linear expansion, the absolute temperature, the density (at 0° C. and no tension), and the mechanical equivalent of heat. If we substitute the value of β given by this equation, in (2), and set $dQ = 0$, we obtain

$$dT = -\frac{aT}{c_P \rho_0 J} \cdot dP, \quad (14)$$

the well-known equation for the change in temperature due to a sudden (adiabatic) change of tension; this equation was derived by Lord Kelvin, and was made the basis of a determination of the mechanical equivalent of heat by Edlund.¹

Equation (12) gives a relation between Young's modulus, its derivative with respect to the temperature (at constant tension), and

the tension (at constant temperature). As a consequence of this equation we may say: *If for a given substance Young's modulus decreases with rising temperature, then the coefficient of thermal expansion increases with increasing tension.* Since it is the general rule that Young's modulus *does* decrease with rising temperature, we may accept it as a general rule that the coefficient of thermal expansion of solid bodies is greater when they are under tension than when free.

According to (12) the coefficient of expansion of a metal wire, when strained to its elastic limit, may differ by several per cent. from its value when the wire is under no tension. In copper at 0° C., for instance, the difference may amount to about 5 per cent.

An equation similar to (12) is given, as a result of slightly different reasoning, by Dahlander,¹ who made experiments with copper, brass, german silver, iron and steel, to obtain the dependence of the coefficient of expansion on tension, and found a rough agreement with the formula. Dahlander determined the coefficient of expansion under different tensions; unfortunately he did not measure Young's modulus and its derivative with respect to the temperature for the *same* wires, but substituted in the formula the values obtained by other observers, with different wires. Accordingly no very close agreement with the formula can be expected, in view of the variation of mechanical properties in different specimens of the same substance.

Equation (13) gives a relation between the derivative of the coefficient of expansion with respect to the temperature, the derivative of the specific heat with respect to the tension, and the absolute temperature; the density and the mechanical equivalent of heat being supposed known quantities. As a consequence of this equation we may say: *If for a given substance the coefficient of expansion increases with rising temperature, then the specific heat (at constant tension) increases with increasing tension.* Since it is the general rule that the coefficient of expansion *does* increase with rising temperature, we may take it as a general rule that the specific heat of solid bodies increases as the tension is made greater.

¹ G. R. Dahlander: Versuch, den Ausdehnungskoeffizienten von Metalldrähten bei ungleichen Spannungsgraden zu bestimmen. Pogg. Ann., CXLV., pp. 147-153, 1872.

For most metals, however, this change in specific heat with the tension is inappreciable. Copper, for example, at 0°C. , gives

$$\left(\frac{\partial c_p}{\partial P}\right)_T = \frac{T}{J\rho_0} \left(\frac{\partial a}{\partial T}\right)_P = \frac{273}{4.2 \times 10^7 \times 8.9} \times 4.04 \times 10^{-8} = 2.95 \times 10^{-14},$$

where P is expressed in dynes per square centimeter. Expressing P in kilograms weight per square millimeter, this becomes 2.89×10^{-6} . If the elastic limit is as high as 30 kilograms per square millimeter, the specific heat under this tension would differ by only 0.00009, or one tenth of one percent from its value under no tension. For steel, at 0°C. , the tension would have to rise to 60 kilograms per square millimeter, before the specific heat would differ one fourth of one percent from its value under no tension. In these computations the values of $\left(\frac{\partial a}{\partial T}\right)_P$ given by Voigt¹ have been used. For other metals the results are of the same order of magnitude.

I am not aware that any attempt has ever been made to study the dependence of specific heat on tension experimentally.

The three equations (11), (12) and (13) were derived with T and P as independent variables. The normal variables of the system are l and T ; T and P are inverse variables. By using the relation $dl = l_0(adT + b dP)$, we could change the independent variables to l and T , or to l and P . Such transformations of course lead to no new *independent* relations, but may bring into prominence relations not otherwise apparent, or may reduce the equations to more convenient form. A thorough investigation of these transformations, however, has not led to expressions possessing either of these advantages. And the use of P as an independent variable, rather than l , has a great advantage, because P (and not l) is the quantity that can be varied directly (or kept constant) during experiments.

¹ W. Voigt : Wied. Ann.

ELASTICITY OF COPPER AND STEEL AT -186° C.

BY J. R. BENTON.

KNOWLEDGE of the dependence of the elasticity of solids on temperature, aside from any scientific value in itself, is of interest also because inferences can be drawn from it regarding the variation of other mechanical and thermal properties,¹ and because of its possible bearing on the theory of intermolecular forces in solids.

The variation of elasticity with temperature has been studied by Wertheim,² Köhlrausch and Loomis,³ Kupffer,⁴ Streintz⁵ and Kiewiet,⁶ and was the subject of very careful researches by Pisati.⁷ These investigators worked mostly above ordinary temperature; Wertheim extended his experiments to -15° C.; the others did not go below 0° . The extremely thorough experiments of Pisati, which furnish such complete and accurate information⁸ for the substances and temperatures he used, were made between 20° and 300° C.

¹ See article: Thermodynamic Formulæ for Isotropic Solids subject to Tension in one Direction, *PHYS. REV.*

² G. Wertheim: Untersuchungen über die Elastizität. *Pogg. Ann., Ergänzungsband II.*, pp. 1-115, 1848.

³ F. Köhlrausch and Loomis: *Pogg. Ann.*, CXLI., p. 481, 1870.

⁴ A. Kupffer: *Mém. de l'Acad. imp. de St. Petersburg (VI. sér.)*, Vol. VI, p. 393, 1857.

⁵ H. Streintz: Über die Änderung der Elastizität und der Länge eines vom galvanischen Strome durchflossenen Drahtes. *Pogg. Ann.*, CL., p. 368, 1873.

⁶ J. Kiewiet: *Wiedemann's Ann.*, XXIX., 617-654, 1886.

⁷ G. Pisati: Sulla Elasticità dei Metalli a diverse Temperature. *Nuovo Cimento*, 3d ser., Vol. I., p. 181; Vol. II., p. 137, 1877; Vol. IV., p. 152, 1878; Vol. V., pp. 34, 135, 145, 1879.

⁸ One criticism may be made on Pisati's experiments. He employed the method of torsional oscillations, using wires about 70 cm. long; for such short wires the uncertain distribution of stress at the points of support may have an influence appreciable within his limits of error.

The low temperatures now available for scientific work make possible an extension of this study of elasticity as influenced by temperature, previously carried on above, but not below, ordinary temperature.

Some of the mechanical properties of matter have already been investigated at liquid air temperature. The ultimate tensile strength of a number of metals was measured by Dewar,¹ who found it greatly increased in most cases, the increase of strength being accompanied by decreased elongation at rupture; decreased elongation appears to accompany the increased strength due to low temperature, much as it accompanies at ordinary temperature the increased strength in iron and steel, due to the presence of other substances in chemical combination or as alloys.

Dewar made no quantitative determinations of the elastic moduli, however, although he mentions several experiments which indicate an increase of elasticity at low temperature. Thus a rod of fusible metal, supported at one end and loaded at the other, offered such resistance to flexure as to indicate a value of Young's modulus four or five times that at ordinary temperature. A spiral of fusible metal wire, which would be pulled straight by an ounce weight at ordinary temperature, supported two pounds at liquid air temperature, and vibrated like a steel spring; thus showing greatly increased modulus of rigidity. Dewar attempted to measure the rigidity modulus of steel by means of a spiral spring, but could detect no change; my experiments show a slight increase. The greatly increased rigidity of rubber is familiar to every one who has used liquid air.

There is no reason, however, to expect that most substances increase in elasticity at liquid air temperature as much as fusible metal or rubber. The metals of Pisati's experiments without exception decreased in rigidity with rising temperature, but at a rate that *increased* with the temperature. The change of rigidity with temperature is then less as a solid is further from its melting point. Furthermore, Pisati's results for different substances at the same

stance is farther from its melting point.¹ If we may extrapolate from Pisati's results, we should expect a substance like iron which melts about 1600° C., to increase much less in rigidity at liquid air temperature than Dewar's fusible metal, whose melting point may have been below 200° C.

This reasoning appears confirmed by direct measurement of the elasticity of copper and steel wires at liquid air temperature.

RESULTS OF EXPERIMENTS.

Each of the numerical values about to be submitted represents only a single complete determination.² Unavoidable circumstances prevented continuing the experiments so as to verify by repeated trials the results obtained, or to make determinations with other substances than copper and steel. It seems worth while, however, to put the experiments on record, even though the results are not numerous, and can not claim the same reliability as if there had been an opportunity to repeat the determinations.

The numerical values given below were obtained from wires approximately one meter long and 1.5 mm. in diameter. Young's modulus was determined by micrometer measurement of the extension under known tension; the modulus of rigidity by measurement of the statical twist due to a known torque; a detailed description of the apparatus follows later. In determining Young's modulus, the tension was varied approximately from 3 to 11 kgs. weight per square mm. For the rigidity modulus, the torque was varied from

¹ I have computed the accompanying table from Pisati's values: the first column of figures gives the melting point, the second gives the change in rigidity modulus per degree Celsius at 0° , expressed in percentage of rigidity modulus at 0° C.; that is,

$\frac{100}{R_0} \cdot \frac{\partial R}{\partial T}$, where R is the modulus of rigidity, and T the temperature.

Aluminum,	600°-650° C.	0.064
Brass,	910	0.035 -0.039
Silver,	920-1040	0.027 -0.041
Gold,	1035-1250	0.0204
Copper,	1050-1330	0.023 -0.025
Steel,	1300-1400	0.0195
Iron,	1500-1800	0.0207

100 to 300 dyne-centimeters per centimeter of length, the (constant) tension being about 11 kgs. weight per square mm.

Denoting by Y_T and R_T Young's modulus and the rigidity modulus at the temperature $T^\circ \text{C.}$, the experiments give the following results :

For the copper wire (hard-drawn), making no allowance for thermal contraction,

$$Y_{-186} = Y_{24} \times 1.177 \pm 0.020,¹$$

$$R_{-186} = R_{21} \times 1.063 \pm 0.003.$$

For the steel wire (piano wire),

$$Y_{-186} = Y_{21} \times 1.085 \pm 0.007,$$

$$R_{-186} = R_{20} \times 1.072 \pm 0.006.²$$

By correcting for thermal contraction, these values are slightly modified ; but the change in them is of the same order as the probable errors, so that it is not necessary to know the coefficients of thermal expansion at low temperatures with extreme accuracy, provided we can be sure they do not increase greatly with decreasing temperature. There is no reason to expect such an increase. The experiment of Zakrzewski³ extending to -103°C. , show diminished coefficient of expansion with decreasing temperature. By extrapolation from his results the average coefficient of expansion of copper between 20° and -186° may be taken as 0.0000156, and that of iron as 0.0000105. Using these values, we obtain the corrected ratios :

For the copper wire :

$$Y_{-186} = Y_{24} \times 1.180,$$

$$R_{-186} = R_{21} \times 1.073.$$

For the steel wire

$$Y_{-186} = Y_{21} \times 1.087.$$

$$R_{-186} = R_{20} \times 1.079.$$

¹ The $\pm$ refers only to the accidental errors, and takes no account of the systematic error mentioned on page 24.

² These ratios refer to the values at room temperature *before* cooling ; those *after* cooling differed slightly. See page 23.

³ J. Zakrzewski : Der Ausdehnungskoeffizient fester Körper bei tiefen Temperaturen. Bull. de l'Acad. de Cracow, 1889. Abstract in Die Fortschritte der Physik, 1889.

According to the theory of elasticity, Poisson's ratio (α) is expressible in terms of Young's modulus and the modulus of rigidity by the formula¹

$$\alpha = \frac{Y - 2R}{2R}.$$

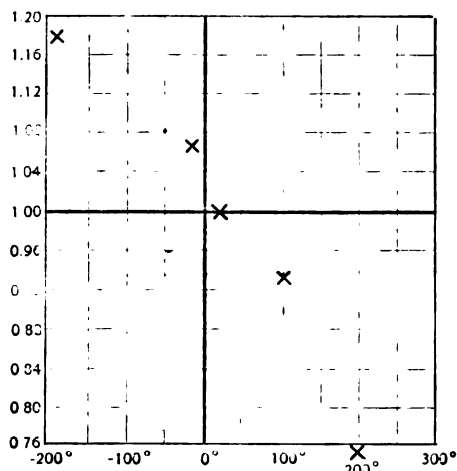
From this equation we find for the copper wire $\alpha_{-186} = 1.098\alpha_{20} + 0.098$; and for the steel wire $\alpha_{-186} = 1.008\alpha_{20} + 0.008$. Poisson's ratio then is greater than at ordinary temperature.

COMPARISON WITH THE VALUE OF ELASTICITY AT OTHER TEMPERATURES.

The mechanical properties of matter in the solid state vary with different specimens of the same substance. Accordingly our numerical values, obtained from a single copper wire and a single steel wire, must be considered as representing simply the behavior of those two particular wires, and not that of copper and steel in general; these values can at best furnish but rough indications of what may be expected with other copper and steel wires. This is more particularly the case because the wires did not undergo special treatment to reduce them to normal elastic condition, such as repeated heating and cooling, which would have been too wasteful of liquid air.

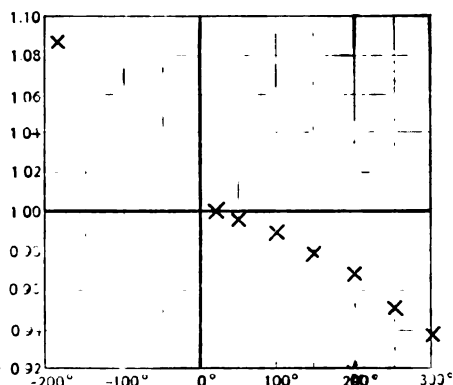
Nevertheless it is interesting to compare the variation of elasticity in these wires with that observed in other wires at higher temperatures by Pisati and Wertheim. I have accordingly plotted some of the values obtained by these observers, together with my own for liquid air temperature (Figs. 1-4, page 22). Regarding the modulus of rigidity, however, it must be remembered that in my experiments torsion took place isothermally, while in Pisati's it must have taken place nearly adiabatically, as the period of torsional oscillations was less than two seconds.

It will be seen that my results are not greatly different from those



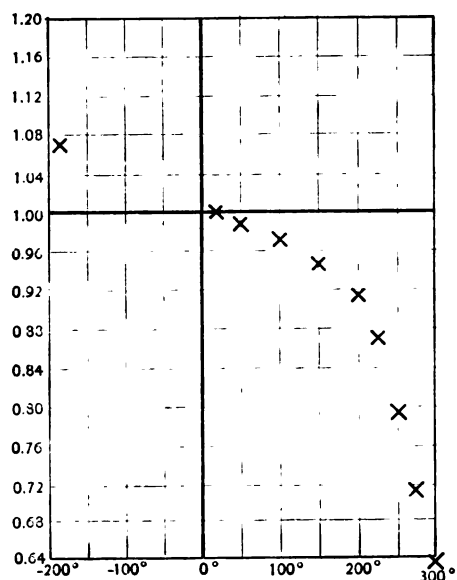
Young's modulus of copper,
in terms of its value at 20°C.
Wertheim's results for high temperatures.
My result for -186°C.

Fig. 1.



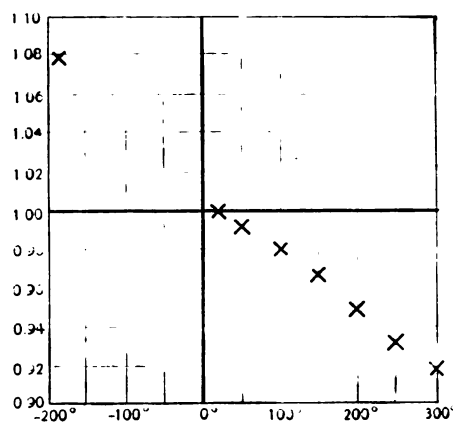
Young's modulus of steel,
in terms of its value at 20°C.
Pisati's results for high temperatures.
My result for -186°C.

Fig. 2.



Rigidity modulus of hard-drawn copper,
in terms of its value at 20°C.
Pisati's values for high temperatures.
My result for -186°C.

Fig. 3.



Rigidity modulus for steel,
in terms of its value at 20°C.
Pisati's results for high temperatures
My result for -186°C

Fig. 4.

from Pisati's experiments. My result, corresponding to an average increase of about 0.04 per cent. per degree C., seems more in accordance with the rather vague statement of Styffe,¹ that Young's modulus for steel increases with decreasing temperature not more than 0.03 per cent., or at most 0.05 per cent. per degree C.

ANNEALING EFFECT OF LIQUID AIR.

Wertheim² observed that subjecting metals to low temperature (-15°C.) leaves a residual effect, similar to that due to annealing when they are heated. The same phenomenon was observable in my experiments. Thus the modulus of rigidity of the copper wire after cooling to -186° and warming again to room temperature, was less than it had been before, in the ratio of $0.974 \pm 0.003 : 1$, while that of the steel wire was greater in the ratio of $1.022 \pm 0.005 : 1$. Sudden cooling from high to ordinary temperature anneals copper and hardens steel; it appears that the same effect occurs in copper as a result of sudden cooling from ordinary to very low temperature, since the modulus of rigidity of copper decreases with annealing.³ From the general impression that the modulus of rigidity of steel is increased by hardening, it would be easy to conclude also that steel is hardened by sudden cooling to -186° ; recent experiments, however, show that the rigidity is slightly decreased, instead of increased, by hardening.⁴ From this it would appear that steel is not hardened, but annealed, by sudden cooling from ordinary temperature; but the data are not sufficient to draw general conclusions.

DESCRIPTION OF THE APPARATUS FOR YOUNG'S MODULUS.

The arrangement used to determine Young's modulus is shown in Fig. 5, which is not drawn to scale, but is simply a diagram; some of the dimensions being exaggerated, and front and side elevations being combined in one figure.

¹ Knut Styffe: *Jern-Kontorets Annaler*, XXI., pp. 11-64, 1866. Abstract in *Dingler's Polytechnisches Journal*, CLXXXV., pp. 205 and 282, 1867. Also in *Kent's Mechanical Engineer's Pocket-Book*.

A was a strong bracket fastened firmly to the wall ; the wire *B*, *B* passed through a hole in *A* and was held there by a set-screw. *C*, *C* was an iron pipe, through which the wire hung ; it was one half inch in external diameter, and 90 cm. in length. On its lower end was soldered a chuck *D*, which was screwed up so as to grip the wire, and at the same time close the end of *C*. The extension of the wires under

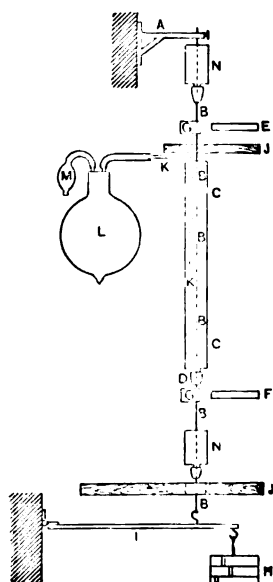


Fig. 5.

varying stress was measured with micrometer microscopes, placed at *E* and *F* ; the distance between the marks on which they were sighted was about 98 cm. When the wire was cooled with liquid air, thick frost deposited rapidly, and obscured the marks ; this difficulty was overcome by fastening on the wire little air-tight vessels *G*, *G*, containing calcium chloride, and closed by plane glass on the side towards the micrometers. The marks were illuminated by light from a distant gas flame, focused upon them by lenses. The stretching forces were applied by the weights *H*, acting through the lever *I*, of mechanical advantage five fourths. *J*, *J* were rigid shelves for supporting the micrometers. Liquid air was introduced into the pipe *C* from the Dewar bulb *L* by means of a force-pump arrangement *M*, devised by Dr. J. S. Shearer. *K*, *K* was a glass tube, reaching to about 50 cm. from the bottom of *C*.

In order that the points on the wire at *E* and *F* should have the same temperature as those within *C*, the short tubes *N*, *N* were fastened on the wire. The intention was to keep them full of liquid air ; I did not succeed, however, in securing a suitable arrangement for keeping them filled ; they were empty during my observations.

When the pipe C was filled with liquid air, the tension of the wire being constant, it was found that the marks at E and F slowly moved up and down in an irregular manner, owing presumably to irregular thermal expansion and contraction in the wire above E , according as it was warmed by air currents from the room or cooled by the air boiling off at the top of C . To measure the extension under these conditions, the following procedure was adopted.

With a given load on H , a reading was taken at F ; the instant it was obtained an assistant changed the load on H , and a reading at F with the new load was taken. The time required for such a pair of readings was less than ten seconds. The difference between the two readings gives the depression of the point F ; by taking the two readings of a pair rapidly, concordant results for the depression were obtained, although the reading for the same load fluctuated continually. Ten such pairs of readings were taken for the lower micrometer; then ten were taken similarly for the upper one; the difference of depression at E and at F gives the elongation of the wire between E and F .

APPARATUS FOR THE MODULUS OF RIGIDITY.

In researches on the modulus of rigidity the method of torsional oscillations has been employed almost exclusively, on account of its great convenience and accuracy, as compared with other methods. In my experiments, where the wire hung through a tube filled with rapidly evaporating liquid, the method was clearly impracticable; I therefore used a statical method.

The apparatus is shown in Fig. 6, which, like Fig. 5, is merely diagrammatical. The method of suspension from A , and the tubes C and N, N , are the same as used for Young's modulus. To the top of C was rigidly fastened a horizontal wooden pulley E , 24 cm. in diameter. Liquid air was introduced into the pipe C by means of a glass tube G , which was

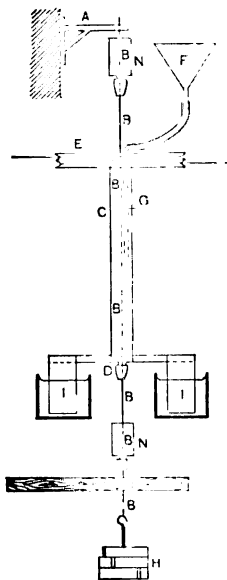


Fig. 6.

connected to a funnel F , supported on the pulley E , and free to turn with it. In order to damp torsional oscillations, the two vanes I, I , dipping in vessels of glycerine, were clamped to the lower end of C . Besides the apparatus shown in Fig. 6, there was at the top of the wire a torsion-head arrangement; and at the bottom of C a system of counterpoise weights for bringing the center of gravity of C below its point of attachment to the wire.

The arrangement for applying a constant twisting moment to the wire is shown on a smaller scale in Figs. 7 and 8. Light flexible threads, a, a , were fastened at one end to opposite sides of the rim of the pulley E , at the other end to the walls of the room about 70 cm. above E . The distance from C to

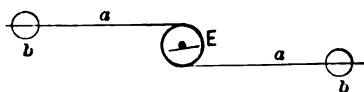


Fig. 7.

each wall was about 2 meters. About 1 meter from C , on each side, very light scale-pans b, b were fastened on the thread. If weights were placed on these scale-pans a couple acted on the pulley E , by reason of the increased horizontal pull in a, a ; E accordingly turned until the couple was balanced by torsion of the wire. To determine the absolute value of this couple, accurate knowledge of the various distances and of the weight of the thread would have been necessary. Since, however, the purpose of the experiments was simply to *compare* the torsional elasticity at -186°C. with that at ordinary temperature, it was not necessary to know the absolute value of the couple, but only to be able to apply the same couple at different times.

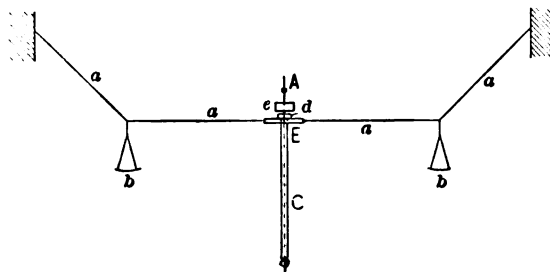


Fig. 8

Since E was rigidly fastened to the pipe C , which in turn was fastened to the lower end of the wire, d turned through the same angle as the lower end of the wire. A second vertical plane mirror e (Fig. 7) was clamped to the wire itself just above the top of C . By means of lenses the light of a gas burner passing through a narrow slit was brought to a focus after reflection from the mirrors d and e , so that two images of the slit were cast upon a scale 359 cm. distant from the wire and 4 meters in length. By reading the distances through which these images were displaced by the addition of weights to the scale-pans b, b , the angles through which the mirrors d and e turned could be computed; the difference between them gives the angle of twist of the wire.

The apparatus was used in the following manner: Weights were chosen, which when placed in the scale-pans b, b , twisted the wire about 12° . A set of twenty readings was made at room temperature, alternately with and without these weights in the scale-pans; the position of the wire was slightly varied two or three times during the set by means of the torsion head at its upper end. Then the pipe C was filled with liquid air, and the same process was repeated, using the same weights. If R_T is the modulus of rigidity and α_T the angle of twist at temperature T° C., then we have

$$R_{-186} = R_{20} \times \frac{\alpha_{20}}{\alpha_{-186}},$$

making no allowance for thermal contraction. After the wire had returned to room temperature, another set of readings was taken.

My thanks are due to Mr. K. P. Beardslee, of Cornell University, for assistance in taking the readings for Young's modulus.

CORNELL UNIVERSITY, June, 1902.

GENERALIZATION OF CARNOT'S CYCLE.

BY SANFORD A. MOSS.

THE ideal heat engine cycle proposed by Carnot comprises the following operations: adiabatic compression from absolute temperature T_0 to T_1 , then addition of heat at constant temperature, next adiabatic expansion, and finally extraction of heat at constant temperature until the cycle is closed. The "efficiency" of this cycle, that is the ratio of the mechanical work done by the working substance, to the heat added, is well known to be $e = 1 - T_0/T_1$.

We shall show that the same expression for efficiency holds for the more general cycle in which the addition and extraction of heat are carried on with varying temperatures, provided that the "specific heat" has the same constant value during these periods. The operations performed on the working substance will then be as follows: adiabatic compression from absolute temperature T_0 to T_1 , then addition of heat during expansion so conducted that the temperature varies directly with the heat added, that is, expansion with "constant specific heat," next adiabatic expansion, and finally extraction of heat with constant specific heat of the same magnitude as during addition of heat.

Before proceeding with the proof of this statement we shall consider the operation of adding heat with "constant specific heat." While a quantity of heat dQ is being added to unit mass of any substance, the volume may at the same time be varied an arbitrary amount dv , so that external work is done having an arbitrary value $1/J p dv$. The difference between dQ and $1/J p dv$ goes to increase the internal energy of the body and the temperature may therefore

the "specific heat" and dv/dQ is the rate of expansion during addition of heat. Hence by properly varying the rate of expansion, we may make the specific heat assume any desired series of values. As a particular case the rate of expansion dv/dQ may be so adjusted that the specific heat dQ/dT has any desired constant value while heat is being added. The line on a " pv " diagram representing such an expansion we may call a "line of constant specific heat."

For solids, lines of constant pressure are approximately lines of constant specific heat. For a perfect gas the curve $pv^n = \text{constant}$ is a line of constant specific heat, n having any value whatever. Although this fact is well known, the following brief proof will not be amiss. A perfect gas is defined as one obeying the relations $pv = RT$ and $dQ = cdT + 1/J p dv$ where c is a constant called the "specific heat at constant volume." From the relations $pv = RT$ and $pv^n = \text{constant}$, we have

$$p dv + v dp = R dT$$

$$np dv + v dp = 0.$$

By subtraction

$$p dr = \frac{R dT}{1 - n}.$$

Hence

$$dQ = cdT + \frac{R}{J(1 - n)} dT.$$

Therefore

$$\frac{dQ}{dT} = c + \frac{R}{J(1 - n)}.$$

This is constant and therefore the curve $pv^n = \text{constant}$ is a line of constant specific heat for a perfect gas for any value of n . For $n = 0$ we have a line of constant pressure, for $n = \infty$ a line of constant volume, for $n = 1$ an isothermal line and for $n = c_p/c = k$ an adiabatic. These are therefore all lines of constant specific heat. As actual gases depart slightly from the perfect gas laws lines of constant specific heat for them will be curves $pv^n = \text{constant}$ where n is not a constant but varies slightly during expansion. The isothermal curve or line of constant temperature, for any substance

whatever, is a line of constant specific heat since dQ/dT has the constant value ∞ .

With this consideration of the properties of lines of constant specific heat we will return to the discussion of the generalized heat engine cycle. Let 0123 be the pv diagram of a heat engine cycle,

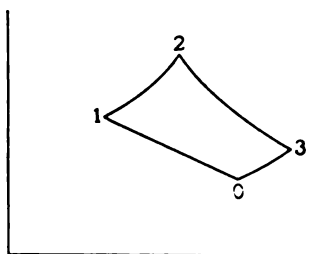


Fig. 1.

any working substance whatever being used ; where 01 and 23 are adiabatics and 12 and 03 are any lines of constant specific heat, with the same value of dQ/dT . Let T be the variable value of the absolute temperature at any point in the cycle and let dQ be the corresponding increment of heat per unit mass. From 0 to 1 dQ is always zero as the compression is adiabatic. From 1 to 2 an amount of heat $\int_1^2 dQ$ is added and this is the amount of energy supplied to our heat engine. From 2 to 3 there is adiabatic expansion and dQ is again zero. From 3 to 0 heat is extracted or a negative amount of heat is added $\int_3^0 dQ$. The mechanical work done by the engine is the area of the cycle diagram and this must be equal to the algebraic sum of the amounts of heat added during the various events, by the law of conservation of energy. Hence the efficiency of the cycle is

$$\epsilon = \frac{\int_1^2 dQ + \int_3^0 dQ}{\int_1^2 dQ}.$$

Since all of the heat is added along curves of constant specific heat

and since the value of the specific heat is the same throughout we have $dQ = CdT$. Hence the expression for efficiency may be written

$$\begin{aligned} e &= \frac{\int_1^2 CdT + \int_3^0 CdT}{\int_1^2 CdT} \\ &= \frac{T_2 - T_1 + T_0 - T_3}{T_2 - T_1} \\ &= 1 - \frac{T_3 - T_0}{T_2 - T_1} \\ &= 1 - \frac{T_0}{T_1} \left\{ \frac{T_3}{T_0} - 1 \right\} \left\{ \frac{T_2}{T_1} - 1 \right\}^{-1} \end{aligned}$$

We may simplify this expression by means of a relation deduced from the application of the second law of thermodynamics to our cycle. As we have a closed reversible cycle we may write

$$\int \frac{dQ}{T} = 0$$

the integration being made over the whole cycle. This becomes therefore

$$\int_1^2 \frac{dQ}{T} + \int_3^0 \frac{dQ}{T} = 0.$$

Since we have always $dQ = CdT$,

$$\int_1^2 \frac{CdT}{T} + \int_3^0 \frac{CdT}{T} = 0$$

or

$$\log \frac{T_2}{T_1} = \log \frac{T_3}{T_0}$$

Substituting this relation in our expression for the efficiency, we obtain

$$e = 1 - \frac{T_0}{T_1} = \frac{T_1 - T_0}{T_1}.$$

This is at once recognized as the value of the efficiency of a Carnot's cycle. In fact Carnot's cycle is included in the general class of cycles considered since the lines of addition and extraction of heat, being isothermals, are lines of constant specific heat for which dQ/dT has the same value, viz., ∞ . For this reason we may call the general cycle we have been considering a "generalized Carnot's cycle."

We have shown therefore that all heat engines having adiabatic compression from T_0 to T_1 and addition and extraction of heat along any lines with the same constant specific heat; have the efficiency $1 - T_0/T_1$.

The temperature variations during the time heat is added or taken away have no influence on the efficiency. It is often supposed that the efficiency of heat engines always increases with the maximum temperature, but this is seen to be erroneous, and cycles of the class considered have the same efficiency whether the temperature is constant or is raised to any value whatever during addition of heat.

We have developed our efficiency theorem for a heat engine using any working substance whatever. It will be worth while to give an independent proof for the case where the working substance is a perfect gas. As we have shown, the curves $pV^n = \text{constant}$ are lines of constant specific heat for a perfect gas. Zeuner has called these "polytropic curves." Our theorem therefore assumes the following form. A heat engine using a perfect gas with a cycle formed by two adiabatic crossed by two polytropic curves with the same index, has the efficiency $1 - T_0/T_1$ where T_0 and T_1 are the initial and final temperatures of the adiabatic compression. This

constant. Let Q be the total heat per unit mass added from 1 to 2 and q the heat extracted from 3 to 0. We have already established the following relation for a perfect gas :

$$dQ = \left\{ c + \frac{R}{J(1-n)} \right\} dT.$$

Hence

$$\begin{aligned} Q &= \int_1^2 dQ = \int_1^2 \left\{ c + \frac{R}{J(1-n)} \right\} dT \\ &= \left\{ c + \frac{R}{J(1-n)} \right\} (T_2 - T_1). \end{aligned}$$

In the same way we may obtain

$$q = \left\{ c + \frac{R}{J(1-n)} \right\} (T_3 - T_0)$$

Since the value of n is the same for both cases,

$$\frac{Q}{q} = \frac{T_2 - T_1}{T_3 - T_0} = \frac{T_1}{T_0} \left[\frac{\frac{T_2}{T_1} - 1}{\frac{T_3}{T_0} - 1} \right].$$

Since 1 and 2 are on the same polytropic curve

$$p_1 v_1^n = p_2 v_2^n.$$

Similarly

$$p_0 v_0^n = p_3 v_3^n.$$

By *division*

$$\frac{p_1}{p_0} \left(\frac{v_1}{v_0} \right)^n = \frac{p_2}{p_3} \left(\frac{v_2}{v_3} \right)^n.$$

where k is the ratio of the specific heat at constant pressure to the specific heat at constant volume.

Hence

$$\left(\frac{v_1}{v_0}\right)^n = \left(\frac{T_1}{T_0}\right)^{\frac{n}{k-1}}.$$

Therefore

$$\frac{p_1}{p_0} \left(\frac{v_1}{v_0}\right)^n = \left(\frac{T_1}{T_0}\right)^{\frac{k}{k-1} - \frac{n}{k-1}}.$$

In the same manner we may show

$$\frac{p_2}{p_3} \left(\frac{v_2}{v_3}\right)^n = \left(\frac{T_2}{T_3}\right)^{\frac{k}{k-1} - \frac{n}{k-1}}.$$

Hence

$$\frac{T_1}{T_0} = \frac{T_2}{T_3} \text{ or } \frac{T_2}{T_1} = \frac{T_3}{T_0}.$$

By means of this relation we may simplify our expression for Q/q . We then obtain

$$\frac{Q}{q} = \frac{T_1}{T_0} \text{ whence } 1 - \frac{q}{Q} = 1 - \frac{T_0}{T_1}.$$

Hence

$$\frac{Q - q}{Q} = \frac{T_1 - T_0}{T_1} = 1 - \frac{T_0}{T_1}.$$

Since $Q - q$ is the mechanical work obtained from the engine, the ratio of $Q - q$ to Q is the efficiency. That is

$$e = 1 - \frac{T_0}{T_1}.$$

This result for the special case of a perfect gas agrees with the result for the general case, as of course it should.

An important application of the theorem we have developed is as follows: Gas engines or "internal combustion" engines, and hot-air engines use a working substance which may be considered as a perfect gas. Hence when heat is added and extracted at constant volume, as in the "Otto" gas engine or Stirling air engine, or when

SOME OPTICAL PROPERTIES OF IODINE.

BY WILLIAM WEBER COBLENTZ.

IODINE is an interesting example of apparently several allotropic states. At low temperatures iodine vapor¹ gives a banded absorption spectrum from red to violet. As the temperature increases, absorption in the blue and violet increases, and finally becomes continuous. At still higher temperatures blue and violet are again transmitted and bands appear.

The relation of the spectra of chlorine and bromine gases² to that of iodine has also been extensively investigated. It has been found that for the halogen group the absorption lines shift toward the red with increasing atomic weight.

According to Gänge³ iodine is the only element which distinguishes itself by peculiar properties towards two groups of solvents. The first group, to which belong water, the alcohols, ethers, etc., with increasing concentration, dissolves iodine with a yellow to a dark brown color. In this group the solutions appear in colors which resemble closely those of solid and liquid iodine by transmitted light. The second group, to which belong carbon bisulphide, chloroform, benzol, etc., dissolves iodine with a violet color, which resembles more nearly that of the vapor, and gives the same spectrum. Hence, the molecular arrangement in the first group of solutions is assumed similar to that of solid and liquid iodine, and in the second group similar to that of the vapor.

The second group is also assumed to have a simpler structure. To test this the CS₂ solution was cooled in a mixture of solid carbon dioxide and ether. It became brown.⁴ In solutions of

¹ Konen, Wied. Ann., 65, p. 258 (1898), gives a very complete bibliography of

iodine in carbon bisulphide above the critical temperature, where the state approximates that of a gas, but the density that of a liquid, Wood¹ found that a continuous or banded spectrum appeared according to the amount of the solvent. These bands seemed to appear only in the presence of an excess of free iodine and each aggregation apparently caused a change of spectrum.

Of a different nature is Tyndall's ingenious work on the diathermancy of iodine in carbon bisulphide. He discovered this property for the infra-red rays² in 1865, and "to make radiant heat an explorer of molecular conditions," he used the carbon bisulphide solutions to filter the obscure from the luminous rays.³

It is the purpose of the present paper to describe some experiments upon the optical properties of this interesting element. The work divides itself into four parts, viz.:

1. The study of the absorption spectrum of iodine in solution.
2. The study of the absorption spectrum of solid and liquid iodine. Comparison of the thickness of iodine as a film, and in solution.
3. Pleochroism of iodine.
4. Anomalous dispersion in solid and liquid iodine.

I. THE ABSORPTION SPECTRUM OF IODINE IN SOLUTION.

So far as the writer is aware no recent or systematic investigations of the optical properties of iodine dissolved in carbon bisulphide have been made, although iodine solutions have been used in the determination of the radiant efficiency of various sources of light for nearly half a century.

Desain and Aymonet,⁴ using a quartz prism and thermopile, report a "cold" band in the infra-red at an angle of $1^{\circ} 30'$ from the extreme red.

In 1883, H. Becquerel⁵ found that a concentrated solution, 2.5 cm. in thickness, absorbed all the rays up to the A line and allowed the less refrangible to pass.

¹ Wood, *Phil. Mag.*, 41, 1896.

In the following paragraphs it is proposed to describe measurements of the absorption spectrum of iodine solutions, which were made for the purpose of obtaining more definite information concerning these and other points.

1. *Measurements in the Visible Spectrum.*

In the study of the visible spectrum a "horizontal slit" spectrophotometer¹ was used. A series of thirteen solutions containing from 3.3 to 0.017 milligrams of iodine per cubic centimeter of carbon bisulphide was prepared, by dilution. Although not very economical, this method reduced the errors in the concentrations. The cell containing these solutions had thin walls of lantern slide glass. The thickness of the liquid layer was 1.5 centimeters.

Measurements of the transmitting power of these solutions were made for 19 different regions of the spectrum between 0.74μ and 0.46μ . The effect of the cell walls and of the solvent were eliminated by observations upon the cell filled with clear carbon bisulphide. The absorption of iodine in the visible spectrum was found to consist of a single broad band with its maximum at wave-length $\lambda = 0.5\mu$, the transmission curve being steeper on the side towards the violet.

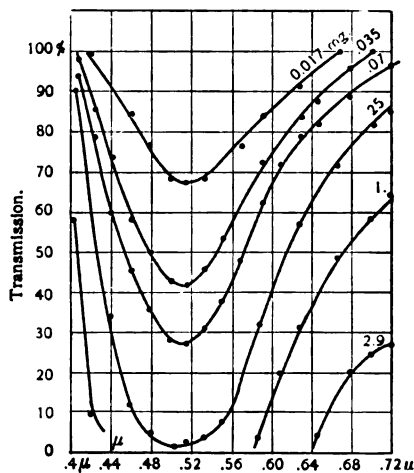


Fig. 1. Iodine in CS_2 .

The gradual development of this absorption band is best indicated in Fig. 1, which shows the transmitting power throughout the visible spectrum of several solutions 1 cm. in thickness. The data for these curves were obtained by substituting the values given in

transmitted light, and are given in Table II. For the weakest solution, 0.017 mg. per c.c., the transmission in the maximum of the band is about 64 per cent., which corresponds to a transmission of nearly 80 per cent. for a layer one centimeter thick. As the concentration increases the absorption band widens and becomes more dense, so that a solution of only 0.25 mg. per c.c. transmits only about 2 per cent. in the center of the band. All stronger solutions are opaque in this region, and as the concentrations increase this opacity extends, until finally the whole of the visible spectrum is absorbed. The strongest solution upon which it was found possible to make spectrophotometric measurements even in the red, contained 3.3 mg. per c.c.

TABLE I.

Transmission of Iodine in Carbon Bisulphide. Thickness of Cell = 1.5 cm.

Wave-lengths.	Transmission, $I_e + I_0$.													Clear CS.
Conc. mg. per c.c.	3.3	2.9	2.0	1.6	1.3	1.0	0.7	0.5	.25	.13	.07	.035	.0175	
Normal Solutions.	$M = .012$	.0117	.0079	.0063	.0051	.0039	.0028	.00197	.00088	.00051	.00028	.00014	.00007	
.406 μ					.22	.43		.76	.86	.90	.91	.98	1.00	
.424					.07	.08		.17	.42	.61	.70	.86	.93	.98
.442					00	00		00	.20	.27	.45	.64	.83	
.46					0	0		0	.08	.19	.32	.47	.77	.96
.48					0	0		0	.04	.12	.24	.37	.70	.94
.50					0	0		0	.01	.10	.17	.30	.65	.90
.517					0	0		0	.02	.11	.16	.30	.63	.87
.534					0	0	0	0	.03	.12	.18	.34	.64	.83
.554					0	0	0	00	.06	.19	.26	.42	.68	.84
.57					0	00	00	.02	.12	.20	.36	.56	.74	.84
.59					00	.03	.03	.14	.20	.36	.53	.66	.80	.85
.61		0	00	00	.03	.11	.16	.24	.38	.51	.66	.80	.86	
.63	0	.01	.05	.08	.10	.19	.27	.35	.47	.64	.76	.84	.91	.88
.645	.01	.03	.08	.14	.20	.30	.40	.47	.60	.71	.81	.88	.94	.92
.665	.02	.06	.11	.18	.28	.38	.47	.54	.66	.74	.85	.93	.99	
.68	.04	.11	.15	.23	.33	.45	.53	.64	.72	.79	.89	.96	1.00	.95
.70	.04	.13	.18	.25	.37	.49	.55	.70	.78	.86	.93	.98	1.00	
.72	.06	.15	.22	.28	.41	.52	.62	.77	.81	.89	.95	.99	1.00	.99
.74	.09	.19	.25	.32	.45	.55	.66	.80	.87	.93	.97	1.00	1.00	

The percentages of light transmitted by the iodine in carbon

ness. In these and the three following tables the concentrations are given in milligrams per cubic centimeter, and also as normal solutions.

TABLE II.

Transmission of Iodine in Carbon Bisulphide. For a layer 1 cm. in thickness. Computed from Table I.

Wave-lengths.	Transmission, $I_e + I_0$.						Extinction Coefficients.					
Conc. mg. per c.c.	2.9	1.	0.25	0.07	.035	.017	2.9	1.	0.25	0.07	0.035	0.017
Normal Solutions.	.0117	.0039	.00098	.00028	.00014	.00007	.0117	.0039	.00098	.00028	.00014	.00007
.406 μ		.58	.91	.94	1.00			.545	.0924	.060		
.424		.09		.79	.86	1.00		2.410		.235	.151	.00
.442		.00	.34	.60	.74	.90		.00	1.080	.510	.369	.105
.46		0	.12	.46	.59	.85			2.12	.779	.528	.165
.48		0	.05	.37	.50	.77			3.020	0.996	.697	.263
.50		0	.02	.28	.43	.69			3.900	1.271	.845	.372
.517		0	.03	.27	.42	.67			3.471	1.312	.868	.402
.534		0	.04	.31	.46	.69			3.230	1.171	.779	.372
.554		0	.08	.38	.53				2.530	.972	.635	
.57		.00		.48		.77		.00		.732		.263
.59		.04	.32	.62	.72	.84		3.230	1.142	.478	.328	.174
.61		.20		.72				1.610		.328		
.63		.31	.58	.79	.84	.92		1.171	.545	.235	.174	.084
.645	.04			.82	.88		3.230			.197	.128	
.665		.49	.72			1.00		.715	.328			
.68	.20			.89	1.00		1.010			.116		
.70	.25	.59	.82				1.388	.528	.193			
.72	.27	.64	.86	.96			1.312	.445	.151	.041		

The absorption of solutions of iodine in alcohol differs very markedly from that of iodine in carbon bisulphide. The solutions are far more transparent in the red and yellow, hence greater concentrations could be used. Measurements with the spectrophotometer show for all concentrations a rapidly increasing opacity toward the violet. This is best shown in Fig. 2 (obtained by computation like Fig. 1), where it will be noticed that a solution of 0.05 mg. per c.c. instead of being nearly transparent at $\lambda = .4 \mu$, transmits only about 20 per cent. of the incident light. In the red, on the other hand, the opacity of the alcohol solutions is relatively much less than that of a corresponding solution in carbon bisulphide. For example, a solution containing 3.2 mg. per c.c. transmits

60 per cent. at $\lambda = .72 \mu$, whereas the corresponding solution in CS_2 allows scarcely 6 per cent. of the incident light to pass through

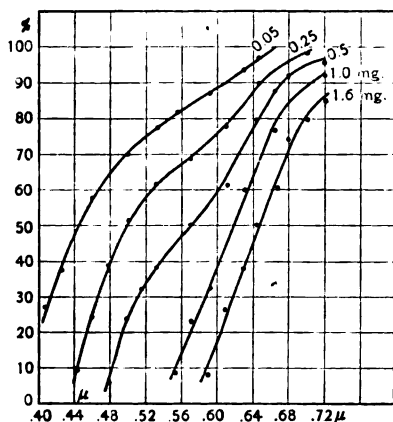


Fig. 2. Iodine in Alcohol.

at that wave-length. The dilute alcohol solutions are dichroic, so that we have an increase in the amount of light transmitted in the region where the CS_2 solutions showed a maximum absorption. From the steepness of the transmission curves, Fig. 2, for the weaker solutions, which begin to trend sharply downward at about $\lambda = .50 \mu$, one would infer the existence of an absorption band located in the ultra-violet.

The very different absorption of iodine in these two solvents is further exhibited in the isochromatic curves in Figs. 3 and 4, which show the increasing opacity of the various wave-lengths as the concentration increases. From the isochromatic curves, for solutions 1 cm. in thickness, where concentrations are plotted as abscissæ and transmission as ordinates, the transmission of normal solutions, $M = .01, .001$ and $.0001$, were obtained and are given in Table IIa.

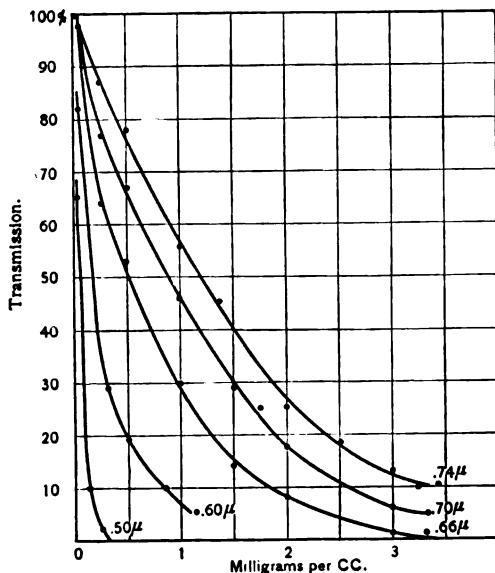


Fig. 3. Iodine in CS_2 .

putation, in the same manner as for the carbon bisulphide solutions,

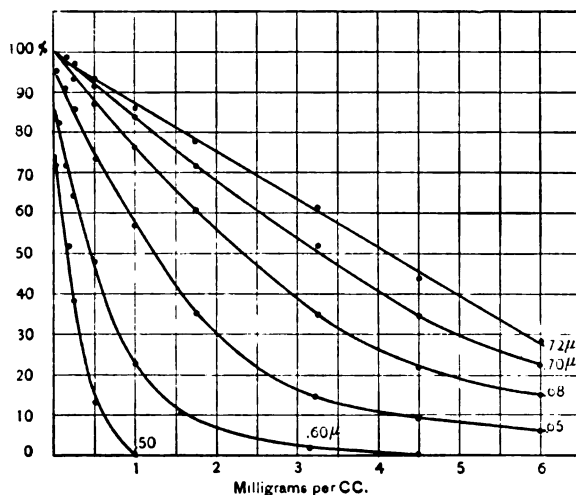


Fig. 4. Solution in Alcohol.

TABLE IIa.

Transmission of Normal Solutions. Cell 1 cm. in Thickness.

Transmission of Carbon Bisulphide Sols. Normal Solutions ($M = 1$).				Transmission of Alcohol Sols. Normal Solutions ($M = 1$).		
Wave-lengths in μ .	$M = .01$.	$M = .001$.	$M = .0001$.	Wave-lengths in μ .	$M = .001$.	$M = .0001$.
.50		2	42	.46	24	60
.56		13	64	.50	50	73
.59		30	72	.534	60	80
.60	0	38	77	.57	67	84
.64	7	60	87	.61	78	89
.66	18	69	93	.645	89	97
.68	24	75	96	.665	92	99
.70	29	82	99	.68	96	100
.72	32	85	100			

From the equation

$$I = I_0 e^{-k d}$$

the extinction coefficient,¹ k , was computed for a thickness, d , ($= 1$ cm.), using the percentages of light transmitted given in

¹ Ångström, Wied. Ann., 36, p. 715 (1889).

TABLE III.

Transmission of Iodine in Ethyl Alcohol. Cell 1.5 cm. in Thickness.

Wave-lengths. Concentra- tions.	Transmission, $I_e \div I_0$.									
	Clear Sol.	6 mg.	c.c. 4.5	3.3	1.6	1.0	0.5	0.25	0.13	0.5
Normal Solutions, $m = 1$.		$M =$.024	$M =$.018	$M =$.013	$M =$.0063	$M =$.0039	.00197	.00098	.00051	.0002
.406 μ	.94								.09	.15
.424								.00	.16	.24
.442								.09	.26	.36
.460								.13	.35	.47
.48	.94						.04	.26	.43	.54
.50							.13	.39	.52	.63
.517	.96					.00	.18	.45	.59	.67
.534						.04	.24	.51	.62	.70
.554	.96					.09	.29	.56	.66	.74
.57					.00	.12	.38	.59	.68	.79
.59	.97				.07	.18	.44	.66	.73	.83
.61	.98			.00	.14	.28	.50	.71	.78	.87
.63	.99	.02	.06	.08	.24	.47	.61	.76	.83	.91
.645		.05	.09	.13	.35	.54	.71	.85	.91	.96
.665	.99	.10	.14	.22	.47	.68	.83	.92	.96	.99
.68		.16	.24	.37	.64	.78	.89	.95	.97	.99
.70	1.00	.22	.35	.53	.72	.84	.92	.97	.99	1.00
.72		.28	.44	.61	.78	.87	.94	.97	1.00	

Tables II. and IV. The corresponding extinction coefficients are given in the same tables.

This work was performed at room temperature, — about 20° C.— on old and new solutions. The principal errors in such work are those introduced in handling the solutions, which evaporate readily, and those due to inaccuracies in the spectrophotometric comparisons, which depend in part upon the physical condition of the observer.

Observations in the Infra-red.

The Radiometer.— This instrument was built according to the design used by Nichols¹ and only the modifications to suit the work in hand need be described. The vanes were of mica blackened by burning camphor gum. To cause the lamp-black to adhere to the mica, a very thin coating of beeswax was first applied by means of a hot wire. When the vanes were held above the burning gum the

¹ E. F. Nichols, PHYSICAL REVIEW, Vol. II, p. 891 (1897).

TABLE IV.

*Transmission of Iodine in Ethyl Alcohol for a Layer 1 cm. in Thickness.
Computed from Table III.*

Wave-lengths.	Transmission, $I_e + I_0$.					Extinction Coefficients.				
Concent'ns, mg. per c.c.	1.6	1.0	0.5	0.25	0.05	1.6	1.0	0.5	0.25	0.05
Normal Solutions.	$M = .0063$	.0039	.00197	.00098	.0002	.0063	.0039	.00197	.00098	.0002
.424 μ					.38					.972
.460				.25	.58				1.389	.545
.50			.24	.52	.71			1.430	.655	.343
.517			.32					1.150		
.534			.38	.62	.77			.972	.478	.263
.554		.09			.82		2.410			.198
.57		.23	.50	.69			1.470	.697	.372	
.59	.08	.32			.87	2.530	1.150	.500		1.39
.61	.27		.61	.78		1.312		.496	.249	
.63	.38	.60			.93	.972	.510			.072
.645	.50		.80	.90	.97	.697		.226	.106	.034
.665	.60	.77	.88			.510	.263	.128		
.68	.74	.85	.92	.96	1.00	.369	.165	.0835	.041	00
.70	.80			.98		.226			.020	
.72	.85	.92	.96			.165	.0835	.041		

wax softened, and the soot deposited in an even layer. The inner window was of mica while the outer one was of plate glass.

The quartz fiber suspension was purposely chosen of medium weight to avoid the effects of temperature changes and earth tremors. There seems to be no criterion for selecting a fiber. Experience shows that a fiber about 8 cm. long, which will float freely in still air without bending over, is a good one to begin with. Such a fiber is quite invisible when held in the light on a cloudy day. The one used in this work was of such a nature, and observations could be made at any time of the day without being affected by outside disturbances. The drift of the zero reading was small, never greater than 1.5 mm. during one reading. Electrification of the vanes in exhausting was the only source of annoyance. This electrification was found greatest, *i. e.*, the vanes were attracted to the mica window the longest, for newly blackened vanes. The vanes no doubt become electrified in handling. The most effective treatment was to wipe off the mica window with alcohol or, better still, with

water, just before exhausting. This trouble was avoided to some extent by exhausting slowly, and during the latter part of the work, by covering the mica window with tinfoil. The spectrometer slits were .4 mm. wide and the maximum deflection, as read on a scale at a distance of 1.5 m., was 40° + cm. The pressure, measured by a McLeod gauge, was about .08 mm. and the period was such that the maximum deflection was reached in 16 seconds.

The Spectrometer. — A mirror spectrometer was used. The mirrors were of 14 centimeters focal length, 6 cm. aperture, and were in good condition. The arrangement of the apparatus is shown in Fig. 5, in which F_1 is the collimator slit; C is the cell containing

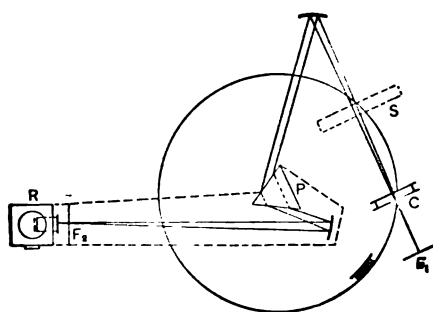


Fig. 5.

the solution; S , a double screen which could be interposed or removed by the observer at the reading telescope; P , the carbon bisulphide prism; and R , the radiometer. To a bracket, rigidly attached to the collimator tube, a cylindrical flame acetylene burner was mounted, with its center directly behind

the collimator slit. In order to avoid the inconvenience of shifting the radiometer in exploring the spectrum, the arm carrying the mirror and the slit, F_2 , which is placed at the vertical focus of the refracted ray, was clamped rigidly and placed in line with the radiometer. This makes the radiometer and spectrometer each a system by itself, and the shifting of either will simply vary the total amount of energy falling upon the vanes.

The spectrometer and radiometer were rigidly fixed upon a heavy slate slab. A bit of white paper on the rear wall of the radiometer indicated whether all the incident light fell upon the mica vane. The sensitiveness of the radiometer was varied by means of the leveling-screw opposite the window, and also by means of the exhaust pump.

In this work ratios are dealt with, and the only errors due to shifting would occur during two successive readings. The consistent

results of successive settings leads to the conclusion that no such shifting occurred. This is best graphically illustrated in Fig. 6, which shows the constancy of the point of maximum deflection. The data for these curves are selected at random from the notebook. They extend over several months, and originally were not taken to illustrate this point.

The prism was set for minimum deviation of the D lines by means of a telescope. By moving the collimator which carried the light, different portions of the spectrum could be brought on the radiometer.

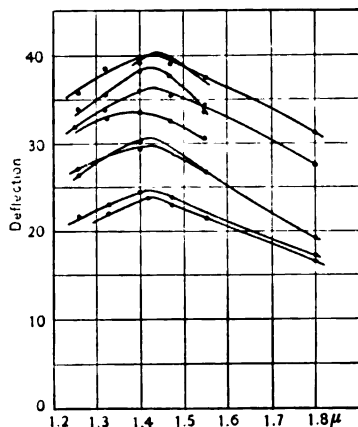


Fig. 6.

Changes of temperature are negligible, since it is known that for a carbon bisulphide prism a change of temperature of 5° is necessary to bring the D_1 line in the place of D_2 .¹ The temperature of the prism was observed at the beginning and at the end of each series of measurements. It was found that during a single series of observations, which took from 30 to 50 minutes, the variation was only about $0^\circ.3$ C., the maximum recorded being $0^\circ.6$ C. The temperature of the room was fairly constant at 18° C. in the morning, rising to 19° or 20° in the afternoon. The solutions stood on an open shelf in the same room and were assumed to be at the same temperature.

Calibration.—The spectrometer was calibrated by using Snow's² values for the position of the sodium lines at 0.59μ , 0.83μ and 1.4μ , and by observing the position of the emission band of CO_2 at 2.7μ . The wave-length of this band has been determined by Paschen. The position of the sodium lines was determined³ by mounting an electric arc of cored carbons, filled with pure sodium

ing at which a deflection was produced by these lines. The position of the CO_2 band was similarly observed by using a Bunsen acetylene burner. The calibration curve plotted from the observed readings is given in Fig. 7. The computed circle readings for the

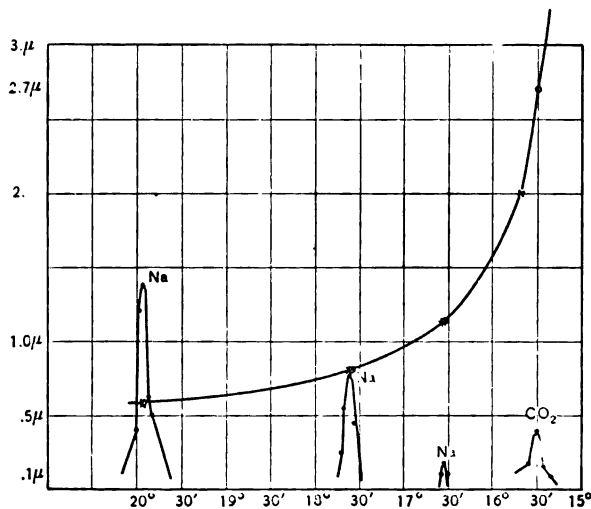


Fig. 7.

same lines—indicated on the same curve by the crosses (* *), were obtained by assuming the indices of refraction of CS_2 as determined by Rubens,¹ the prism in these computations being taken at minimum deviation for each line.

3. *Curves and Results.*—Observations were made with and without the cell of iodine solution interposed in the path of the incident light, and the ratio of the deflections gave the percentage of transmission. At first the entire spectrum to 2.7μ was explored, using a single solution. This was found impracticable far in the infra-red on account of the absorption band of CS_2 found by Paschen² at 2.7μ , the increased absorption of the glass, and the very narrow dispersion of the spectrum in this region. Between 1.8μ and 2.7μ a slight variation in the circle reading caused a great variation in transmission through the same solution.

To avoid errors in setting the collimator, from $1.8\ \mu$ to $2.7\ \mu$, a less convenient method had to be adopted. The collimator arm was set so that a given region of the spectrum fell upon the radiometer vane, and the transmission of the entire group of solutions was found without changing the setting. This method gave consistent results. The influence of the walls of the cell and the solvent were eliminated as in the case of the spectrophotometric observations already described, by determining the transmission through the cell when filled with clear carbon bisulphide.

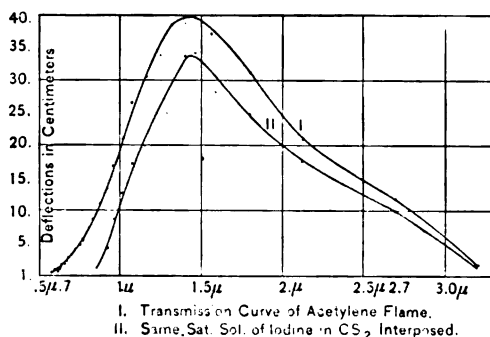


Fig. 8.

The absorption of the cell when filled with a saturated solution of iodine in CS_2 is best illustrated by the curves in Fig. 8. The ordinates are deflections in centimeters, and the abscissæ are wavelengths. Curve I. shows the deflections in various parts of the spectrum when the radiometer vane was exposed to the uninterrupted rays of the acetylene flame, while curve II. shows the corresponding deflections after the light has passed through the cell containing the iodine solution. The ratio of the two curves beyond $.76\ \mu$ is about .77.

Table V. gives the percentage of radiation transmitted by the cell filled with clear CS_2 , and with various concentrations of iodine ranging from 2.5 milligrams per c.c. to saturation.

The data contained in this table are shown graphically in Fig. 9, from which it will be seen that the cell containing clear CS_2 is not uniform in its transmitting power for different rays, the curve being a minimum at about $1.0\ \mu$, and a maximum at $1.8\ \mu$. The

TABLE V.
Transmission of Iodine in Carbon Bisulphide.

Wave-lengths.	Transmission, $I_e \div I_0$.					
Concentrations.	Cell + Clear CS ₂ .	2.5 mg. per c.c.	5 mg.	10 mg.	20 mg.	Saturated Solution.
.73 μ	63	2	00	00	00	00
.76	64.5	34	18	7	00	00
.78	65	43	31	18	5	00
.81	66	53	44	34	17	00
.84	65	55	50	46	35	00
.88	62	59	54	52	48	11
.92	61	—	58	54	56	32
.97	61	57	—	60	59	48
1.02	63	—	59	62	61	54
1.08	64	61	—	63	—	—
1.16	65	65	62	66	63	64.5
1.26	67	68.5	70	69	66	68
1.40	70	71.5	69	71.5	70	71
1.56	74	75	74	74	75	76
1.80	78	78.5	75	76.5	78.5	79
2.12	73	72.5	72.5	72	73.5	73.5
2.70	59	57	55	59	57.5	57.5

curves for the transmitting power of the cell when filled with the iodine solutions rise rapidly from zero in the region between 0.7 μ and 0.9 μ and join the curve for the clear CS₂ at about 1.1 μ . It ap-

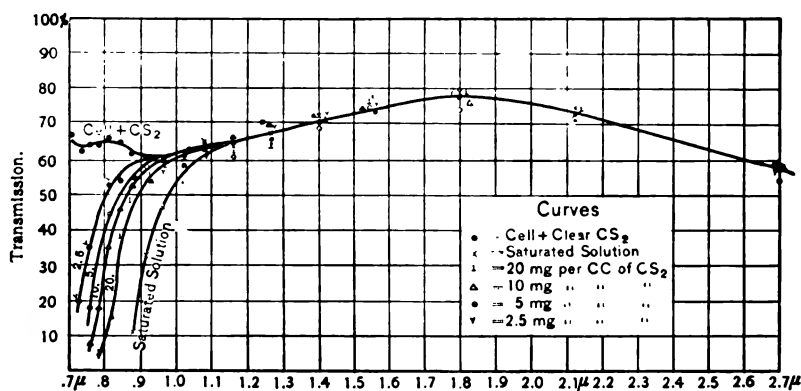


Fig. 9.

pears that beyond this point the addition of iodine to the solution has no effect whatever upon the transmitting power. This effect

will be noticed again in considering the transmission through solid films of iodine, while the absorption band is of interest in the work on anomalous dispersion.

Similar measurements of the transmission of iodine in ethyl alcohol were made, but it was found impossible to carry the observations

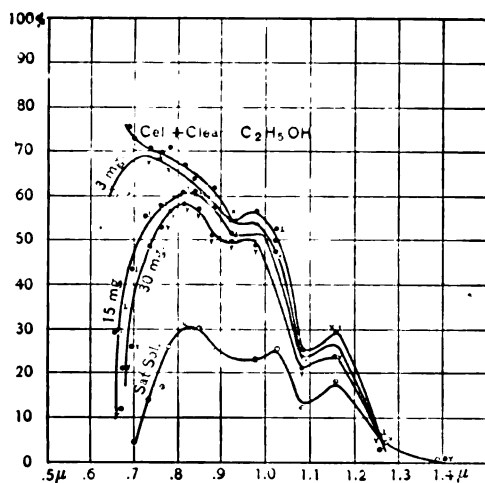


Fig. 10.

beyond 1.4μ on account of the increasing opacity of the alcohol, the transmission of which at that wave-length was only 3 per cent.

The transmission of iodine in alcohol is shown in Fig. 10, from which it will be seen that the alcohol was by no means free from water, as is evident from the presence of the narrow absorption bands at 0.93μ and 1.08μ . This gives an excellent check of the spectrometer calibration for this region. Abney and Festing¹ have published photographs of the infra-red absorption spectrum of ethyl alcohol, which shows these bands. The observations of E. F. Nichols² also agree with the above, showing that alcohol becomes opaque at about 1.4μ . It seems probable that the presence of these bands and the rapidly increasing opacity of the alcohol for the longer wave-lengths is due to the presence of water. Puccianti³

measurements of iodine dissolved in alcohol, for solutions containing respectively 3, 15 and 30 milligrams per c.c. — also a saturated solution.

TABLE VI.

Transmission of Iodine in Ethyl Alcohol.

Wave-lengths.	Transmission, $I_t + \%$.			
Concentrations.	Cell + Clear C_2H_5OH .	15 mg. per c.c.	30 mg.	Saturated Solution.
.68 μ	76	40	21	00
.70	73	44	26	4
.73	73	56	49	14
.76	70	58	53	17
.78	71	59	57	26
.81	67	60	58	31
.84	64	61	57	30
.88	62	57	51	25
.92	54	51	49.5	25
.98	56	—	49	23
1.02	52.5	52	40	25.5
1.08	25	23	21.5	12.5
1.16	30	29.5	24	18
1.28	5	5	4	4
1.40	3	3	4	3

The question of the difference in absorption of new and old solutions, *i. e.*, "velocity reaction," is illustrated by the curves in Fig. 11 which show the transmission through a saturated solution

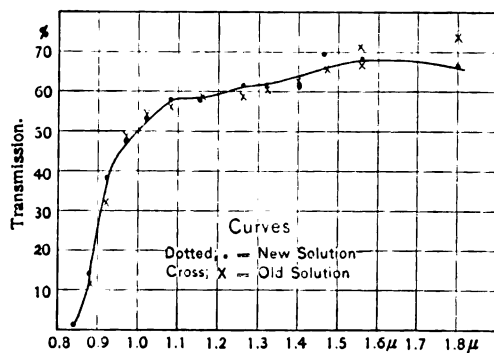


Fig. 11.

while new, curve I., and after standing thirty-one days, curve II. The comparison is not quite a fair one in the regions where the energy transmitted is small, *e. g.*, beyond 1.6 μ , because for the first curve a very stiff fiber was used, while for the second curve the radiometer was almost

THE EFFECT OF HIGH TEMPERATURES ON THE CHANGE OF RESISTANCE OF BISMUTH IN A MAGNETIC FIELD.

BY FRANK B. JEWETT.

IN a series of experiments carried on at Ryerson Physical Laboratory, University of Chicago, during the winter of 1899-1900, some interesting results were obtained regarding the effect of high temperatures on the change of resistance of bismuth wire in a magnetic field. The experiments were originally undertaken simply to determine the allowable temperature variations in the ordinary form of commercial spiral, as used in measuring field strengths. As the results soon showed, however, that at the higher temperatures the change of resistance curve varied greatly from its mean temperature form, the experiments were carried on much beyond the originally set limits. A few of the results, together with a short description of the method employed in forming the spirals, are appended in the following article.

The commercial spiral as prepared by Hartman & Braun, consists simply of a flat, non-inductive coil of comparatively fine wire having a resistance of from 12 to 20 ohms. The manner of making the wire would seem to point to a crystallographic formation such that any direction through the coil is one of mean resistance; this was the assumption though no microscopic examination of a wire section was made.

At the time of the experiments the impossibility of procuring bismuth wire made the employment of a quick and simple process for its manufacture necessary. After a number of trials the following scheme was adopted as giving more satisfactory results than the ordinary mode of casting in capillary tubes. Two long pieces of cast iron were first planed smooth and fitted with machine screws for clamping them firmly together; shallow, slightly trapezoidal grooves were then planed in one of the pieces and a reservoir cut

in one end (Fig. 1); the grooves were highly polished in order to facilitate the removal of the castings. These latter were formed by standing the mould, heated almost to redness, upon an iron plate and pouring molten bismuth into the upper or reservoir end. Since bismuth is malleable at about 200° C. the cap plate was taken off the mould while still quite hot and the castings removed. The wires thus formed, while of the desired length, were neither of the right shape nor cross section to be easily worked; to reduce the amount of metal, the strips were clamped between pieces of

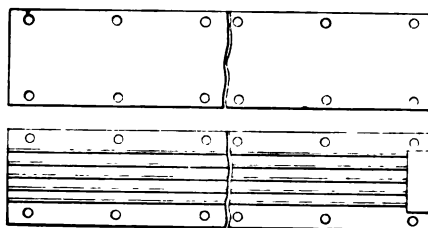


Fig. 1.

heavy sheet brass and the whole filed down the requisite amount. This filing breaks up the crystalline continuity and renders the metal extremely friable; recrystallization is accomplished by transferring the now more or less square filament to a metal plate whose temperature is slightly above the melting point of bismuth. This process serves a double end, for not only does it recrystallize the metal, but at the same time the surface tension causes the wire to assume a form of almost circular cross section. Care must be exercised lest the plate be too hot, in which instance the surface tension causes the wire to break and collect in globules. With the aid of a hot plate the wire formed as above can be readily worked into any desired shape. The spiral once formed the only remaining difficulty is in fastening on the terminal wires—for low temperatures this can be readily accomplished by the use of ordinary soft solder, but as such a joint is in reality simply a piece of fusible metal, other methods must be employed when the temperature is to be above 90° C. After many fruitless attempts pure bismuth was found to answer better than any other solder, although its application is somewhat difficult. To facilitate handling and at the same time protect the

spiral from chance injury it was enclosed in a shallow fiber capsule with screw top.

In order to obtain consistent results, it is of prime importance that the bismuth employed be as pure as possible, slight admixtures of foreign metals causing great variations in behavior under identical conditions. In the following work the castings were made from metal electrolytically deposited from an acid nitrate solution with a rod of Kohlbaum's C. P. for an anode. The variations in the behavior of specimens *A* and *B* (Tables I. and II.) are probably due to the fact that the wires of which the two spirals were constructed had considerably different diameters, and as is well known, the size of the crystals vary with the dimensions of the casting, when the latter is not made under pressure.

For keeping the spiral at any desired temperature, a bath of heavy mineral oil was used; the box containing the oil was of brass, furnished with insets at the sides to permit of the close adjustment of the pole pieces. The heating was accomplished by means of Bunsen burners and fairly constant and uniform temperatures were maintained by means of an electrically driven propeller. The magnet used was one designed to give a uniform field of 35,000 C.G.S. units over a considerable area for pole pieces 3 mm. apart.

Discussion of the Results.—In making the determinations the process was simply to bring the spiral to the desired temperature, and after constant conditions had been attained, to measure the resistance in fields of various strengths. A condensed outline of the results is given in Tables I. and II. and the corresponding curves are plotted in Figs. 2 and 3—Table I. and Fig. 2 refer to specimen *A*—Table II. and Fig. 3 to specimen *B*. In plotting the curves, abscissæ were taken to represent field strengths in C.G.S. units and ordinates to represent percentage changes of resistance— $\frac{R - R_0}{R_0}$, where R is the resistance of the spiral at any temperature in field H and R_0 is the resistance at the same temperature outside the magnetic field.

Upon examining the plates it will be noticed that in every case, with the possible exception of the one for 24°, the curves seem to

consist of two straight lines representing regions of uniform variation, connected by a more or less extended curve denoting a region of non-uniform variation — in each case also the percentage change is greater in the second half of the curve. As the temperature of

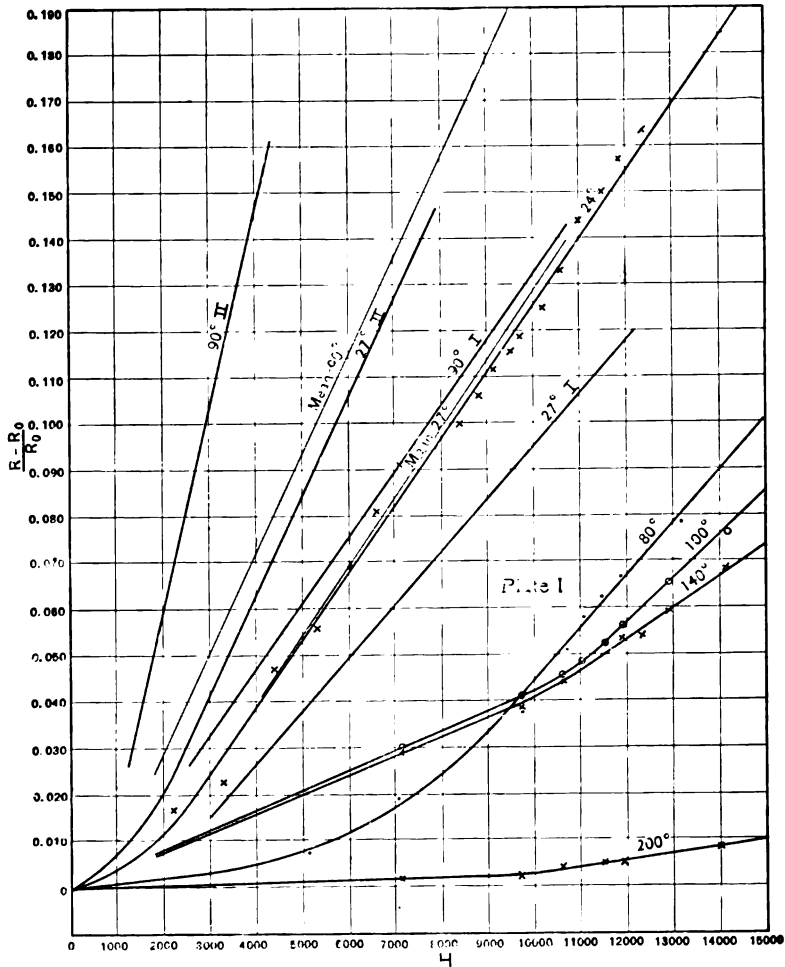


Fig. 2.

the spiral is raised this variable portion seems to travel toward the region of denser magnetic flux.

Even more striking is the apparent existence of fairly well defined temperatures of subsidiary minimum and maximum variation in the

region below $H = 10,000$. For spiral B the number of readings was too small to give anything but a general idea as to the location of these points; in the case of A , however, if the readings for vari-

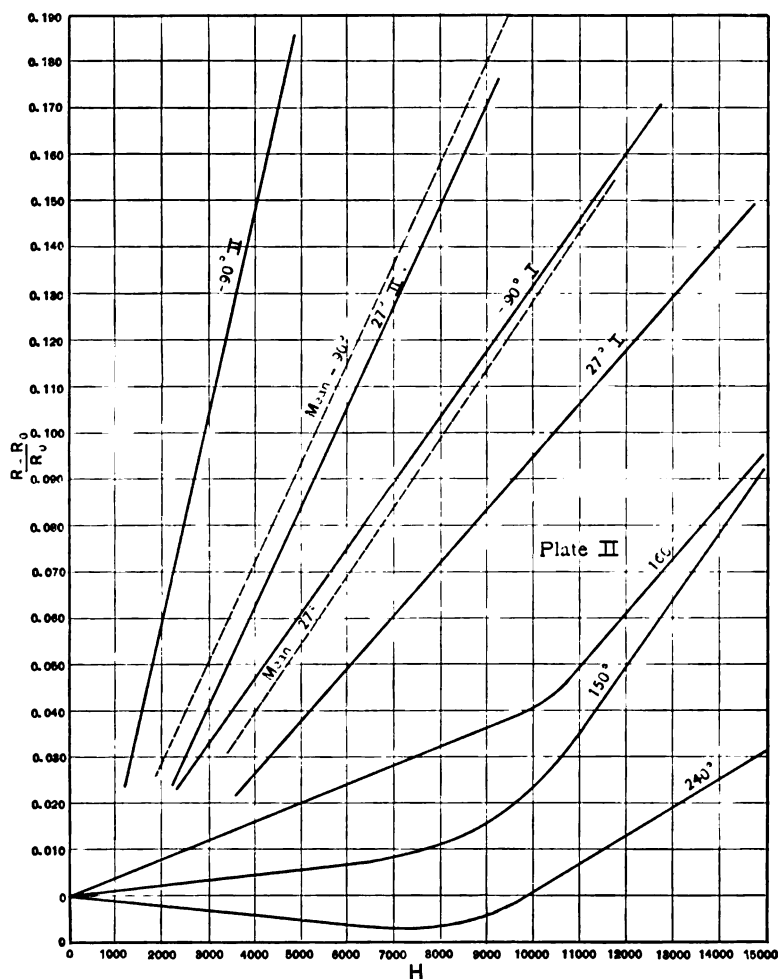


Fig. 3.

ous values of H are plotted with $\frac{R - R_0}{R_0}$ as ordinates and temperatures as abscissæ (Fig. 4), it is seen that the point of minimum variation is in the neighborhood of 80° , while the succeeding maxi-

imum occurs somewhere between 100° and 115° . Beyond this point further increase in temperature causes a continuous diminution in the percentage change of R with respect to H . It is also evident that the greatest variation occurs for $H = 6,000$ and that the minimum disappears at about $H = 9,600$.

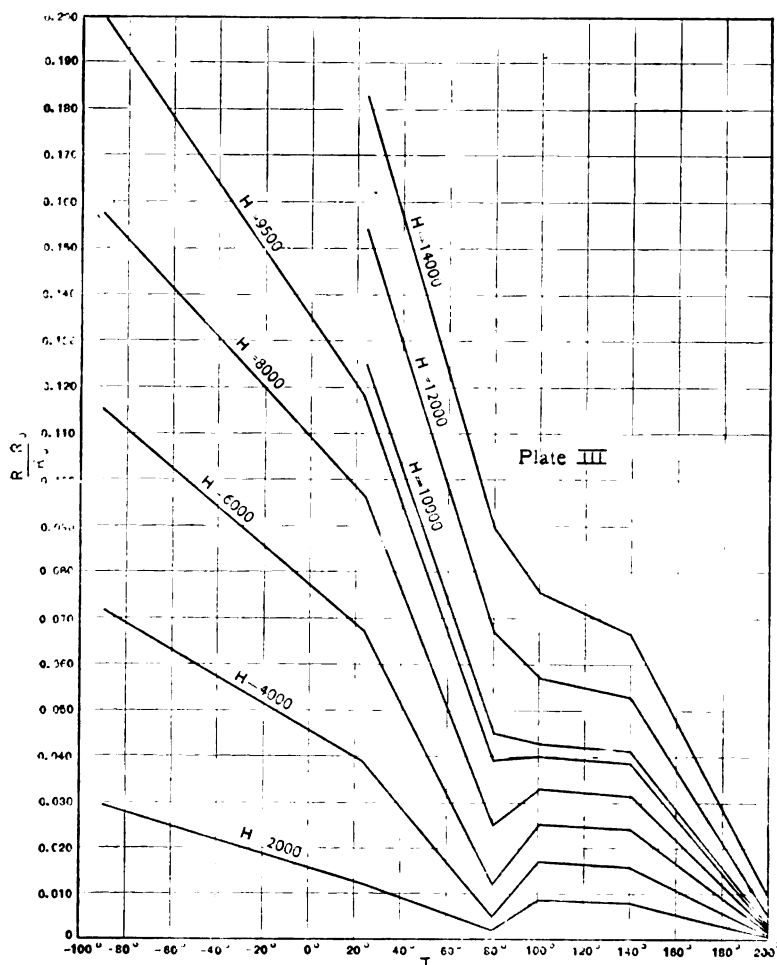


Fig. 4.

While not plotted the corresponding curves for specimen B would apparently be quite similar — so far at least as regards the point

of maximum field influence; the point of minimum disappearance for this specimen is somewhere near $H = 18,000$.

In both Figs. 3 and 4 are plotted curves showing the variation of resistance for 27° and -90° ; these curves are from a few readings taken by Dr. E. Van Everdingen, Jr.,¹ while experimenting on the Hall effect in crystalline bismuth plates at very low temperatures. The curves marked I. were from readings taken with the direction of maximum resistance in the rod coinciding with the lines of force. Curves II. have this direction perpendicular to the lines. In both plates the mean of the two curves are shown by the dashed lines and the proximity of the mean for 27° to the curve for 24° would seem to indicate that the previously stated assumption regarding the direction of mean resistance in the spiral was justifiable.

The values of $\frac{R - R_0}{R_0}$ for -90° in Fig. 4 are those of the mean curve for that temperature.

While the curve for spiral *B* at 240° shows a negative value for $\frac{R - R_0}{R_0}$ between $H = 0$ and $H = 10,000$, it is doubtful if much reliance can be placed on it, as two or three sets of readings on *A* at neighboring temperatures above and below this point, showed that at about 245° the metal loses entirely the property of change of resistance in a magnetic field. This is what would be expected since bismuth loses its crystalline structure at about $245-250^\circ$. (In this connection it is interesting to note that with thin plates of non-crystalline bismuth, formed by means of the cathode stream, no evidence of any magnetic effect on the resistance was observable even at field strengths as high as 27,000.)

It is to be regretted that a larger number of readings at shorter temperature intervals was not made, but as at the time none of the results were plotted, the peculiar inversion effect was inadvertently overlooked. In case the experiments were to be repeated it would seem advisable to discard the cast spiral form entirely and use slender rods cut from electrolytically deposited plates, as was done by Dr. Van Everdingen; this would permit of determinations being made for both positions of the axis of maximum resistance.

¹ E. Van Everdingen, Jr., Verslagen van de Afdeeling Waternkunde der Kon. Akademie van Wetenschappen te Amsterdam, Oct. 28 and Dec. 30, 1899.

TABLE I.

Temp.	H	R	$\frac{R - R_o}{R_o}$	Temp.	H	R	$\frac{R - R_o}{R_o}$
24.0	0	1.322	0	100	0	1.358	0
24.0	2,200	1.345	0.017	100	7,100	1.399	0.030
24.0	3,300	1.353	0.023	100	9,700	1.414	0.041
24.0	4,400	1.385	0.047	100	10,600	1.420	0.045
24.0	5,300	1.397	0.056	100	11,000	1.424	0.048
23.5	6,000	1.414	0.069	100	11,500	1.429	0.052
23.5	6,600	1.430	0.081	100	11,900	1.434	0.056
24.0	7,100	1.443	0.091				
24.0	8,400	1.454	0.100	140	0	1.383	0
24.0	8,800	1.463	0.106	140	7,100	1.423	0.029
24.0	9,100	1.469	0.112	139.5	9,700	1.436	0.038
24.0	9,500	1.476	0.116	140	10,600	1.445	0.044
24.0	9,700	1.480	0.119	140	11,000	1.452	0.049
24.0	10,200	1.488	0.125	140	11,500	1.455	0.052
24.0	10,600	1.498	0.133	140.5	11,900	1.456	0.053
24.0	11,000	1.512	0.144	139.5	12,300	1.458	0.054
24.0	11,500	1.521	0.150				
24.0	11,900	1.530	0.157	200	0	1.353	0
24.0	12,400	1.537	0.163	200	7,100	1.356	0.0022
				199.5	9,700	1.356	0.0022
79.5	0	1.334	0	201	10,600	1.358	0.0037
80.5	7,100	1.360	0.019	199.5	11,000	1.360	0.0052
80.0	9,700	1.386	0.038	200	11,500	1.360	0.0052
80.0	10,600	1.401	0.050	200	11,900	1.360	0.0052
80.0	11,000	1.410	0.057				
79.5	11,500	1.417	0.062				
79.5	11,900	1.422	0.066				

TABLE II.

Temp.	H	R	$\frac{R - R_o}{R_o}$	Temp.	H	R	$\frac{R - R_o}{R_o}$
148	0	1.288	0	240	0	1.227	0
148	8,200	1.303	0.012	240.5	7,100	1.218	-0.007
147	9,700	1.320	0.025	240	9,700	1.225	-0.001
149.5	10,600	1.320	0.025	239.5	10,600	1.250	0.019
149	11,300	1.340	0.040	239	11,500	1.255	0.022
149.5	12,300	1.400	0.087(?)	241	11,500	1.250	0.019
				241	11,900	1.240	0.012
160	0	1.297	0				
160	7,100	1.335	0.029				
160	9,700	1.348	0.039				
159	10,600	1.355	0.044				
158.5	11,000	1.364	0.051				
160	11,500	1.372	0.057				
160	11,900	1.373	0.058				

In conclusion I wish to express my appreciation of the interest shown throughout the work by Professors Michelson and Stratton, and to thank Professor Stratton for many suggestions given. My thanks are also due to Dr. Earhart for help rendered in the preliminary readings.

BOSTON, MASS.,
October 23, 1902.

NOTE ON MAGNETOSTRICTION IN BISMUTH.

BY EDMOND VAN AUBEL.

MR. A. P. WILLS published in the issue of the *PHYSICAL REVIEW* for July, 1902, an interesting memoir on magnetostriction in bismuth. Using intense magnetic fields up to 3200 C.G.S. units, this physicist undertook to determine whether bismuth undergoes elongation when submitted to the action of magnetism. Such an elongation was found by Shelford Bidwell, in 1888, and was considered as very perceptible, even with the magnetic field of 470 C.G.S. units.

It seems well to call attention to several articles published previously on the same subject, and which have escaped the notice of the American physicist. Taking them altogether, we can draw from them the same conclusion as that reached in the experiments of Mr. Wills.

Tyndall¹ was probably the first to examine the question, but he observed no variation in length. Later, G. P. Grimaldi² undertook to find out whether the great influence exercised by magnetism on the electric resistance and on the thermoelectric power of bismuth could be explained by certain deformations which this metal underwent in the magnetic field. Even with a very sensitive apparatus he was unable to find any variation of length in bars of bismuth about 30 to 40 cm. long.

In 1892 I also examined the influence of magnetization on the length of a bar of bismuth,³ using Fizeau's method of interference fringes, and a bar of bismuth 31 cm. long. This was made of the absolutely pure metal that had been used by Classen for the deter-

¹ Tyndall: On Some Mechanical Effects of Magnetization, published in his *Re-*

mination of atomic weight, and which I had used in my researches on electric resistance. Nevertheless no elongation could be found. In the report of the meeting of the Royal Society of Edinburgh,¹ May 16, 1899, we find the following statement: "Dr. C. G. Knott gave a short note on Magnetic Strains in Bismuth. A slight indication has been obtained that there was a change of form in bismuth when strongly magnetized, but the indication was so slight that it was more prudent meanwhile to reserve judgment." Several days later Shelford Bidwell called attention in the same review² to the fact that he had observed, in 1888 (Philosophical Transactions), an elongation in a bar of bismuth amounting to 1.5 ten-millionths of the length for a magnetic field of 840 C.G.S. units. He has, since that time, repeated the same experiment with another sample of bismuth furnished by Messrs. Johnson and Matthey, but, although the magnetic field was now carried up to about 1500 units, he could not even then observe the least indication of any change in the length.³ Nevertheless an elongation ten times less than that previously observed could have been noticed. So Shelford Bidwell adds: "After this experiment I should hesitate to attach importance to any such observations, unless the bismuth employed had been proved by analysis to be free from traces of magnetizable metals."

This interesting article is followed by several lines by C. G. Knott, who recognizes the necessity of working with the pure metal, but declares that he has not paid especial attention to this point in his experiments. In a letter which he afterwards did me the honor to send to me, the same physicist wrote that he considered the bismuth with which he had worked as certainly impure, although the metal had been as pure as possible. The effect observed was, moreover, very weak and doubtful.

In 1899,⁴ *Nature* published a note by me on the same subject, in which note I called attention to the fact that I had obtained similar results with a piece of bismuth whose purity was beyond doubt, and by the use of a very sensitive method.

¹ *Nature*, London, June 22, 1899, page 101.

This bibliography shows that all the experiments made up to this time agree to establish the fact that a bar of pure bismuth does not change in length in a magnetic field.

Mr. Will's experiments, also performed apparently with the pure metal, and in powerful magnetic fields reaching up to 3200 C.G.S. units, give additional confirmation of the previous results.

UNIVERSITY OF GHENT,
August, 1902.

NEW BOOKS.

The Electric Arc. By HERTHA AYRTON. Pp. xxv + 479. New York, Van Nostrand; London, The Electrician Publishing Co., 1902.

At the Electrical Congress in Chicago in 1893 Professor Ayrton read a paper on the electric arc, the manuscript of which was destroyed by fire before publication. Those who had the pleasure of listening to his paper will recognize in Mrs. Ayrton's volume the restatement in fuller form of its salient features, together with a wealth of new experimental material of her own. The book is in form and character indeed a scientific memoir in which the results of the author's own investigations are given in great detail. This method of publishing the results of an extended investigation, while unusual, possesses many advantages. It is possible in the first place, as Mrs. Ayrton has done, to give nearly a hundred pages to a history of the scientific literature and to describe in an adequate manner the work of one's contemporaries, instead of merely alluding to them in footnotes, which is all that is possible to the author who publishes his results in the scientific journals. In this way only can the bearing of one's work to that which has gone before be properly brought out and the real nature of the advance which has been made be clearly indicated.

The difference between Mrs. Ayrton's volume and the ordinary scientific compilation will be apparent from the statement that with the exception of the introductory chapters on the appearance, and history of the arc, and three brief appendices, there is no chapter in which her own work does not occur as the central feature around which the investigations of others are grouped. The inevitable result is, of course, a certain incompleteness, due to the fact that many topics which would naturally enter into a treatise on the electric arc are omitted altogether. Of arc lamps as such, for example, nothing is said. The volume deals solely with the physics of the arc and almost exclusively with the physics of the direct-current arc, since it is with this form chiefly that Mrs. Ayrton's own experiments deal. Of the spectrum of the arc, which in itself would afford material for a second memoir not less extensive than that which Mrs. Ayrton has written, nothing is said, and there are a dozen other topics which one might reasonably look for concerning which she has remained silent. The ten chapters which form the substance of the vol-

ume, deal with the following topics: The phenomena connected with the striking of the arc and with sudden variations of current; the steady potential difference between the carbons when the length of the arc is fixed and the current varied, and when the current is fixed and the length of the arc is varied; the area of the crater; the depth of the crater; the relations between the diameters of the carbon and the potential difference between them; the apparent resistance of arc with constant current; the relation between the current and length of arc for constant potential difference; the relations between the power and length of arc for constant currents and between power and current for constant lengths of arc; the relations between the E.M.F., the resistance of the outside circuit; the length of the arc and the current; the ratio of the power expended in the silent arc to the power expended by the generator; phenomena of hissing arcs; the light emitted by the arc under various conditions: its luminous efficiency and the conditions of maximum luminous efficiency; the mechanism of the arc; its true resistance; the question of a back E.M.F.

The most striking and perhaps the most important contributions to our knowledge in this work are those which relate to hissing arcs and to the effects produced by what Mrs. Ayrton calls *arc vapor* and *arc mist*. While the really important portions of the memoir might have been condensed into a paper of 100 or perhaps less pages, the service which Mrs. Ayrton has done to the science of physics in presenting the results of her investigation in this fuller form is certainly greater than though she had contented herself with the usual brief and cursory method of publication. The growth of the literature of science has in recent years been such as to make it almost impossible for even the most indefatigable of readers to keep in familiar touch even with those topics in which he is chiefly interested. Any one, therefore, who having completed an investigation will take the trouble to present in connection with his own work a fair and lucid description of what has been done before in the same field, even though this may necessitate the expansion of the memoir to several hundred pages is, like the present author, deserving of special recognition and of the thanks of all collaborators.

E. L. N.

THE
PHYSICAL REVIEW.

NOTE ON THE VARIATION OF THE SPECIFIC HEAT
OF MERCURY WITH TEMPERATURE. EXPERI-
MENTS BY THE CONTINUOUS-FLOW
METHOD OF CALORIMETRY.¹

BY H. T. BARNES AND H. L. COOKE.

SINCE the communication to the Royal Society in 1900, of the work of one of the authors on the specific heat of water, it has been possible to continue the experiments by the continuous-flow method of calorimetry to the case of mercury. These measurements, which include an interval of temperature between 0° and 90° C., have been carried out with essentially the same apparatus as previously used for water, and with the same electrical standards. The first determinations which were made by mercury by this method were made by Professor Callendar and one of the authors during the summer of 1897, just previous to the Toronto meeting of the British Association. The results then obtained with our preliminary apparatus were entirely revised and recalculated three years later. It was the intention of one of the authors to include them in his paper on the specific heat of water since they afforded a most satisfactory verification of the theory of the method. Deeming it advisable, however, to redetermine the value of the specific heat and to extend the experiments over as large a range of tem-

tained from our preliminary experiments, stand in close agreement with the authors later results, the error being less than 1 part in 1,000, although carried out with different electrical standards, and a calorimeter differing materially in design from the one recently employed.

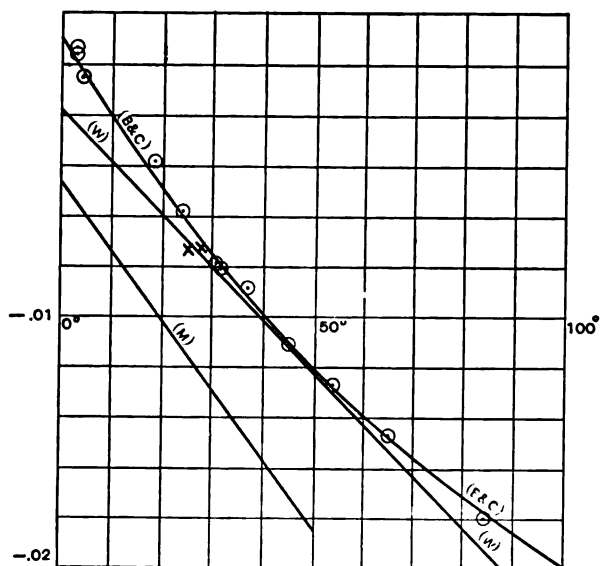


Fig. 1.

○. Observations of Barnes and Cooke, 1892. ×. Observations of Callendar and Barnes, 1897. *B. & C.* Formula proposed by Barnes and Cooke. *W.* Formula proposed by Winkelmann. *M.* Formula proposed by Milthaler.

The present series of experiments was made with a calorimeter in every way similar to the "water" calorimeter described by Professor Callendar and one of the authors (B. A. report, 1898) with the exception that the flow tube was smaller in bore. The rest of the apparatus was essentially the same, with a few minor changes.

PRELIMINARY RESULTS.

These results obtained in 1892 are compared here with the

eter employed had a flow tube 1 meter long and 1 mm. in diameter coiled in a spiral as described by Professor Callendar (Phil. Trans., Vol. 199, 1902). The thermometers were pair *A* described by one of the authors (Phil. Trans., Vol. 199, 182, 1902, Sec. 3c). The Clark cells were S_1 and S_2 and the resistance standard was one of the manganin coils described in Section 3b, *supra*.

The only three determinations made at room temperature are as follows :

Date.	Mean Temperature.	J_s	s
July 9, 1897	25.0	.13898	.033174
" 14 "	27.4	.13899	.033177
" 19 "	28.0	.13900	.033179

RECENT RESULTS.

The later results were obtained with improved apparatus. The calorimeter had a flow tube about 1 mm. in diameter and 45 cm. long. The thermometers were pair *E*.

The mica-frame resistance standards and Clark cells X_2 and X_{11} were used. Redeterminations were made of the value of the resistances in terms of the standard platinum-silver coils in the possession of the laboratory, which showed that they had remained unchanged to 1 in 100,000 since the series of tests taken for the measurements for water, and already described. The Clark cells were found to have lowered a little in value since 1900, and a correction was necessary in order to reduce to the mean of a number of new cells. The history of these original crystal cells, made in 1895, to which X_2 and X_{11} belong, shows that from comparisons with new cells made from time to time, they have steadily lowered in value. A new comparison was also made to several Weston cadmium cells. There is no reason to doubt that the values assumed for the electrical standards are the same as those assumed in the measurement of the specific heat of water, to much better than 1 in 10,000. The thermometers were also recalibrated several times, as well as further determinations made of the box coil corrections. Hence, even assuming that any error exists for the values assigned to the units used in the calculations for water,

the specific heat of mercury may be expressed from these measurements in terms of water, and any possible error eliminated from the relative value. Since also the same apparatus has been used, and as far as possible the same experimental arrangements systematic sources of error in the method are to a large extent avoided.

The absolute values of the thermal capacity of mercury are given in the following table for different temperatures, calculated by the method already described by one of the authors.

The results are in terms of the nitrogen thermometer and are uncorrected as yet for the temperature gradient in the flow tube as described by Professor Callendar.

Since this correction is small, and is nearly balanced by the correction to the standard hydrogen scale, the results will not be changed much by its application, and the variation with temperature will remain practically unaffected.

Mean Temp.	J_s	$S(J = 4.1891)$	Formula.
2.85	.140056	.033435	.14003
2.93	.140085	.033440	.14002
4.42	.139927	.033405	.13996
18.37	.139459	.033291	.13939
24.52	.139182	.033224	.13915
31.68	.138890	.033156	.13889
32.14	.138873	.033151	.13888
32.41	.138846	.033141	.13887
36.59	.138753	.033121	.13873
45.00	.138447	.033050	.13847
53.39	.138225	.032997	.13822
65.22	.137942	.032929	.13791
83.89	.137479	.032818	.13752

In Fig. 1 is given a plot of the observations the vertical scale being the value $\frac{J_s - 0.140}{0.140}$. The preliminary results are also included.

It will be seen that the specific heat decreases with rise in temperature, suggesting that portion of the curve for water between 10° and 20°.

Results over a wider interval of temperature could not be conveniently taken with the present apparatus. We purpose continuing

the work to much higher temperatures immediately, in order to trace the course of the curve as the boiling point of the mercury is reached.

The equation representing the change of J_s and shown by the curve in Fig. 1, reads :

$$J_s = J_0 - 4.462 \times 10^{-5}t + .0157 \times 10^{-5}t^2$$

where

$$J_0 = .140154.$$

The value of the specific heat in terms of water has been calculated, taking the value of J equal to 4.1891 for a thermal unit at 15.5° , which was the temperature recommended by Griffiths at the Paris Congress in 1900 as being suitable for the selection of the thermal unit.

The temperature expression of the specific heat reads :

$$S_t = S_0 - 1.074 \times 10^{-5}t + .00385 \times 10^{-5}t^2,$$

or

$$\frac{S_t - S_0}{S_0} = - .000321t + .00000115t^2$$

where

$$S_0 = .033458.$$

This gives for the temperature coefficient at any temperature t the expression,

$$\frac{d}{dt} \left(\frac{S_t}{S_0} \right) = - .000321 + .00000230t,$$

and for the average change per degree at 50° , the value $-.0000069$.

COMPARISON TO THE WORK OF OTHER OBSERVERS.

The value of the specific heat obtained by Regnault¹ and so universally adopted, *i. e.*, .03332 is found to be in agreement with our determination at 13.3° and very close to the value at the ordinary temperature of laboratory work. The value was obtained over a range 10° to 100° in terms of a thermal unit at 10° . and is consid-

Experiments to determine the variation with temperature indicated an increase in specific heat with increasing temperature.

Winkelmann¹ carried out an extensive series of experiments by the method of mixture, to determine the temperature coefficient and was the first to show that the specific heat decreased as the temperature increased. This result being so exceptional at the time and so contrary to the results for other liquids, the author was inclined to regard it as doubtful until he finally verified the first experiments by a second series, in which he eliminated certain sources of error. Two ranges of temperature were taken, the first between 20° and 50° C. and the second between 26° and 142° C. The average variation from these measurements was found to be .0000069. No absolute measurements were made, but the value obtained by Regnault was assumed.

The formula proposed by Winkelmann is shown in Fig. 1; this reads:

$$S_t = S_0 - .0000069t.$$

The remarkable agreement with the average change from our determinations is shown very clearly in the figure, a total decrease of 2 per cent. between 0° and 100° being given both by Winkelmann's results and our own.

The position of the curve in the figure shows that the temperature at which Winkelmann assumed Regnault's value, which in one case was the mean from 16° to 20° and in the second case from 13° to 25° is very nearly equal to the temperature at which our value is exactly equal to Regnault's, *i. e.*, 13.3° C.

Pettersson and Hedelius² obtained .033266 between 0° and 5°, .033262 between 5° and 16°, .033300 between 5° and 26° and .033299 between 5° and 36° C. These results were all expressed in terms of a thermal unit for water at 0°.

The measurements were repeated more accurately by J. Milthaler³ who used a similar method to the previous investigators, *i. e.*, the

showed that the specific heat apparently decreased with rise of temperature, the average decrease from his measurements being .0000092, which is somewhat greater than Winkelmann obtained.

The formula proposed by Milthaler is shown in the figure (*M*). The slope of the curve is about the same as the slope of our curve at the low temperature, but since Milthaler supposed this formula to apply as high as 200° its agreement for the low temperatures is more a matter of coincidence. The position of the line in the figure shows that the value of Pettersson and Hedelius which he assumed, is too small at 0° compared to our values, but this value will be increased when corrected to a thermal unit for water at 15.5° instead of 0° and will then be brought into much closer agreement.

MCGILL UNIVERSITY, MONTREAL.

SOME OPTICAL PROPERTIES OF IODINE. II.

BY WILLIAM WEBER COBLENTZ.

II. THE ABSORPTION OF SOLID AND LIQUID IODINE.

1. *Solid Iodine.*

IN all this work commercial, "resublimated," iodine was used. The films were made by melting large iodine crystals between pieces of plate glass or rock salt. For the region of the spectrum up to $2.7\ \mu$ the radiometer-spectrometer arrangement used was the one employed for the iodine solutions. On account of the difficulty in obtaining a large uniform film of iodine, the specimens to be examined were placed between the slit F_1 and the source. They were mounted in vertical ways which kept them parallel to the slit and at the same time prevented lateral shifting. This necessitated the shifting of the source, which decreased the deflections, so that a more sensitive fiber, giving a period of 1.5 minutes, was used throughout all the succeeding work. The pressure in the radiometer was about .02 mm.

In all, four films were examined, the thickness varying from 0.015 mm. to 0.055 mm. All but the last were transparent, the color of an incandescent lamp filament by transmitted light being similar to that of the dense iodine solutions in CS_2 . The absorption of the glass plates was found, and corrections made. From the resulting curves, Fig. 12, it will be seen that the absorption rapidly decreases for all films and becomes quite constant from $1.2\ \mu$ to $2.7\ \mu$. The absorption of the thinnest film becomes zero at about $1.0\ \mu$. Table VII. gives the transmission for different wavelengths.

For the region beyond $2.7\ \mu$ the entire apparatus was rearranged. To avoid sudden changes of temperature, the whole apparatus was enclosed in tin boxes. The drift of the zero reading was never

very large, but was unsteady until after the radiometer had been enclosed and packed in wool. The movable arm F_1 was protected

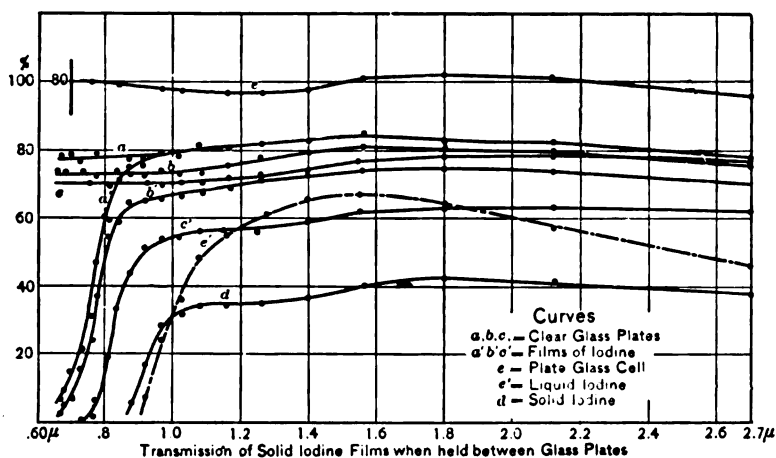


Fig. 12.

$a' = .015$ mm., $b' = .020$ mm., $c' = .025$ mm., $d' = .055$ mm., $e' = .062$ mm.

by several shields to avoid stray radiations. The whole is shown in Fig. 13, in which N is the source, C the film, S_1 , S_2 the shutters, and P the prism. The lower figure, a , is a tin-foil covering for the inner window, to prevent electrification of vanes. Later on it was found that part of this electrification came from the exhaust pump. This was prevented by placing a tube containing gold foil in series with the radiometer.

The windows of the radiometer were of rock salt, cut parallel to the cleavage planes to avoid polishing. The radiometer slit subtended an angle of $10'$ of arc $= .13\mu$ at 1μ .

A 70 mm. rock salt prism having an angle of $59^\circ 55' 47''$ was used instead

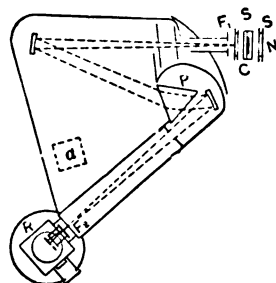


Fig. 13.

TABLE VII.

Transmission through Solid and Liquid Iodine. (Glass Included.)

Wave-lengths in μ .	Solid Films.					Liquid.
	Clear Glass.	Thickness of Films in mm.				
		Transmission, $I_e + I_o$.				
		$a = 0.015$.	$b = 0.020$.	$c = 0.025$.	$d = 0.0055$.	$e = 0.062$.
.66 μ	78					
.67	78	7	2			
.68	78	10	6			
.70		14	8			
.73	78	21	16	1		
.76		33	24	2		
.78		47	38	7		
.81	78	60	54	19		
.84		70	59	34		
.88	78	75	64	44	5	
.92		77	65	51	17	7
.97	79	79	67	53	29	24
1.02	79	81	67	55	32	36
1.08	81	82	68	56	34	48
1.16	81	80	69	57	35	55
1.26	82	82	71	57	35	57
1.40	83	83	73	54	37	59
1.56	84	85	74	62	40	62
1.80	83	83	74	63	43	63
2.10	82	83	74	64	41	58
2.70	78	77	70	62	38	46

Calibration. — The prism was set at minimum deviation for the sodium lines, by using a telescope, and calibrated by observing the absorption bands of quartz, found by Nichols¹ at 5.3 μ , 6.25 μ , 6.68 μ and 8.1 μ , and the emission bands of the bunsen acetylene flame, found by Paschen² at 1.9 μ , 2.7 μ , 4.4 μ , and 6.62 μ . The quartz plate was 2 mm. thick, and was cut perpendicular to the optic axis. The results are shown in Fig. 14, where wave-lengths and circle readings are plotted. The work was greatly facilitated by computing the approximate positions of the various absorption bands. The minimum deviation for different wave-lengths was then computed by

¹ E. F. Nichols, PHYSICAL REV., Vol. 4 (1897).² Paschen, Wied. Ann., 52, p. 209 and 53, p. 334 (1894).

using the indices of refraction found by Rubens & Trowbridge,¹ by Paschen,² and by Langley.³ When changed to circle readings and

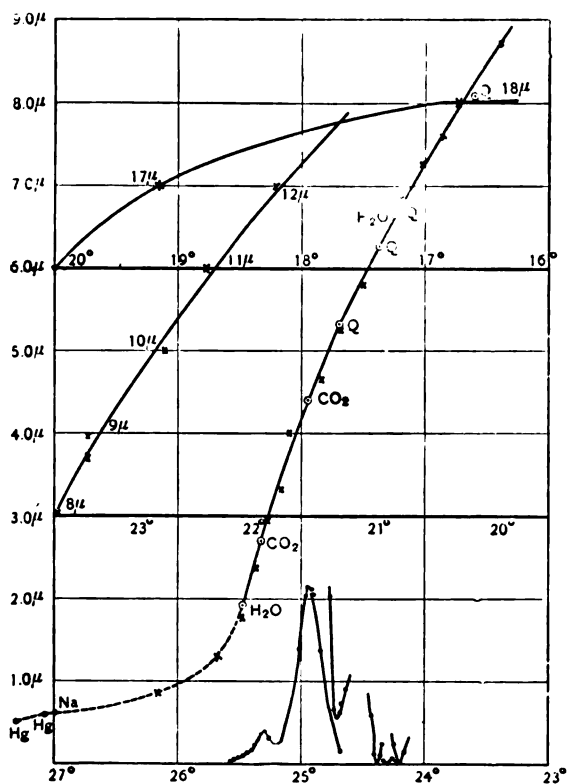


Fig. 14.

plotted on the calibration curve (represented by * * *), they are seen to follow the latter very closely. Finally the march of the radiation through the prism was computed, thus finding the actual deviation of the rays. The error introduced by not keeping the prism at minimum deviation for different wave-lengths is shown in Table VIII.

TABLE VIII.

Angle of prism 59° 57' 15".

Wave-length.	Computed Minimum Deviation.	Computed Actual Deviation.	Correction, $\Delta\lambda$.
1.0 μ	39° 55'.6	39° 55'.6	0.000 μ
2.945	39 16.3	39 16.9	+ 0.032
7.22	38 1.9	38 3.4	+ 0.063
12.0	35 50	35 55.3	+ 0.150

The actual error agrees well with the theory as worked out by Wadsworth.¹

Beyond 10 μ the calibration was continued by computing the minimum deviation settings for the different wave-lengths. Since no absorption bands of iodine occur beyond 8 μ the errors in wave-lengths introduced by using minimum deviation settings are of minor importance.

For a source of radiation an acetylene flame was used for the shorter wave-lengths. This was very unsatisfactory beyond 7 μ . For measurements at longer wave-lengths the "glower" of a Nernst lamp, and subsequently the two "heaters" of the same lamp were tried. These last proved to be the most satisfactory for the long wave-lengths. Beyond 16 μ , with a slit-width of 1 mm., these "heaters," placed at a distance of about 10 cm. from the slit, gave a deflection of 6 mm. on a scale placed at a distance of 1.5 meters.

The films were melted between plates of rock salt, which were ground but not highly polished. Considerable difficulty was experienced in making films of even fairly uniform thickness and free from air bubbles, so that the chief error lies in not knowing the true thickness. This work was done during warm weather, and on account of evaporation the transparency of the film varied from day to day. To explore the entire spectrum required about four hours. In order to lessen the time and to avoid any appreciable variation in the transmission that might occur during this interval, thus

the remaining part, the method of exploration adopted was to take a few readings for a few points throughout the entire spectrum, and also take the mean of a great many readings at a certain region of the spectrum,—*e. g.*, at 7μ . Consequently curves *b* and *c*, Fig. 15, represent most accurately the absorption band region at 7μ ,

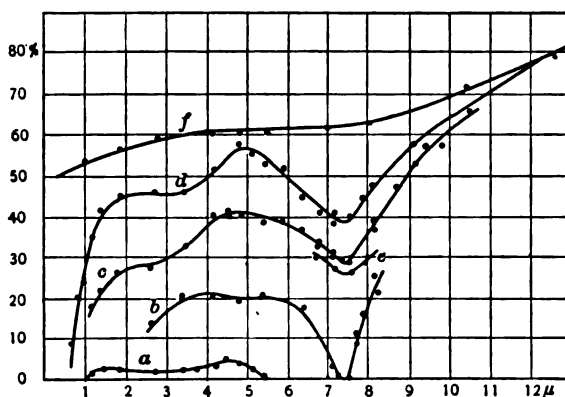


Fig. 15.

Solid iodine films. *a* = .15 mm., *b* = .09 mm., *c* = .06 mm.,
d = .027 mm., *f* = rock salt.

while *d* shows the region of short wave-lengths. The latter also verifies the absorption spectrum of the films between glass plates. Curve *c* indicates the increased transparency of *b* after standing three days. The rock salt used for the plates came from Louisiana. The plates contained impurities, probably gypsum. The transmission curve of two of these plates, used to determine curve *d*, is shown in curve *f*, Fig. 15. It will be noticed that the transmission increases steadily from 54 per cent. at 1.0μ to 66 per cent. at 9μ .

Results.—An absorption band was found, Fig. 15, which begins at 6μ and extends to 10μ , with a maximum at about 7.4μ . The curve is asymmetrical, with its steeper side towards the long wave-lengths where the transparency rapidly increases. Beyond 12.5μ , where deflections were very small (an acetylene flame having been

marked, gave a transmission of 77 per cent., which, on eliminating the absorption of the rock salt plates, makes this region transparent.

Table IX. gives the transmission for the different wave-lengths

TABLE IX.

Transmission of Films of Solid Iodine and Rock Salt.

Wave-length $\lambda =$	Transmission.			
	Layer a .15 mm.	Layer b .09 mm.	Layer c .06 mm.	Layer d .027 mm.
.87 μ				20.4
1.0				24.4
1.15			18.0	35.2
1.4	1.7		21.8	42.0
1.85	2.4		25.4	45.0
2.7	2.5	13.5	28.7	45.5
3.45	2.1		32.4	46.2
3.73	2.8	20.8		
4.17	3.8	20.8	40.3	52.0
4.50	4.9	19.4	41.6	58.0
4.80	4.4		40.1	55.7
5.42	0.8	20.5	38.1	53.0
5.66	0.0		39.1	
5.90				51.5
6.36		18.0	37.2	45.0
6.75			34.6	41.0
7.14		2.9	31.0	38.0
7.33		0.9	29.6	
7.54		0.0	29.3	40.0
7.93		9.0	29.0	44.3
8.13		16.5	39.5	
8.70			47.6	
9.40			57.5	59.5
9.80			58.0	
10.5			66.0	77.0
12.5				

2. Liquid Iodine.

In this work, the greatest difficulties met with were those found in providing a suitable cell, which would not be attacked by the iodine when in a liquid state. The different parts of the cell had to fit well, and had to be subjected to great

bulbs of glass, by flattening tubing, by using mica, etc., but each was found defective. Finally two pieces of plate glass were selected, thick enough to withstand a high pressure. In the face of one of them a flat cavity b was made by grinding and polishing, which, when the two plates were clamped together as shown in Fig. 16, gave a cell of suitable depth. In the other plate a deeper circular pocket (a) was ground, which, when the plates were placed together, connected with b in the manner shown. In the figure, which is about two thirds of the actual size, g, g , are the glass plates (each 5 mm. thick), w is a shield of hard wood, and t is a thermo-junction for the determination of the temperature.

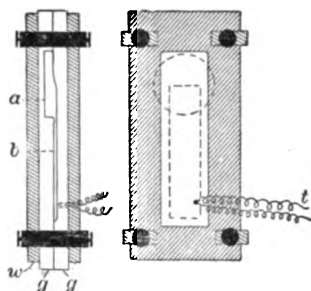


Fig. 16.

The grinding was done on a power drill press, using an extension arm, which gave freedom of motion in all directions in the plane of the glass plate. The grinding tool was a small lead cylinder, with which washed emery and water were used. The polishing was done by covering the lead with beeswax, and applying rouge. In this manner a quite plane, highly polished surface was produced.

The cell was only 0.062 mm. deep, and entirely enclosed. It was filled by placing in the reservoir, a , solid iodine crystals, which, when melted ran down into the cell.

The cell was mounted in vertical ways, between the collimator slit and the acetylene flame, Fig. 5. The spectrometer apparatus used was the same as that for the study of the solid iodine.

(a) *Transmission at a Variable Temperature.*—The cell was placed in an oven and heated far above the melting point of iodine—in one case to 155°C .

The cell was then placed before the slit, and allowed to cool. The rate of cooling and the period of the radiometer was such that

wire. The latter was placed in a small hole drilled in the cell wall, and was held fast by means of tinfoil, covered with plaster of paris.

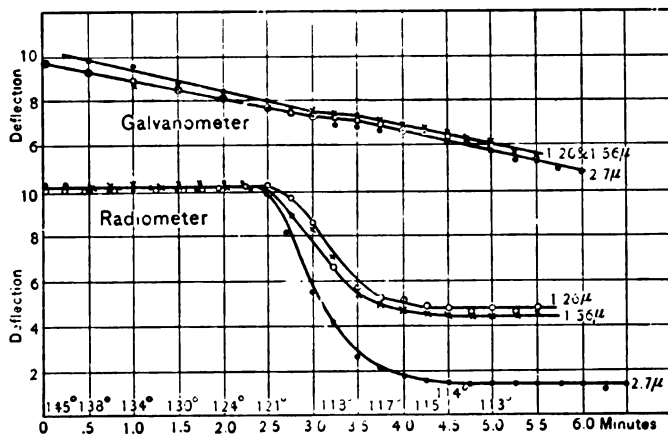


Fig. 17.

The transmission through liquid iodine with variation of temperature was studied by this method at 1.26μ , 1.56μ , 2.7μ . The curves in Fig. 17 serve best to show the result. Here the ordinates represent deflections of the galvanometer (upper curve), and of the radiometer (lower curve). The abscissæ represent the time in minutes, also the temperatures as indicated by the thermo-

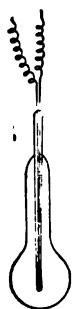


Fig. 18.

utes, also the temperatures as indicated by the thermo-element. From these curves it is evident that the apparatus in this form had a considerable temperature lag, for the melting point of the iodine must correspond to the sudden drop in the radiometer deflections. From Fig. 17 it may be seen that this is several degrees too high. In order to check this interpretation of the discrepancy, the thermo-element was subsequently placed in a thin glass tube, and sealed in a glass bulb containing a larger quantity of iodine, Fig. 18. Conditions as regards pressure were thus quite similar to those in the experiment. The melting point as obtained from the curve of cooling of this apparatus was $114^{\circ}.1 \text{ C.}$, which is in close agreement with previous determinations.¹

¹ Ostwald, *Allgemeine Chemie*, gives $113^{\circ}.9$.

Considering the fact that the thermo-junction is at the side of, and not in contact with, a layer of iodine 0.062 mm. thick, one would expect such a lag. Nevertheless, in spite of the lag of the thermo-junction the experiment just described serves perfectly to establish the fact that the transmitting power of liquid iodine does not change materially with temperature between about 145° and the melting point for any one of the three wave-lengths selected.

It also shows that the transmission to be the same for all these regions ($1.26\ \mu$, $1.56\ \mu$, and $2.7\ \mu$), and much greater than that of solid iodine for those wave-lengths.

It will also be seen from the curves that the transmitting power of the solid iodine varies but little for several degrees below the melting point.

(b) *Transmission at a Constant Temperature.*—After it had been found that the temperature had no effect on the transmission, the spectrum of liquid iodine up to $2.7\ \mu$ was further explored by heating the cell until the iodine had melted, and then placing it before the collimator slit. The iodine remained liquid for over two minutes, which gave ample time to make a series of measurements at a given region of the spectrum. Since the absorption is so different in the two states, there was no difficulty in knowing when crystallization occurred.

In Fig. 12 curve *d* represents the transmission through the solid film and the glass cell. Iodine contracts on cooling so that it no longer fills the cell, being only 0.055 mm. thick, while the depth of the cell was 0.062 mm. In its liquid state (curve *e*, which represents the mean of two series of observations) the transparency rapidly increases; the transmission curve crossing that of the solid at about $1.0\ \mu$. It becomes a maximum at $1.5\ \mu$, where the transmission is about 90 per cent. greater than that of the solid, and then gradually decreases to $2.7\ \mu$, where the difference in transmission is only about 2 per cent.

All the observations have been corrected for the deflection due to the hot glass, which at no part of the spectrum was greater than 3 mm.

In Table VII. is given the transmission through liquid iodine and cell.

(c) *Transmission through Turbid Media.*—A thin film of iodine is far more transparent to the visible rays when liquid than when solid. The appearance of an incandescent lamp filament, by light transmitted through a solid film, is similar to that which it presents when viewed through the CS_2 solutions; but through a thick layer the filament appears much broadened and blurred, probably due to internal reflection.

Ångström¹ has shown that the absorption of media containing discrete particles follows the same law as that for perfectly homogeneous bodies, so long as the particles are small in comparison to wave-lengths. He found that, in the case of long wave-lengths, the absorption is like that of a homogeneous medium.

At the time when the absorption of films of solid iodine was measured the sensitiveness of the radiometer was slowly decreasing from day to day. This did not affect the absorption measurements. The application of Ångström's work to the relation of transmission to thickness, with reference to the blurring effect just mentioned, was not thought of at that time. To apply this test, the radiometer must have the same sensitiveness for all films. By means of the exhaust pump the sensitiveness had been kept approximately the same for all films. Selecting measurements where the direct deflections were the same for the different films,

and applying them in the formula $\log \frac{I_0}{I_n} = n \log \frac{I_0}{I_1}$, where I is the intensity, represented by the radiometer deflections, and n the ratio of the thickness of the two films compared, the results agree sufficiently to conclude that for long waves (not tested beyond 2.7μ), the solid films absorb like an optically perfect medium. Beyond 2.7μ the films were between plates of rock salt which were not plane, consequently the thickness of the films is not known accurately enough for this work.

3. *Comparison of the Absorption of Iodine as a Film, and in Solution.*

A comparison of the absorption of the CS_2 solutions and the films, Figs. 9 and 12, shows a great similarity between them. The thinnest film of the solid is transparent in the infra-red, like the CS_2 ,

¹ Ångström, Wied. Ann., 36, p. 715 (1889).

solution. Furthermore, the thickest solid film and the saturated solution are very similar for the short wave-lengths. The question then arises whether or not the transparency of the CS_2 solutions is due to the small quantity of iodine dissolved.

A computation of the thickness of iodine in a 20 mg. per c.c. solution of iodine in CS_2 (if the iodine were deposited in an even layer), shows that it is about 0.07 mm. The saturated solution would make a much thicker film, at least 0.1 mm. thick. Nevertheless the CS_2 solution is perfectly transparent in the region where much thinner films of solid iodine show considerable absorption.

Rubens,¹ using a cell with fluorite windows, has measured the absorption of CS_2 up to $3\ \mu$, and has found that it is almost entirely transparent. A comparison of the absorption of glass in Fig. 12 and the absorption of CS_2 + cell in Fig. 9 shows that the bisulphide is almost transparent.

It will be interesting to learn the action further out in the infra-red

III. PLEOCHROISM.

The unequal transmission of the two polarized rays through a doubly refracting crystal was first observed in tourmalin, by Biot² in 1819. This was followed by the publication of a list of some sixty dichroic crystals, by Brewster.³ During the latter part of the century our knowledge of selective absorption has been extended by the researches in the ultra-violet, by Agafonoff⁴ and in the infra-red by Merritt⁵ and others.

An examination of the literature on the optical properties of iodine shows that, if mentioned at all, little more than the crystalline form, and the ratio of the axes of symmetry is given. This, however, is what one would expect, considering the physical state in which this element is found. Nothing is known about the position of the optic axes. Consequently in the following work the expressions "maximum" or "minimum extinction" have been used.

¹ Rubens, *Wied. Ann.*, 45, p. 260 (1892).

² Biot, *Bulletin de la Soc. Philomathique*, 109 (1819.)

³ Brewster, *Phil. Trans.*, p. 11 (1819).

⁴ Agafonoff, *Archives des Sci., Phys. et Nat.*, t. 2, p. 349 (1896).

⁵ Merritt, *PHYS. REV.*, Vol. 11, p. 424 (1895).

A transparent film of iodine shows that, when the Nicol prism is turned for maximum extinction, the transmitted light is deep red, while for minimum extinction the light is greenish-yellow.

The iodine was melted between pieces of plate glass, and usually crystallized in long narrow films, in all directions. This crystalline structure is not likely to be due to stresses in cooling. The glass cell used in the absorption work, and the prism used in measuring indices of refraction, exert no pressure on the iodine when cooling. Yet the double refraction was present just as in the films between glass plates. Films of cyanine did not show double refraction.

For this work glass plates showing little or no double refraction were selected. After melting the crystals, and the solid film had formed, there were always air bubbles present, through which polarization in the glass could be detected if present. A region was then selected where the film was uniform and no double refraction appeared in the glass.

It may not be out of place to mention that the glass showed temporary double refraction immediately after cutting. A fan-shaped effect could also be produced by drawing a sharp file across the edge of the plate. Placing a plate, thus treated, between crossed Nicols the light would be restored, but would disappear again at the end of three or four minutes.

For the visible part of the spectrum the horizontal slit spectrophotometer, already mentioned, was used. Since there was considerable difficulty in obtaining uniform films sufficiently large, it was found most convenient to place the crystal with its long side parallel to the slit, and then rotate the polarizer.

For a source of polarized light a large Foucault prism was employed.

The crystal used in the spectrophotometric work was extremely thin, and by ordinary light appeared yellowish-brown in color. From previous measurements of thicker films its thickness was estimated to be 0.005 mm., or even less than this. In convergent polarized light it had a greenish tint, and gave uniform extinction over an area of about 1×10 mm. All other thicker films were too opaque for photometric work.

The infra-red region was explored by means of the radiometer and CS_2 prism arrangement used in I. For an analyzer the Fou-

cault prism, already mentioned, was used. It was mounted on the collimator arm between the slit F_1 , Fig. 5, and the first mirror. It was necessary to rotate the prism, since a large uniformly polarizing film was not obtainable. The film of iodine was thicker, 0.012 mm., but was still transparent. It was introduced between the source and the slit F_1 of the collimator.

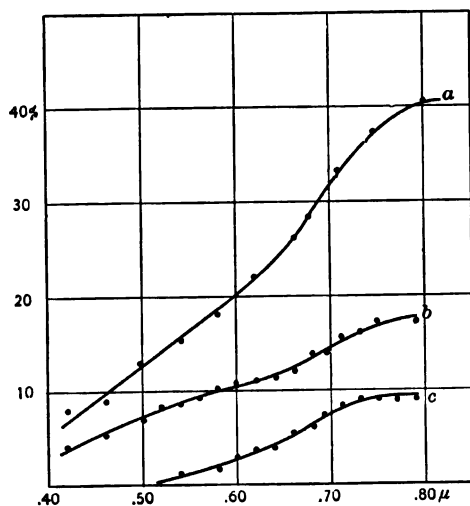


Fig. 19.

An examination of the transmission curves, Fig. 19, shows that for maximum extinction (curve c), where the color of the film is tinged with red, the absorption becomes complete at the end of the yellow. On the other hand, for minimum extinction (curve b) light is transmitted throughout the spectrum.

Curve a gives the transmission of ordinary light through the same film.

These curves are quite similar to those obtained for the alcohol solutions.

In the infra-red the absorption is interchanged. Here, for minimum extinction, the transmission is less (curve, b') throughout the spectrum than for maximum extinction (curve c). In Fig. 20 curve d gives the absorption of unpolarized light through a film 0.015 mm. in thickness, while b' and c' show the absorption of the polar-

ized rays. In each case the curve has been corrected for the absorption of the glass.

The curves in Fig. 20 are connected by dotted lines simply to show their trend. This is permissible since both were transparent so that

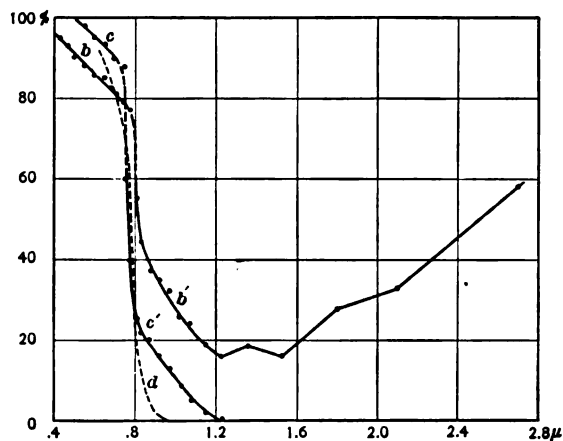


Fig. 20.

the position of maximum and minimum extinction could be distinguished. For a proper comparison, the curves must be corrected for thickness, which, on this scale, would make those for the visible spectrum quite insignificant.

Extreme accuracy was not possible by this method. As indicated by Merritt¹ the action of stray light from the prism must be eliminated by using two spectrometers in series. Since the radiometer is fixed, such an arrangement is not so easily used as with a bolometer. The loss of polarized light at the prism could have been avoided by placing the polarizer and the film of iodine between the second mirror and the radiometer slit. This likewise was not practicable on account of the shifting of the spectrum, due to inaccuracies in setting the apparatus, such as turning the polarizer. Since all the films used were so dense that nothing could be determined about the position of the axes of the crystal, it seemed sufficient to arrange the apparatus as indicated.

The irregularities in the curves are, at least in part, due to errors in making the observations, and since they are not very marked, it

¹L. c., p. 433.

may be assumed that stray radiations did not seriously affect the results.

This is confirmed in part by a comparison with the absorption of ordinary light, by a slightly thicker film (curve *d*, Fig. 20), which becomes transparent at 1.1μ . These curves, Fig. 20, have been corrected for the absorption of the glass and the Foucault prism.

IV. ANOMALOUS DISPERSION OF SOLID AND LIQUID IODINE.

The remarkable absorbing power of iodine vapor was discovered by Le Roux¹ in 1860. He found that only the red and violet rays were transmitted, and that of these the red is the more refracted, contrary to what takes place in the ordinary cases of refraction.

The indices of refraction for the red and violet rays for iodine vapor were measured by Hurion² in 1878. He found for the red, *C* line, $n = 1.0205$, and for the violet, *C*₁ line, $n = 1.019$. From these, assuming that the refracting power of a body is independent of its physical state, he found for solid iodine, $n = 1.89$ for the red, and $n = 1.83$ for the violet.

In 1886 Dazebrink³ found the anomalous dispersion of iodine in CS₂.

1. *The Indices of Refraction of Solid Iodine.*

In finding the refractive indices for solid iodine, the chief difficulties consisted in obtaining plates sufficiently plane-parallel, and in making prisms which were transparent enough, yet at the same time free from rills due to contraction and crystallization.

The size of the glass plates was about 2 x 5 x .5 cm. They were made from a good quality of plate glass, which had a slight yellow tinge.

The prisms were made by placing several large iodine crystals between two plates, and clamping one edge (previously ground straight), with a flat-jawed metal clamp, having a long handle.

The wedge thus formed was held above a Bunsen flame, and when melting began, a thin piece of cardboard was inserted in the open edge, and the whole clamped tight. It was found that cooling

¹ Le Roux, *Ann. de Chemie et de Physique* (3), tom. LXI., p. 285 (1861).

² Hurion, *Jour. de Physique*, 7, p. 181 (1878).

³ Dazebrink, *Beiblätter*, Bd. X., p. 493.

the whole quickly in cold air gave the best results. In all this work the glass plates had to be perfectly clean in order to get a thin edge which was free from holes or rills. Unlike cyanine, iodine is brittle, so that it was impossible to break off the glass and yet keep a smooth refracting edge. In addition to this, evaporation must be contended with. This, however, was of minor importance since the work was performed during zero weather.

By proceeding as just indicated a prism could be made very quickly, and after several hundred trials five were obtained, of different sizes and having different angles and refracting qualities. The angles of the prisms were from 8' to 30'. The refracting edges were from .5 cm. to 2.5 cm. in length.

For producing the spectral lines to be measured, cored carbons, filled with the salts of Li, Na and Th elements, were used. An Arons mercury lamp was also used. The spectrum was produced by means of a flint-glass prism.

TABLE X.

Prism.	Angle, α .	Deviation, δ .	n for Li_2 .
1	30'.2	28'.5	1.95
2	27'.5	29'.3	2.07
3	24'.0	24'.5	2.06
4	8'.4	7'.9	1.94

The refractive index was measured by placing the iodine prism between the one of flint glass and the observing telescope. A portion of light thus passed through the glass undeviated, while another part passed through the iodine and was deviated. Two images were therefore visible in the telescope, and by means of the micrometer eye-piece their distance apart could be measured. Then from the known length of the telescope, 24.5 cm., and the calibration of the filar micrometer, 1 turn = .243 mm., the angular deviation was computed. This gave the tangent of the angle. The angle of the prism was found by measuring the thickness of the wedge inserted, and its distance from the refracting edge. The error introduced by so doing was small, since the thickness of the wedge inserted could be measured quite accurately, while its dis-

tance from the refracting edge was great in comparison, and a slight variation in the latter affected the results in the third decimal place. The indices of refraction were computed from the well-known formula $n = (a + \delta) \div a$ and are given in Table X.

The large variation in the refractive indices is to be explained in part by the fact that in prisms 1 and 4 the wedges were of paper, which is difficult to measure, *e. g.*, in No. 4 the thickness was only 0.106 mm. In prisms 2 and 3 the wedges were of hard material which kept a sharp edge.

Furthermore, the transparency and definition must be considered. Thus No. 3 had a refracting edge of about 2.5 cm. and gave excellent definition. In No. 2 the edge was of about the same length but was full of air bubbles, and its definition was poorer. The others were full of air bubbles and rills, while the refracted band was blurred for No. 4, and almost invisible for No. 1. A better result for solid iodine is given on a subsequent page.

2. *Liquid Iodine.*

The hollow prism for the liquid iodine was made by grinding and polishing a wedge in a piece of plate glass by the same method as for the absorption cell. The refracting edge was quite straight, as shown by the straight interference bands, and although not highly polished, the transmitted spectral lines were well defined. In fact, a more highly polished prism was rendered useless on account of the lack of planeness caused by excessive polishing. In making the hollow wedge the plate of glass is rendered prismatic. This causes a deviation of the spectral lines in the opposite direction to that by the iodine prism. Hence the deviation of each line by the empty prism was carefully measured, and added to that produced by the iodine prism.

The same spectrometer arrangement was used as in the preceding work. The angle of the prism was computed from the measurement of the distance between the deviated and undeviated So-

light from the collimator slit, one from the refracting edge, the other from the plane glass. The distance between them, measured with the filar micrometer, gave the tangent of twice the angle. The mean of 21 readings gave an angle of $4'.43$, which is in close agreement with the preceding measurement. This cell was also used for the indices of refraction of solid iodine.

The results for both solid and liquid iodine are given in Table XI. The same formula was used as in the preceding work. The refractive indices of the Li_α , Li_β and Na lines are given for the value of the angle α_1 of the prism obtained by reflection and by refraction. In II. the refracting edge was much denser and the Li_β line could not be seen.

TABLE XI.

I. Telescope, focal length 24.5 cm.

	Deviation of Glass Wedge. Each a Mean of 10 Readings.	Deviation of Liquid Iodine. Mean of 26 Readings for Each Line.	Total Deviation = δ .	Liquid.		Deviation of Solid Iodine. Mean of 23 Readings.	Solid. n by Reflection.
				n by Reflection.	n by Refraction.		
Li_α .	.126 mm.	.216 mm.	$4'.67$	2.05	2.07	.217 mm.	2.07
Li_β .	.151	.170	$4'.37$	1.99	2.00	.170	2.00
Na .	.153	.161	$4'.27$	1.96	1.98	.160	1.96

II. Telescope, focal length 24.3 cm.

Li_α .	Mean of 7 readings.	$4'.81$	2.07	2.08
Na .	Mean of 11 readings.	$4'.36$	1.98	1.99

It is interesting to note in the table that the deviation δ for the solid iodine is the same as that for the liquid, although the readings were made exactly a week apart, the iodine having been melted several times during this interval.

Considerable difficulty was experienced in obtaining a solid prism by this method, on account of the small angle which prevented an even crystallization of the solid iodine.

The sources of error here are the determination of the angle of the prism at the refracting edge, and the rigidity with which the two plates are held together. Unlike cyanine, which is somewhat viscous when melted, the iodine flows freely like water, and, in order to prevent its going beyond the refracting edge, the plates must be subjected to considerable pressure. Thus, in one series of measure-

ments it was found that all the indices were very much smaller than those obtained previously. An examination of the prism showed that the iodine had forced itself between the plates, and decreased the effective refracting angle.

The green thallium band could not be measured on account of the absorption of the prisms. An Arons lamp was used as a source for the yellow Hg lines, which appeared very faint. Only a few readings were made, which were very uncertain, and gave a refractive index of 2.08. An attempt was made to use polarized light (Foucault prism), but no successful measurements were obtained on account of the increased absorption.

Professor A. C. Gill, of the Department of Mineralogy, kindly examined several films of solid iodine. All but the one used in III. were too thick for the purpose. This one was extremely thin. Assuming the thickness to be 0.005 mm. and that the dispersion does not change with the normal order of the interference colors, the double refraction ($\gamma - \alpha$), is at least 0.5.

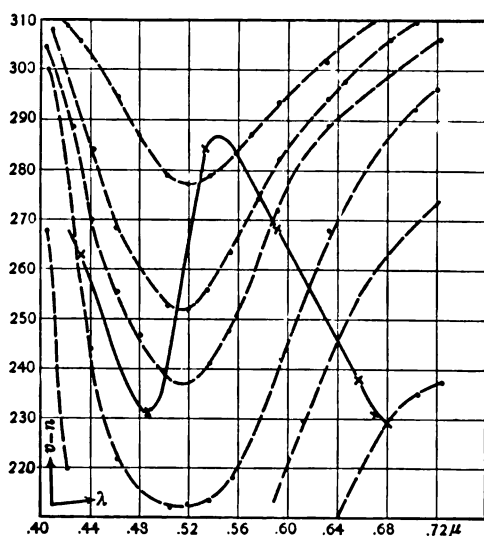


Fig. 21.

In Fig. 21 the curves show the relation between the transmission and anomalous dispersion of CS_2 solutions of iodine. The ordinates represent the differences of the refractive indices, ν and n , of the

iodine solutions and of carbon bisulphide. The data for the dispersion curve are due to Dazebrink, and have been taken from Wüllner's *Experimental Physik*, Vol. 4, p. 202. According to Dazebrink¹ the shifting of λ_m for the different concentrations, is of no consequence, is often opposed to theory, and is also to be found in solutions of colorless salts.

The values of the refractive indices agree well with those found by Hurion.² He assumed that the refractive power of a mixture is the sum of those of its constituents (Wüllner's formula),³ and found for solid iodine :

$$n \text{ for red, } C \text{ line,} = 2.07.$$

$$n \text{ for blue, } G \text{ line,} = 1.98.$$

Since this formula holds only where the constituents have undergone no chemical change, one would infer that the CS_2 solutions of iodine have that property, and that the change in the transparency of iodine in the solid state and in solution is yet to be explained.

It will be noticed that the refractive indices are much smaller when computed by Hurion, by assuming the refracting power of a body to be independent of its physical state. Since at a high temperature iodine vapor dissociates from I_2 into 2I it is possible that the discrepancy is due to this fact.

SUMMARY.

1. The absorption of iodine in solution of carbon bisulphide and ethyl alcohol was measured to $\lambda = 2.7 \mu$. For the visible spectrum a horizontal slit spectrophotometer was used, while the infra-red was explored by means of a radiometer.

In the visible spectrum the CS_2 solutions have a strong absorption band, maximum at about 0.515μ , while that of the alcohol solution lies in the ultra-violet. The latter solution becomes dichroic for weak concentrations.

All the CS_2 solutions become transparent beyond 1.5μ , while the

2. The transmission of solid iodine was found to 13μ . Two absorption bands exist, one in the visible spectrum, the other at 7.4μ .

Liquid iodine was explored to 2.7μ . It was found that its absorption does not depend on the temperature, but is less than that of solid iodine. A comparison of the thickness of iodine, as a film, when in solution of CS_2 , and as a solid film shows that the total quantity of iodine in a saturated solution would make a film almost ten times thicker than the thinnest, transparent, solid film. Yet the solution is transparent, while the same thickness of iodine, in a solid film, is quite opaque. For the long waves the relation of transmission to thickness of solid iodine fulfils the law for optically perfect media with a degree of approximation equal to that of the knowledge of the thickness of the films used.

3. The two polarized rays are unequally absorbed. One is more transparent to the visible rays, while the other is more transparent in the infrared.

4. Like the vapor and the CS_2 solutions, solid and liquid iodine shows anomalous dispersion. The indices agree well with those computed from solutions for spectral lines lying close to those measured, and are the same for iodine in a solid and a liquid state.

This investigation was made under the direction of Prof. E. L. Nichols to whom the writer expresses his obligations for the interest he has taken in the work, his timely suggestions and the liberal supply of apparatus made accessible. He is also grateful to Prof. E. Merritt for his friendly criticisms during the various phases of this investigation.

THE PHYSICAL LABORATORY OF
CORNELL UNIVERSITY, May, 1902.

THE RADIANT EFFICIENCY OF THE MERCURY ARC.

BY WILLIAM C. GEER.

I N the year 1860 Way¹ observed that an intense light was produced when an electric current was allowed to flow through a fine thread of mercury which was falling from an upper reservoir to a lower one. He had this "mercurial electric light" on the mast of a yacht off the Isle of Wight where its brilliant radiation was observed from a distance of many miles. The newspapers of the day commented upon its intensity and suggested it as a means of artificial illumination.

The interest thus aroused in the discovery naturally stimulated investigation and as a result many patents² have been issued, in this and other countries, whose main specifications describe a lamp for obtaining an arc between mercury terminals. Rapiéff in 1879 took out a British patent (No. 211, 1879) for an inverted U-tube having the ends dipping into mercury, and the arc playing either in air or in a vacuum. Rizet in 1880 obtained a French patent (certificate d'addition of March 20, 1880, for Rizet's patent No. 132,426 of August 27, 1879). The design was similar to the above while the U-tube is filled with nitrogen. Langhans in 1887 obtained a German patent (No. 45,880) for a U-tube with legs filled with any metal or metalloid. By far the most interesting and thorough attempt to render this principle of commercial value has been made by Cooper-Hewitt³ with what he calls a mercury "vapor lamp." He has patented a large number of designs.

None of these mercury vacuum tubes has proved to be practicable as a means of artificial lighting and consequently they have failed of commercial success. However, the mercury arc has rendered valuable service to pure science. The green lines, produced by

¹ Chem. News, 2, 167, 1860.

² von Recklinghausen, Zeit. f. Elektrotech., 23, 492, 1902.

³ Cooper-Hewitt, Elect. World and Eng., 37, 679, 1901; 38, 503, 1901; 39, 80, 1902.

this means, are employed in spectrometric work as a source of monochromatic light of high intensity which can be maintained for a long time, when properly cared for, and which is comparatively easy to produce. The device commonly used in the production of the arc is some form of the tube, or "lamp," described by Arons¹ in 1892 and in 1896. Many slightly different designs have been made by Lummer² and others. These lamps all embody the same principles, viz., a U tube of some sort having the two legs filled with mercury, through the ends of which platinum wires are sealed, the whole being exhausted as far as possible.

From the published results of the work of Arons³ and the fact that the arc gives out such a brilliant light, one would infer that the radiant efficiency, or the ratio of the luminous radiation to the total radiation, must be large. The following paragraphs will describe the results of an attempt to measure the efficiency of that part of the radiation of the mercury arc which was transmitted through the glass tube within which it was formed. Any radiation from the glass itself was carefully corrected for and left out of account in the calculations.

The vacuum tube used, Fig. 1, was of the form designed by Arons. The distance between the mercury terminals was about 2 centimeters while the arc itself measured some 5 centimeters. The electrical energy was supplied by a 110-volt direct-current circuit.

The measurements were made according to the method described by Professor Merritt⁴ and frequently used in this laboratory. The calculations of the efficiency were based on the deductions of Professor Nichols,⁵ who found that the radiant efficiency of a source of luminous energy is approximately expressed by the equation,

$$E = \left(1 - \frac{e'}{a'}\right) \frac{a'}{A}$$

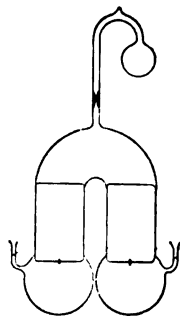


Fig. 1.

in which A is the total energy of the source being measured, a' is the energy transmitted by a cell of distilled water, and e' is the energy transmitted by a cell of water and a cell containing a saturated solution of iodine in carbon disulphide.

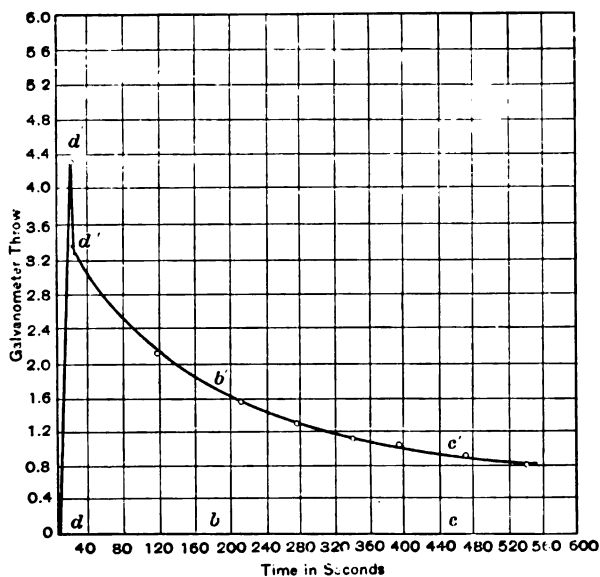
The radiation from the lamp was allowed to fall upon the face of a thermopile and the first throw of the galvanometer read.¹ This reading is proportional to the total radiation, A . The throw was then observed with a cell of distilled water interposed between the lamp and the thermopile, which gave the reading proportional to the luminous radiation a' . Since a portion of the longer waves is transmitted by the water, the correction reading e' was taken by observing the throw when a cell, containing a saturated solution of iodine in carbon disulphide, was interposed together with the water cell.

The value of A , taken under these conditions, does not represent the total radiation of the arc alone, for it was found that the glass tube, in which the arc was playing, became so highly heated that it emitted a very considerable radiation of its own. Even after so short a run as one minute the mercury boiled in the tube and there was danger of its destruction. The galvanometer throw, then represented the true total radiation of the arc, A , plus a large radiation of the heated glass, which may be designated d'' . It was found, therefore, that no true value of A could be obtained directly by means of the apparatus at our disposal.

When one considers a curve which shows graphically the energy, in terms of galvanometer throws, that the lamp is radiating from the time the arc is started until it is extinguished and the glass has cooled nearly to its original temperature, a ready method of finding the value of A becomes evident. Such a curve is seen in Fig. 2. The arc was started with the lamp at or near the room temperature. The radiation rapidly increased in intensity, until at the end of 30 seconds it was measured by the throw d' . On the instant that this throw was reached the circuit was broken, the arc extinguished and the time of the observation recorded. This throw measured the arc radiation, A , plus that of the glass, d'' . While the lamp was cooling several successive throws were taken and the time of each recorded. Thus the remainder of the curve, $b'c'$, is a time curve

¹ Merritt, American Journal Science [3], 41, 417, 1891.

of cooling of the hot glass. By extrapolation of this cooling curve to the time d an ordinate, d'' , is found which evidently measures the energy of the hot glass at that time. Since the arc was broken



at the time d , the difference between the ordinates d' and d'' is the throw proportional to the arc radiation alone, which is the desired quantity A .

The full detail of the method is seen by a reference to Fig. 3. The frame resistance consisted of four tin-strip resistance frames which were used to regulate the current for the different determinations. The lamp was mounted in the position shown in two holes in a block of wood, one end of which was fastened to the table by means of a small hinge. It could then be tilted, by means of a cord, from the observer's position at the galvanometer telescope. The mercury then flowed from one leg to the other, thus striking the arc. The screen was of sheet iron, which also was controlled from the observer's position. The iodine solution and water were contained in plane glass cells one and one half centimeters in thickness. The thermopile, placed 25 centimeters from the lamp, was a 24-couple instrument connected to a d'Arsonval galvanometer,

Upon the table which supported the telescope were the ammeter, the voltmeter and the key for operating the chronograph relay. This key, the chronograph relay, the chronometer, which was beating two-second intervals, and the circuit breaker were connected as shown. In order to determine A , the chronometer and chronograph were put in operation and the iodine and water cells removed. The lamp was then tilted, thus striking the arc, the ammeter and voltmeter read, while as soon thereafter as possible the screen was drawn away and the first throw of the galvanometer observed. On the instant that this throw was read the key was pressed, thereby actuating the chronograph relay, which recorded the time of the observation, and actuating the circuit breaker which simultaneously

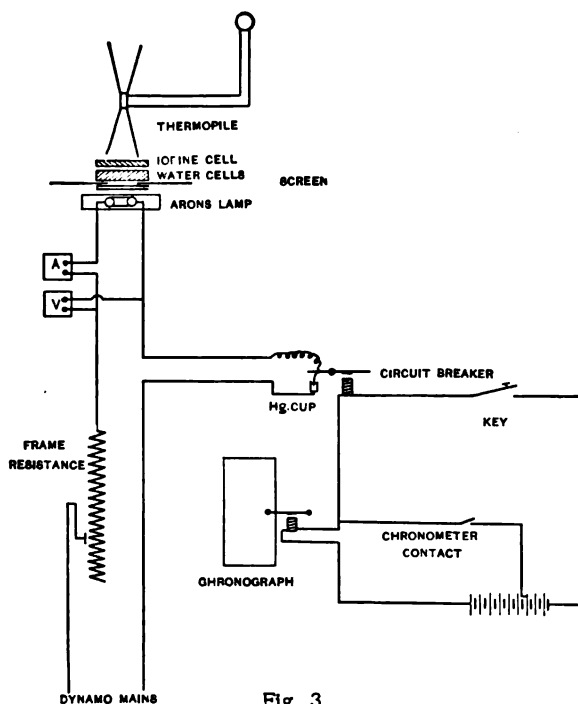


Fig. 3.

extinguished the arc. As the lamp cooled several successive throws were read, for each of which the time was recorded by pressing the key. By this means the time curve of cooling was obtained, one of which has been shown in connection with Fig. 2. On extrapola-

tion to the ordinate axis d'' was obtained. The difference between this value and d' , which was the throw actually observed, gave the quantity A which was the throw proportional to the radiation of the arc alone.

No difficulty was experienced in obtaining the readings with the water cell nor with the iodine cell. When these were being read the heat waves, except the shorter lengths, were absorbed by the water. These three quantities, A , a' and e' , were determined for several currents between five and nine ampères and the efficiencies for each current strength calculated therefrom. In making the readings the plan followed was to make several determinations of A , then several of a' and of e' and finally several additional determinations of A . The mean values of a' and e' were used with each value of A in calculating E from the equation given above. Lower currents were tried but the arc would not maintain during the time necessary to make a reading. With higher currents the lamp became so highly heated that the results were wholly unreliable and consequently were rejected.

An annoyance which was of common occurrence was caused by the arc going out before the first throw was read. This was especially frequent when working with the lower currents. In order to be certain that the arc was playing throughout each throw a small mirror was mounted near the telescope of the galvanometer by means of which the light from the lamp was reflected into the free eye. The breaking of the arc was at once recognized and the readings repeated until satisfactory.

TABLE I.

Time.	Seconds.	Galvanometer.		d	$1/d$
		Zero.	Throw.		
11:47:49	000	35.36	39.72	4.36	
49:28	99	35.60	37.72	2.12	0.472
51:1	192	35.50	37.06	1.56	0.640
52:6	257	35.60	36.90	1.30	0.769
53:9	320	35.60	36.73	1.13	0.885
54:6	377	35.60	36.62	1.02	0.980
55:22	453	35.50	36.42	0.92	1.088
56:16	528	35.60	36.40	0.88	1.220

The full data for one determination of E is given in Table I. When the reciprocals of the galvanometer throws, $1/d$, are plotted as ordinates and time as abscissas the curve of cooling is found to be a straight line from which the extrapolation is easily accomplished. The curve from the data given is shown in Fig. 4, while

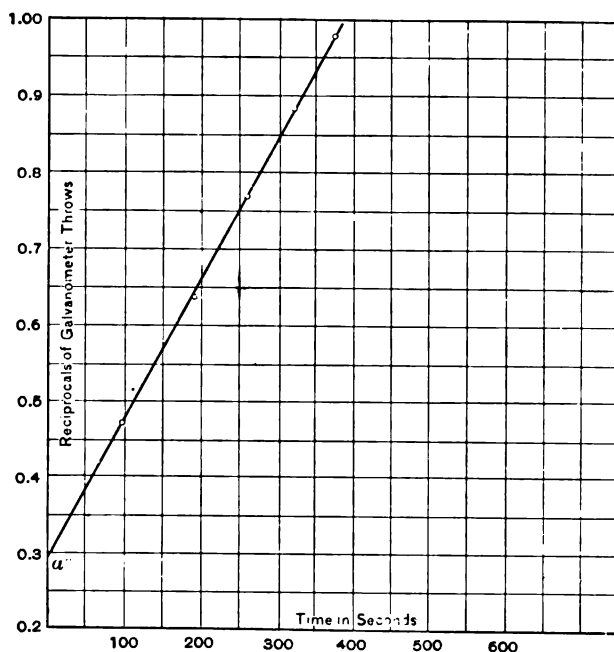


Fig. 4.

the complete curve has already been discussed, since it is seen in Fig. 2. The extrapolated intersection with the ordinate axis a'' , is the reciprocal of d'' , the throw due to hot glass at the instant that the arc is extinguished. A is the difference between the throw first obtained and this value d'' . E is then calculated from the equation.

A summary of all the results is to be found in Table II. The separate determinations are seen to vary considerably even at the same current. This variation may be due to several causes. In the first place the arc was extremely unsteady, the cathode being in constant motion. Arons¹ states that the arc is a discontinuous

¹Wied. Ann., 47, 768, 1892.

TABLE II.

Volts.	Amp.	$A + d''$	a''	A	a'	d'	$a' A$	E
14.7	5.06	4.53	3.18	1.35	0.534	0.030	0.396	0.374
14.7	5.06	5.66	4.44	1.22	"	"	0.438	0.414
15.2	5.08	5.72	4.54	1.18	"	"	0.453	0.428
14.5	4.98	4.87	3.77	1.10	"	"	0.485	0.458
14.9	4.98	5.12	3.76	1.36	"	"	0.393	0.371
14.7	5.00	4.68	3.64	1.04	"	"	0.513	0.484
14.7	5.03	4.59	3.59	1.00	0.456	0.0375	0.456	0.419
14.5	4.75	4.36	3.38	0.98	"	"	0.465	0.427
14.5	5.11	5.70	4.65	1.05	"	"	0.434	0.398
14.7	5.02							0.419
14.2	6.03	5.06	4.00	1.06	0.535	0.030	0.505	0.477
14.5	5.79	5.10	4.08	1.02	"	"	0.524	0.494
14.4	5.97	5.20	3.76	1.44	"	"	0.372	0.351
14.3	5.98	4.13	2.95	1.18	0.523	0.040	0.443	0.409
14.5	6.09	4.56	3.35	1.21	"	"	0.432	0.399
14.5	6.35	5.10	4.00	1.10	"	"	0.476	0.440
14.4	6.03							0.428
14.4	7.05	7.57	5.88	1.69	0.623	0.0375	0.375	0.353
14.5	7.00	5.49	4.00	1.49	"	"	0.426	0.401
14.6	7.23	5.86	4.12	1.74	"	"	0.365	0.344
14.6	7.15	6.39	5.13	1.26	"	"	0.504	0.474
14.5	7.21	5.57	3.92	1.65	"	"	0.385	0.362
14.6	6.99	4.94	3.54	1.40	"	"	0.453	0.427
14.8	7.14	5.43	3.95	1.48	"	"	0.429	0.404
14.5	6.98	4.52	3.17	1.35	"	"	0.470	0.442
14.4	7.13	4.50	3.12	1.38	"	"	0.460	0.433
14.7	7.01	5.67	4.24	1.43	"	"	0.444	0.418
14.6	7.09							0.406
14.3	8.07	4.60	3.40	1.20	0.678	0.035	0.555	0.526
14.5	8.15	5.47	4.00	1.47	"	"	0.461	0.437
14.6	8.07	6.82	5.21	1.61	"	"	0.421	0.400
14.5	8.16	4.55	2.70	1.85	"	"	0.366	0.347
14.5	8.20	5.30	3.61	1.69	"	"	0.401	0.380
14.5	8.07	5.26	4.03	1.23	"	"	0.552	0.523
14.4	8.07	6.37	4.95	1.42	"	"	0.477	0.452
14.5	8.11							0.438
14.6	8.97	5.97	4.44	1.53	0.826	0.050	0.540	0.507
14.7	9.20	7.66	6.17	1.49	"	"	0.554	0.520
14.7	9.13	6.55	4.78	1.77	"	"	0.467	0.438
14.6	8.97	9.43	7.94	1.49	0.750	0.050	0.504	0.470
14.7	8.97	5.44	3.92	1.52	"	"	0.493	0.461
14.7	9.04							0.479

one, basing his conclusion on the sound that is given by a telephone receiver when properly connected. The writer has observed that when one places his ear close to the lamp an unsteady sound of low pitch can be distinctly heard. This unsteadiness is undoubtedly one of the causes for the variation in the efficiency determinations. Then again, the mercury which distilled from the terminals condensed upon the cooler sides of the tube forming an irregular layer which absorbed considerable energy. Furthermore, the temperature of the tube at the time when the throw of the galvanometer was read varied a good deal with the different determinations, even in spite of great care. Since the total electrical energy¹ used has been found to change with the temperature of the tube, it is quite probable that the efficiency may be influenced by this cause. Also the temperature of the tube would influence the rate of cooling and as a consequence it would be possible that the extrapolated values of the hot glass deflections may be in error in some cases. These errors could not be corrected with the apparatus at hand. The mean of a large number of determinations is doubtless quite near to the true efficiency.

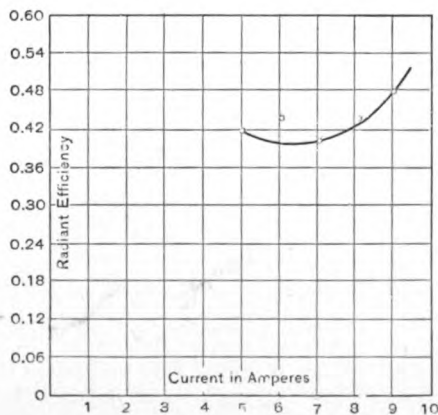


Fig. 5.

creases more rapidly than that of the invisible part. This does not accord with the work of Ångström on the intensity of the radiation from vacuum tubes who said¹ that "bei constantem Druck ist für ein und dasselbe Gas die Strahlung der Stromstärke proportional, und weil dies nicht nur für die Gesamtstrahlung, sondern auch für einen bestimmten Theil derselben gilt, so ist die spectral Vertheilung der Energie bei constantem Druck von der Stromstärke unabhängig." To be sure, the case of the Arons lamp is not exactly similar to a vacuum tube. The condition here is that of a saturated vapor in contact with its liquid phase, so that with the changing temperature the pressure in the tube changes as well. As the current is increased the temperature of the electrodes should increase and consequently the pressure in the tube should rise. According to the authority quoted the efficiency falls² with an increase of the pressure. If our measurements are correct the efficiency rises slightly under these conditions. In any event there is a field here which merits further and more thorough investigation.

The following are a few of the more common light sources and their radiant efficiencies, which were measured by this general method:³

Source.	Observer.	Efficiency.
Illuminating gas, argand burner.	Rogers.	0.0161
Incandescent electric lamp.	Merritt.	0.060
Arc light.	Tyndall.	0.104
Acetylene.	Stewart and Hoxie.	0.105
Geissler tubes.	Staub.	0.320
MERCURY ARC.	Geer.	0.409 to 0.479

From these figures and the full data given in Table II., it is seen that the mercury arc is quite remarkable. It has the highest known radiant efficiency. It is to be remembered, however, that these results indicate the efficiency only of that part of the radiation from the mercury arc which was transmitted through the glass tube within which the arc was formed.

¹ Wied. Ann., 48, 509, 1893.

² Wied. Ann., 48, 530, 1893.

³ Nichols, PHYS. REV., 11, 221, 1900.

When one recalls that the spectrum of mercury is a line spectrum it is to be inferred from the efficiency values that some well-defined lines or bands are to be found in the region of the infra-red. Indeed the high efficiency here recorded may in part be due to the presence of energy of such large wave-length that the glass walls of the tube absorbed the waves, and, therefore, they were not taken account of in this method of determining the efficiency. The high heating of the walls of the tube may, at least to a slight extent, be ascribed to this cause. Several lines were predicted by Kayser and Runge,¹ but they have not been found as yet, and, so far as the writer is aware, nothing has been written which includes a systematic study of this portion of the mercury spectrum. In the near future it is purposed to publish the results of an investigation in this field.

In conclusion, it is with pleasure that sincere thanks are expressed to Prof. E. L. Nichols and to Prof. E. Merritt for the encouragement and many valuable suggestions which were given in the course of this work.

CORNELL UNIVERSITY, ITHACA, N. Y.,

November 15, 1902.

¹Wied. Ann., 43, 405, 1891.

THE CHANGE OF VOLUME IN CLARK AND CADMIUM CELLS AND ITS RELATION TO CHANGE OF ELECTROMOTIVE FORCE DUE TO PRESSURE.

BY ROLLA R. RAMSEY.

WHEN a Clark or Cadmium cell is put under pressure its E.M.F. is increased, as I have shown in a previous paper.¹ Since if the current is sent backwards through the cell a greater amount of energy is required to maintain the current constant in the case when pressure is applied, than that required when the cell is not under pressure, one would be led to expect that the volume of the cell was increased or that energy was used up in overcoming pressure.

Gilbault² has calculated this change in certain cells by means of a large number of measurements of densities of different per cent. solutions of the salt contained in the cell. It seemed to me to be interesting to measure the actual change and to compare this value with the value obtained by theoretical calculation.

Theoretical.—Suppose a cell whose E.M.F. is E , at zero pressure to be placed under a pressure, P say, and the E.M.F. be found to be increased by an amount, e . If in the first place a current i is allowed to pass through the cell the energy developed is

$$Eit,$$

or

$$EQ.$$

where Q is the quantity of electricity.

The same amount of energy would be used up in overcoming the E.M.F. of the cell if the same quantity of electricity were forced backwards through the cell. Place the cell under pressure

¹ PHYS. REV., Vol. 13, p. 1, 1901.

² Lum. El., Vol. 42, pp. 7, 63, 175, 220, 1891.

and force the same quantity backwards through the cell and the energy used in overcoming the E.M.F. of the cell is,

$$(E + e)Q,$$

or

$$EQ + eQ.$$

Since the first term is the energy required to overcome the ordinary E.M.F. of the cell, the second is the extra required due to the fact that the cell is under pressure, or the amount used up in external work in overcoming pressure (unless the nature of the electrolyte is changed by the pressure). This would require a change in volume. The external work in overcoming pressure would be,

$$Pdv.$$

This quantity should equal the added quantity of energy

$$Pdv = eQ.$$

It will be apparent that when e is positive the volume of a cell will increase when current is forced backward through the cell of will decrease when allowed to flow in the usual manner.

Knowing e , Q , and P , dv can be calculated.

For Clark cell :

Let $e = 11.6 \times 10^{-6}$ volts or 1,160 C.G.S. units, the increase of E.M.F. per atmosphere pressure.

$$Q = 1 \text{ C.G.S. unit quantity.}$$

$$P = 1 \text{ atmosphere, or } 1,050,000 \text{ dynes.}$$

$$dv = \frac{eQ}{P},$$

$$= \frac{1,160 \times 1}{1,050,000} = .0011 \text{ cu. cm. per C.G.S. unit quantity,}$$

$$\text{or } dv = .11 \text{ cu. millimeter per coulomb.}$$

For cadmium cell :

$$e = 7.6 \times 10^{-6} \text{ volts.}$$

$$dv = .072 \text{ cu. millimeters per coulomb.}$$

Apparatus and Methods. — The method of measuring the change of volume was to observe by means of a cathetometer, the rise or fall of the liquid in a capillary tube which was attached to the cell. The cell was connected in series with four to seven gravity cells and an instrument for measuring current. The cell was made of glass tubing in the **H** form, Fig. 1. The **H** was blown and the electrodes placed as directed, in the case of the Clark cell, by Kahle;¹ in the case of the cadmium cell by Jaeger and Wachsmuth.² To one stem of the **H** a stop-cock was fused to the other a capillary stem of known cross-section. (In later cells the stem was heated and drawn to a capillary and the diameter measured afterwards by cutting the tube in two and placing under a micrometer microscope.) The cell was now filled through the stop-cock with the saturated electrolyte. By means of the stop-cock the electrolyte could be kept at any desired height in the tube. The cathetometer was one made by the Société Genevoise reading to $\frac{1}{80}$ millimeter. For current measurement a tangent galvanometer was used at first. In the later observations a Weston mil-ammeter was found to answer all requirements. Several forms of voltmeters and electrolytic ammeters were tried but were found to be useless for the work.

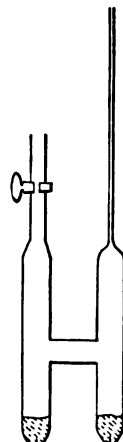


Fig. 1.

The apparatus was set up in a basement room. A thermostat was not attempted. To avoid rapid changes of temperature the cell was immersed in a bath of coal oil, which was immersed in a six-gallon jar of water. The jar was placed in a wooden box and packed with excelsior. By this means the temperature changes were not rapid, but there were variations of a degree or more from day to day. Attempts were made at first to run the current through the cell several days at a time and thus to get a large change in volume. This failed for two reasons. After the current had been running for some time, from a half hour to three hours, depending upon the size of the electrodes, the cell would polarize,

¹ Wied. Ann., 51, p. 203, 1894.

² Wied. Ann., 59, p. 575, 1896.

rendering the current intermittent, making the readings of the ammeter uncertain. Electrolytic ammeters were substituted, but the observations were uncertain on account of the temperature variations. Even after the coefficient was known the largest calculated change of volume fell within the limits of errors of the temperature coefficient. In one case the cell was packed in ice but the index gradually rose in the tube. This change was comparable to the change due to the current (the current strength being very small after polarization of the cell). The probable cause of this apparent change of volume was the aging of the glass of which the cell was composed. The best results were found to be obtained by short runs with the temperature as nearly constant as possible. Since the changes of volume were small and the temperature variable, the accuracy was not as great as was hoped to be obtained.

The readings were taken in the following order: The reading of the cathetometer was noted, the current started and readings taken at stated intervals, every minute or two. As soon as the ammeter began to show fluctuations the current was broken and the height of the index again noted by means of the cathetometer.

TABLE I.
July 26. Cell No. IV.

Time.	Temp.	Height.	Current in Milliamperes.
5.42 p. m.	0°	35.344	1.5
44			1.53
46			1.48
47		35.260	
48			1.4
49		35.240	
50			1.39
52			1.37
55		35.180	1.32
58			1.2
59		35.100	1.0
$dh = 2.44$	millimeters.	$dv = .254$ cu.	millimeter. $Q = 1.48$ coulombs.

Table I. contains a set of readings which will serve as as ample. The quantity of electricity was obtained from the area of a curve plotted with time as abscissas and current as ordinates. The current always

fell on one half or less of its initial value before the polarization fluctuations set in. These fluctuations were quite erratic. At times the galvanometer would drop to zero for an instant, even with seven cells on. In one case a large number of cells were connected on and the volume change became erratic. Hydrogen bubbles were found in the cell afterwards. The probable cause of the polarization is the small solubility of the depolarizer, mercurous sulphate. The solubility of mercurous sulphate being, according to Dolezalek,¹ .05 per cent. The E.M.F. of the cell as shown by means of a potentiometer was lowered when polarized. In the case of a Clark cell 1.417 volts at one time. After standing a few hours the cell regained its initial E.M.F. In one case after a continuous run of a week, upon resting 24 hours a hasty comparison showed the E.M.F. to be 1.425 volts at 24.4° temperature. The E.M.F. should be 1.421 volts, assuming Jaeger's² values. The comparison was made with a cadmium cell assuming Jaeger's values as found upon the same page.

Results.—Table II. contains a tabulated list of the various sets of observations upon Clark cells.

TABLE II.

Clark Cell.

Date.	Cell.	Temp.	Cross Section in sq. mm.	dh.	dv.
Jan. 25	I	16.0°	1.11	.1 mm.	.111
April 21	II	19.6	1.11	1.54	1.72
21	II	19.6	1.11	3.02	3.37
22	II	19.6	1.11	1.2	1.34
May 31	III	21.7	.368	.76	.276
June 3	III	22.6	.368	.40	.147
12	III	23.3	.368	.16	.054
13	III	23.8	.368	.42	.154
16	III	23.4	.368	.22	.081
July 26	IV	0	.104	2.44	.254
28	IV	0	.106	.16	.017
29	IV	0	.107	.46	.049
Aug. 1	IV	0	.111	.24	.026
1	IV	0	.111	.88	.097

¹ Jaeger, Die Normalelemente, p. 10.² Jaeger, Die Normalelemente, p. 118.

TABLE II.—*Continued.*

Approx. Curr. Milliampere.	Direction.	Time.	Q Coulombs.	dv per Coulomb.	
				Obs.	Calc.
.18		81 min.	.872	.127	.11
.12		180	13.	.132	
		37	30.5	.111	
		7	5.8	.232	
6.3		7	2.68	.104	
		8	3.87	.038	
		7	3.28	.018	
		5	2.3	.067	
		3	1.31	.062	
		17	1.48	.171	
1.45		4	.276	.062	
1.15		14	.531	.092	
.635	Reversed.	8	.413	.064	
.86		10	.443	.218	
.74	Reversed.			Mean .105	

Table III. contains the work done on a cadmium cell. All the results on this cell fell below the calculated value. Judging from the general behavior of the cell I would say that this was probably due to a bubble of air in the cell. Owing to the crowded condition of the department it is necessary to tear down my apparatus, so I give my results as they now stand.

TABLE III.

Cadmium Cell.

Date.	Cell.	Temp.	Cross Section sq. mm.	dh .	dv .
Sept. 2	V	22.°6	.160	.8 mm.	.128 cu. mm.
Sept. 2	V	22.°5	.160	.14	.022
Sept. 4	V	22.°	.160	.10	.016
Sept. 5	V	21.°5	.160	.5	.080

Approx. Curr. Milliampere.	Direction.	Time.	Q Coulombs.	dv per Coulomb.	
				Obs.	Calc.
1.5		30 min.	2.37	.054	.072
1.9	Reversed.	5	.57	.039	

The observations marked "reversed" are taken with the current forced backward through the cell. The results from these observations are the same as those obtained when the current is flowing forward through the cell, showing that the effect is reversible.

PHYSICAL LABORATORY, UNIVERSITY OF MISSOURI,
COLUMBIA, MO., Sept. 16, 1902.

ON THE DIMENSIONS OF LARGE INDUCTANCE COILS.

BY JAMES E. IVES.

IT is sometimes desired to construct large coils of known self inductance. Having recently had occasion to calculate the inductance of a number of such coils, I have thought it desirable to record the results.

The inductance of a coil of rectangular cross-section may be calculated, to about one per cent., from Maxwell's formula :

$$L = 4 \pi a n^2 \left[\log_e \frac{8a}{r} + \frac{1}{12} - \frac{1}{8} \left(\theta - \frac{\pi}{4} \right) \cot 2\theta \right. \\ \left. - \frac{1}{8} \pi \operatorname{cosec} 2\theta - \frac{1}{8} \cot^2 \theta \log_e \cos \theta - \frac{1}{8} \tan^2 \theta \log_e \sin \theta \right]$$

where (see figure)

L = inductance of coil,

a = mean radius of coil,

n = number of turns in coil,

r = diagonal of the cross-section,

θ = angle made by diagonal with lower edge of the cross-section.

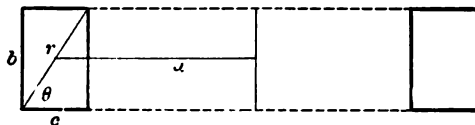


Fig. 1.

In this formula only the first term of a series is used. If greater accuracy is required the second term must be added.¹

This expression may be written in the form

$$L = 4\pi a n^2 \left[\log_e \frac{8a}{r} - f(\theta) \right],$$

¹ See PHYSICAL REVIEW, Vol. XIV., p. 297, 1902.

where $f(\theta)$ is a term depending only upon the shape of the cross-section. For a square cross-section, $f(\theta) = .83$. For a cross-section in which $\theta = 73^\circ$, $f(\theta) = .72$.

The inductances of seven coils of varying dimensions, constructed with No. 18¹ (Brown and Sharpe gauge) copper wire, were calculated by this formula, and the results are given in the accompanying table. The wire was double cotton covered, and the centers of the wires were assumed to be one twentieth of an inch apart. Coils I.-VI. were wound with 5,000 feet of wire, and coil VII. with 10,000 feet. Only two values of θ were used, viz., $\theta = 45^\circ$ and $\theta = 73^\circ$. It may be noted, that for the same θ , the cross-sections, though not all of the same size, are all similar. Coils III. and VII. have the dimensions for maximum inductance for 5,000 and 10,000 feet, respectively, of No. 18 wire. The dimensions of a coil of maximum inductance for a given length of wire are found by taking $\frac{2a}{b} = 3.7$, where a is the mean radius of the coil, and b is the length of a side of the *square* cross-section.²

TABLE I.

	Length of Wire in Feet.	Number of Turns.	a in Inches.	b in Inches.	c in Inches.	θ	Inductance in Henries.	Remarks.
I.	5,000	2625	3.13	2.75	2.75	45°	.72	Maximum* for 5,000 feet.
II.	5,000	2625	3.75	5.00	1.50	73°	.85	
III.	5,000	2230	4.36	2.36	2.36	45°	1.05	
IV.	5,000	2230	4.36	4.30	1.29	73°	.92	
V.	5,000	1940	5.00	2.20	2.20	45°	1.04	
VI.	5,000	1940	5.00	4.00	1.20	73°	.93	Maximum for 10,000 feet.
VII.	10,000	3530	5.50	2.97	2.97	45°	3.33	

From the values given in the table it is evident :

1. That a coil of maximum inductance must have a square cross-section.
2. That the inductance of a coil, with a given length of wire, increases rapidly, as the mean radius is increased, up to the maxi-

¹ No. 18 wire has a diameter of .0403 inch.

² See Maxwell, Electricity and Magnetism, § 706.

mum inductance, and then, as the mean radius is increased still further, decreases very slowly. Therefore it is better to make the mean radius too great than too small.

3. That for coils of maximum inductance, the inductance increases very rapidly as the length of wire used increases; not quite as fast as the square of the length.

UNIVERSITY OF CINCINNATI,
October, 1902.

A CONVENIENT STORAGE BATTERY INSTALLATION.

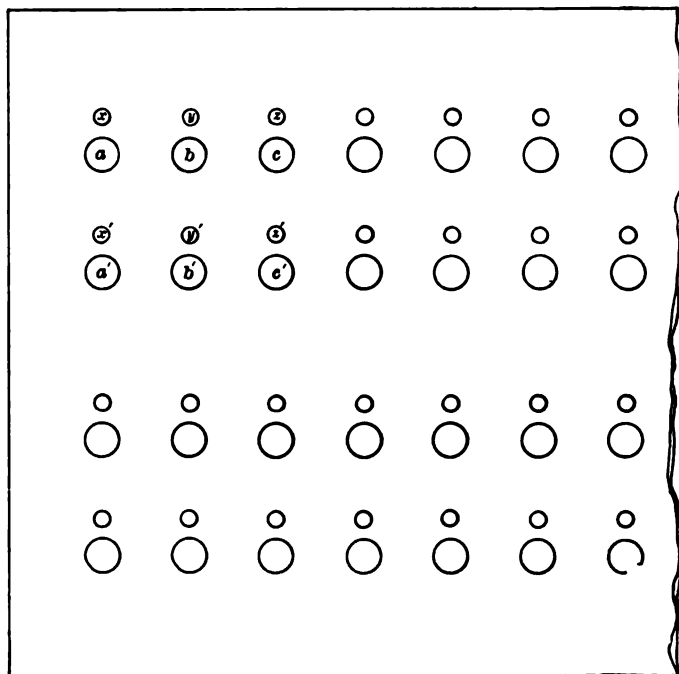
BY W. J. HUMPHREYS.

A GOOD storage battery is justly regarded as one of the most essential things in the equipment of a physical laboratory. Its value, however, both as a reliable and convenient source of electric currents and as an instrument for instruction, depends largely on the way in which it is set up; and while it would seem that there could be no trouble whatever in adapting so simple a thing to the use of students, I suspect that others, like myself, have found the problem not wholly free from difficulties; at least such was my own experience when I undertook, some four years ago, to so fit up a storage plant that it would meet the various needs, other than those of lighting and power, of our new laboratory. Nor was I able to get helpful suggestions from ~~any~~ of several sources to which one would naturally turn for such directions. But I did obtain much kind and valuable information from Professor Hering who had just completed an excellent storage installation in the physical laboratory of the Johns Hopkins University. I should probably have tried to duplicate his work in every particular but for the fact that the means at my disposal and some of the objects in view compelled modifications and simpler plans. The mounting finally used cost but little, is convenient in every particular and suited to teaching and general laboratory work. A test of four years has not suggested desirable changes, and besides it has so commended itself to others that it is now adopted in several institutions. I therefore, in response to many requests, offer a brief description of its essential features, trusting that in this way I may serve some one who has occasion to install a storage plant for similar purposes.

The battery I shall describe consists of thirty-six cells of the "chloride accumulator," type E, with glass jars, but evidently the same plans can be used for any number and any type of cells.

The frame on which these cells are placed is accessible from all sides, is eight feet four inches long, six feet high and one foot six

inches wide, and is divided into three shelves, each carrying one dozen cells. The cross-sections of the corner posts and of the railings along each shelf are two and three quarter by three and one half inches. The frame is still further strengthened by an upright at the middle of each side, and the whole is mounted on six heavy glass insulators, one under the foot of each post. The paint with which the entire woodwork is covered is the kind known as "tileite," selected because of its resistance to acids and alkalies. Each cell



is separately mounted in a wooden tray about three quarters of an inch deep that stands on three small glass insulators. The trays are painted with asphalt varnish and filled with ground mica of medium fineness, the flakes varying in area from one to three or four square millimeters. Still further insulation is secured by covering the cells with vaseline which prevents any liquid that might get on the sides from forming a continuous layer. The mica is intended to absorb stray drops of water or acid which will be taken up by the top layers, leaving the lower ones still dry and non-conducting. Good white sand would probably do nearly as well as the mica.

It has been suggested that carbonate of soda be added to the sand or mica since this would unite with the acid, should any get on it, and produce dry non-conducting sulphate of soda, but there is a chemical action between the carbonate of soda and the mica which, though it takes place slowly, soon produces a damp, unsatisfactory mass that can do no good and is likely to do much harm.

The liquid in each cell is covered with a layer, about a quarter of an inch thick, of paraffine oil. This prevents excessive evaporation during very dry weather, and absorption of moisture that might cause disastrous overflowing when the air is particularly damp. Besides, the disagreeable and corrosive acid spray, caused by the bursting of bubbles while the cells are being charged, is well nigh wholly avoided. Apart from the comfort this gives it makes it safe to have apparatus near the battery, avoids the necessity for extra ventilation, prevents fouling of the battery frame and preserves the insulation.

The above precautions against leakage may be excessive, but they are easily made, and I have had no trouble whatever from imperfect insulation during a constant use of the battery for four years, though it is placed in a small basement room that has often been very damp, nor has it been necessary to renew or clean any of the insulating materials.

Two large copper wires, of very low resistance, run from each cell to the switch board which is placed just beyond a partition near one end of the battery frame. These wires are soldered to the terminal lugs of the elements, and thus the trouble due to corroded and bad connections is avoided. Each wire is also soldered to a fuse block conveniently located above the switch board. According to my experience it is far better to use fuse links provided with flat copper terminals, since fuse wires, when clamped down with screws, frequently get loose and give bad connections. The links seem to be free from this objection.

The switch board which lies horizontally in a suitable frame, consists of a close-grained white marble slab, two inches thick, twelve inches wide and forty inches long. This slab is drilled through with holes the proper size to take the wires from the battery that come up from beneath, bend over and terminate in nearby larger holes about three quarters of an inch deep, that contain mercury.

I will say for the benefit of those who have occasion for the first time to drill marble, that it is an exceedingly easy thing to do. I use for this work a sensitive drill press and ordinary twist drills. It is only essential to use plenty of water as a lubricant, but of course it is also desirable to keep the drills reasonably sharp.

For the sake of compactness and convenient spacing these holes are arranged in two double rows as illustrated in Fig. 1, which shows a part of the slab as laid off or drilled, but without connections. The cups, a, b, c , and so on, are connected by wires through $x, y, z, \dots$, say to the positive sides of the cells, while a', b', c' and so forth, are similarly connected through $x', y', z', \dots$, to the negative sides of the corresponding cells. In this way the slab, by its pairs of mercury cups, becomes a simple representative of the entire battery, one that any beginner can understand and by means of which he can readily test the voltage of all sorts of combinations.

To connect the cells in multiple or series straight heavy wires with their ends bent down are used. The mercury cups are so spaced that the wires connecting them in multiple, that is, of sufficient length to unite a and b say, are too short to connect the positive and negative sides of a cell, while those used for joining in series, and therefore reaching from a to b' , are too long to connect opposite sides of the same cell. In this way the chances of short-circuiting are materially reduced, a feature by no means undesirable in a laboratory battery.

The wires of the various laboratory circuits are soldered to a row of fuse blocks above the switchboard, and connection between them and the mercury cups is secured by means of flexible cords, also soldered to the fuse blocks that terminate with a short section of solid copper. I used, for these terminals, heavy wires about an inch long, drilled axially deep enough to admit the ends of the flexible cords, which, of course, were soldered in place. As a matter of convenience each circuit is correspondingly numbered above the switchboard and at the terminal end.

A rheostat, a circuit breaker and an ammeter in the charging circuit, and a voltmeter for testing the cells, all close beside the switchboard, complete the outfit.

UNIVERSITY OF VIRGINIA, July, 1902.

NOTE ON THE SELECTIVE ABSORPTION OF
FUCHSINE AND CYANINE.

BY WILLIAM W. COBLENTZ.

IF we except the investigations of materials suitable for prisms, our knowledge of the selective absorption in the infra-red region of the spectrum is rather limited.

A knowledge of the absorption spectrum of fuchsine and cyanine is of twofold interest: (1) the work of Pflüger¹ on the anomalous dispersion of solid dyestuffs shows that for cyanine and fuchsine there is a single absorption band extending from $.45\mu$ to $.65\mu$; (2) fuchsine and cyanine are chemically different in composition. Fuchsine ($C_{20}H_{19}N_3HCl + 4H_2O$), contains a methyl group, while the chemical constitution of cyanine, ($C_{30}H_{39}N_2I$), is unknown. Nietzki² says of it that "Possibly the cyanines possess a structure analogous to that of the phenylmethane dyestuffs," *e. g.*, fuchsine.

In the present work the radiometer rock-salt apparatus previously described³ was employed.

The cyanine film was obtained by melting the crystals on a plate of rock salt. Considerable difficulty was experienced in obtaining a film of fuchsine which was thin enough, yet at the same time would not crack and peel off the rock salt. The method adopted for obtaining a thin film was to reduce the fuchsine crystals to a fine powder, add a little alcohol to form a thick paste, and apply it to the rock salt. After it had dried, it was reduced to the proper thickness by wiping off the surface with a cloth dampened with alcohol.

The percentage of transmission was obtained by noting the deflection of the radiometer vane caused by the light passing through the rock salt and film, and also by that which passed through a clear part of the same plate.

¹ Pflüger, Wied. Ann., 65, p. 173, 1898.

² Nietzki, Chemie der Organischen Farbstoffe, p. 262, 1901.

³ Phys. Rev., Vol. XVI., p. 35, 1903.

Two cyanine films, thickness .05 mm. and .182 mm., and one fuchsine film thickness .025 mm., were examined.

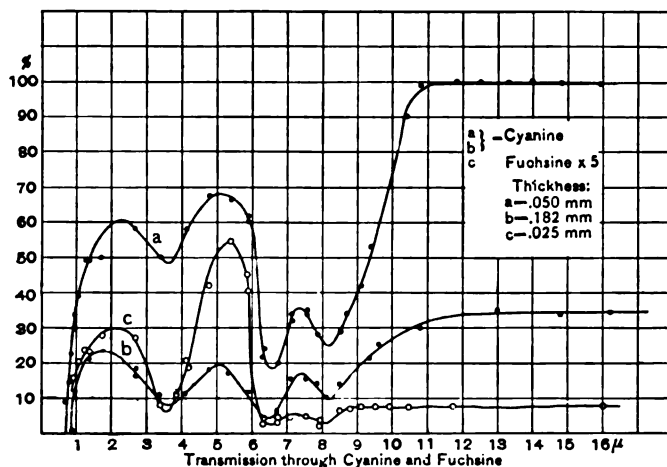


Fig. 1.

The similarity of the transmission curves, Fig. 1, of these two substances is very striking. The transmission of a film of cyanine, 0.05 mm. thick, rises suddenly from zero at 0.7μ to 60 per cent. at 2.3μ , where a shallow absorption band begins and extends to 4.5μ —maximum at 3.5μ . At 5.5μ a deep double absorption band begins and extends to 9.5μ . The first maximum is at 6.5μ while the second is at 8.2μ . Beyond 10μ to 17μ the absorption is constant, and for this film is zero.

As a whole, the absorption of fuchsine is far greater than cyanine, so that a film 0.025 mm. had to be used. In Fig. 1 the ordinates of the fuchsine curve are magnified 5 times. Beyond 5μ the results represent the mean of a number of readings, repeated on different days. In the fuchsine curve the mean and the single readings have been plotted for several wave-lengths.

Fuchsine has three absorption bands whose maxima coincide with those of cyanine. Beyond 9μ the absorption again becomes constant. The transmission for the different parts of the spectrum is given in Table I.

TABLE I.
Transmission through cyanine and fuchsine.

Wave- Lengths in μ .	Cyanine.		Fuchsine $c = 0.025$.	Wave- Lengths in μ .	Cyanine.		Fuchsine.
	$a = 0.05$ mm.	$b = 0.182$ mm.			0.05 mm.	0.182.	0.025.
.7 μ	9			6.75	20	5.6	0.90
.8	14			7.14	34	15.5	0.75
.9	23.5	12	3.3	7.54	35	15.3	0.85
1.0	34	21		7.93	28	14.5	0.73
1.1	39		4.0	8.12		10	
1.4	48	23	4.6	8.50	29	14	1.30
1.75	50	18.5	5.7	8.70	34		1.30
2.7	58		5.4	9.1	42	18	1.50
3.45	50	10	1.6	9.4	53	22	1.40
4.17	65	10.9	4.0	10.45	90		1.46
4.82	67	18.5	8.4	10.8	99	30	
5.42	63	17	11.	11.8	100	35	1.50
5.90	62	12	9.	12.5	100		
6.36	24	9.9	0.72	16. +	100	34	1.40

Pfänger¹ has found the constants a , g and d' of cyanine in the Kettler-Helmholtz dispersion formula.² Using the upper values of g and d' , and finding the extinction coefficient,

$$x = \frac{1}{l} \frac{\lambda}{4\pi} \frac{1}{\log. e} \log \frac{I_0}{I},$$

for the first absorption band of cyanine, at $\lambda = 6.5 \mu$, the index of refraction is 1.43. In Table II. all the indices except the last one are due to Pfänger.

TABLE II.

$\lambda = .44 \mu$	$n = 1.69$
.57	1.48
.62	1.94
.64	2.10
.65	2.23
.66	2.19
.67	2.08
6.50*	1.43

¹ *L. c.*

² Dispersion formula :

Julius¹ examined a large number of compounds, and found that ~~all those~~ containing the methane group have a maximum absorption band at $\lambda = 3.45 \mu$ and a second band between 8 and 9 μ . But his prism calibration is wrong beyond 5 μ , ~~since he used~~ the straight line extrapolation from this point, so that the second band lies between 6 and 8 μ . The work of Abney and Festing² on the absorption spectra in the infra-red also shows that the methyl group has a distinctive selective absorption.

In Fig. 1 the maximum of the first absorption band is indicated at 3.55 μ . After the curves had been sent to the engraver this region of the spectrum was reexplored more thoroughly, at every 0.1 μ , with the result that the maximum occurs at 3.5 μ . Table III. gives the transmission.

TABLE III.

Wave-Lengths in μ .	2.5	2.93	3.14	3.30	3.38	3.49	3.56	3.68	3.92	4.05	4.65
Transmission.	61	56.7	48.3	42.8	40.8	39.8	40.3	41.2	47.8	53	63

Considering the similarity of the absorption bands of fuchsine and cyanine, and the experiments of Julius, one is tempted to assume that there is more than a mere possibility of a relation between the chemical constitution of cyanine and fuchsine, *i. e.*, that cyanine has a structure analogous to that of the phenylmethane dyestuffs.

Cyanine contains one atom of iodine. But there is no similarity between the curves³ of cyanine and iodine, and like Abney and Festing, one concludes that: "The detection of the presence of iodine in a compound is at present undecided."

PHYSICAL LABORATORY, CORNELL UNIVERSITY,
June, 1902.

¹ Julius, *Beiblätter*, 17, p. 34, 1893.

² Abney and Festing, *Phil. Trans.*, 172, p. 887 (1881).

³ Iodine curves are given in *PHYS. REV.*, Vol. XVI., No. 2, 1903.

THE SPECTRAL ENERGY CURVE OF THE ACETYLENE FLAME.

BY G. W. STEWART.

IN a previous paper¹ on this subject, I used a modification of Paschen's method of obtaining the actual distribution of energy in a pure spectrum from the observations taken with a spectrometer and radiometer or bolometer. This modification proves to be a mistake, and, since the correct curves for the distribution of energy in the spectrum of acetylene are of some service, I have redrawn the curves, using the proper method of correction. Moreover, in view of the fact that this modification of Paschen's method has received favorable comment from some quarters, I shall point out the error made in the previous paper and at the same time present experimental proof of the correctness of his formula.

There are two articles, one by Runge² and the other by Paschen,³ which give an accurate formulæ for the correction of observed curves in spectral energy work. The two equations which they present are identical save that Runge has a^2 where Paschen has a . This, together with the fact that Runge says, "Die Wirkung auf den Bolometerstreifen wächst daher annähernd proportional ab " (that is, a^2 when the bolometer strip b equals the spectrometer slit width a), led me to the hasty conclusion that the two equations were not the same, one of them stating that the energy falling on the radiometer or bolometer strip is approximately proportional to the square of the slit width, and the other that it is approximately proportional to the slit width. A closer examination of Runge's article shows that his formula is identical with Paschen's; that he defines another term in the formula, $f(x)$, in such a way that his $af(x)$ is the same as Paschen's $f(x)$; and that the above quotation is true,

¹ PHYSICAL REVIEW, Vol. XIII., No. 5, November, 1901.

² Zeitschr. für Math. und Phys., 42, 1897, p. 205.

³ Annal. der Phys., 60, 1897, p. 712.

if one bears in mind that he imposes the condition that the spectrometer slit width, a , in wave-lengths is the same for all wave-lengths, *i. e.*, is a constant in the formula.

The errors made in my discussion in the article already cited are found on pages 275 and 276. On page 275 the statement, "In different parts of this spectrum, the slit has different widths when expressed in wave-lengths," is true, but one should at this point consider only the actual width, which is constant. On page 276 occurs the sentence, "If the radiometer slit is at x , then an image of the spectrometer slit due to the wave-length x will fall entirely within the radiometer slit, and the intensity of this image is $af(x)$."

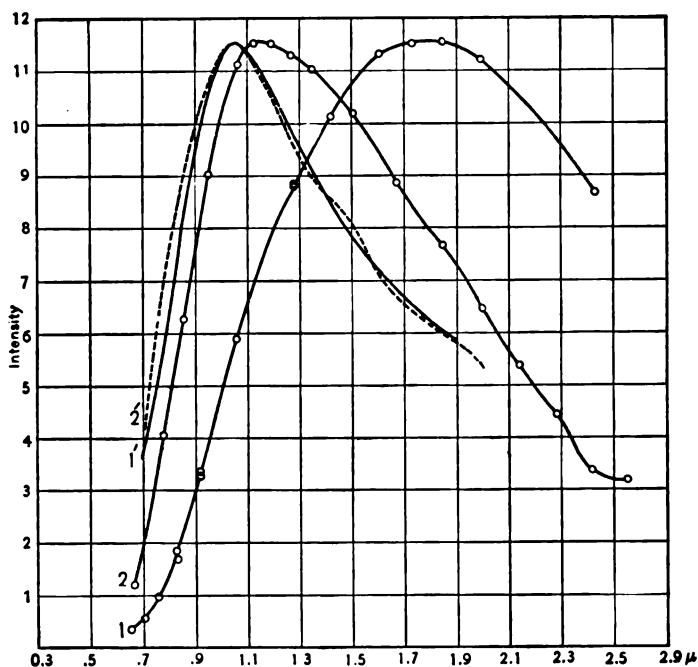


Fig. 1.

If a were the actual width in centimeters, say, the intensity would be proportional to $af(x)$ or to $f(x)$, a then being constant. But a is the slit width in wave-lengths. Hence instead of $af(x)$ I should have written $f(x)$. This change makes the resulting equations identical with Paschen's.

In order to demonstrate experimentally the correctness or incorrectness of Paschen's method, I found the distribution of energy of the acetylene flame using a rock-salt prism, angle $59^{\circ} 47' 53''$, in place of the fluorite, which was used in the previous work. The apparatus otherwise was precisely the same as before.

The proper correction applied to the observed fluorite and rock-salt curves should give the same result. In Fig. 1 curves 1 and 1' are the observed and corrected curves, respectively, when rock-salt was used, and 2 and 2' are those obtained with the fluorite. Curves 1' and 2' will be seen to agree very closely, particularly when one considers the very great difference in the observed curves.

The water emission bands at 1.46μ and 1.90μ do not appear in curve 1' because enough observations were not taken to bring them out. In fact the curve represents one of the two series of observations which were made. This, together with the small deflection in the region 0.7μ to 1.0μ would account for the discrepancy in that portion of the curve. The refractive indices of rock-salt used are those of Rubens.¹

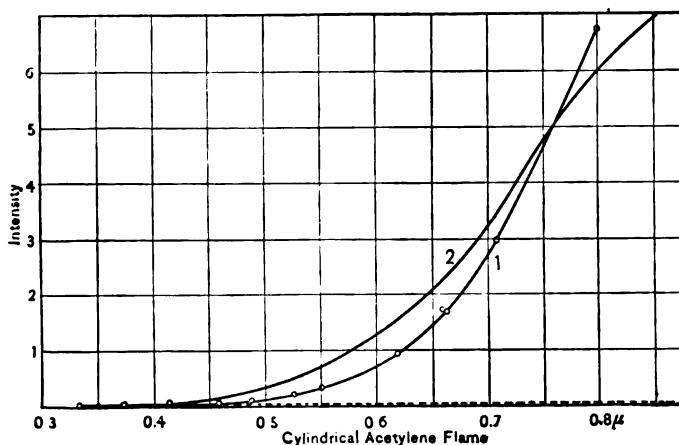


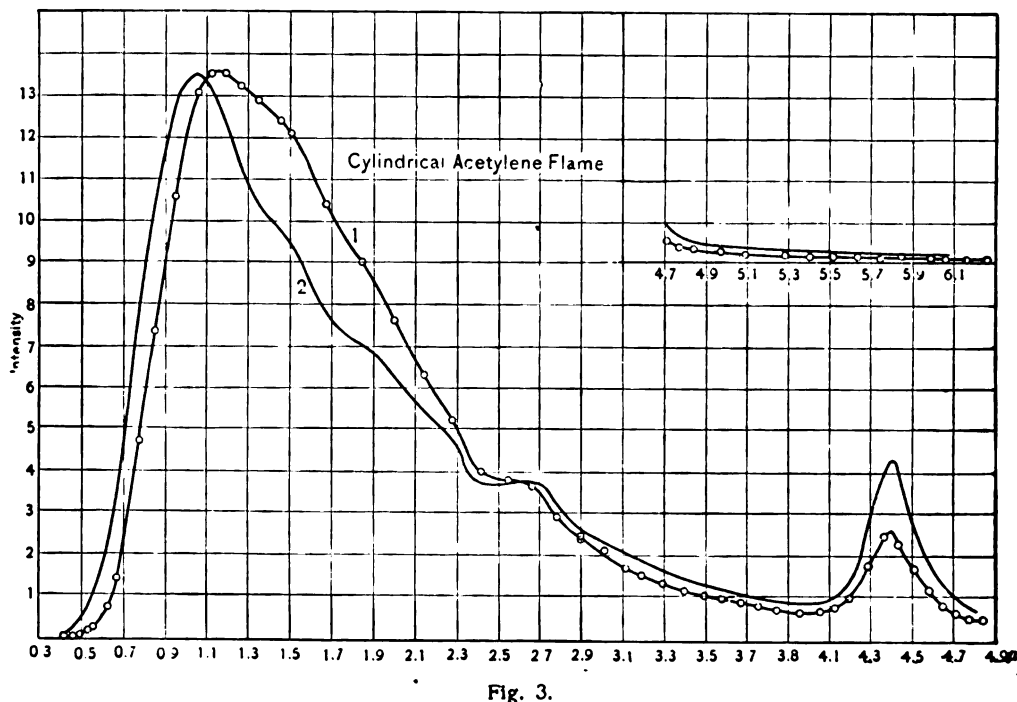
Fig. 2.

The new curves for the distribution of energy in the spectrum of

correction only the first two terms of Paschen's formula was used, viz.,

$$af(x) = F(x) - \frac{1}{8}F_1(x).$$

In a recent article¹ Ångström finds the efficiency of the acetylene flame to be 5.5 per cent. The efficiency as given by the ratio of the area of curve 2, Fig. 2, as far as 0.76μ , to the total area, is 3.9 per cent. While the accuracy of this method in practice is questionable, yet in this case the observations in the visible spectrum were made with great care, as will be seen by Fig. 3. The errors



in the infra-red due to CO_2 and H_2O absorption increase the value

NEW BOOKS.

Light, a Consideration of the More Familiar Phenomena of Optics.

By CHARLES S. HASTINGS, Professor of Physics in Yale University.
New York, Charles Scribner's Sons, 1901.

A series of publications were prepared with the approval of the president and fellows of Yale University by a number of the professors and instructors and issued in connection with the bicentennial anniversary as a partial indication of the character of the studies in which the university teachers are engaged. This volume on light, by Professor Hastings, belongs to this series.

"There is a very large number of phenomena in the domain of optics which may be regarded as familiar, in the sense that all of them may be observed without the accessories, which are found only in the collections of philosophical apparatus characteristic of physical laboratories. To describe and explain this class of phenomena is the aim of this book."

The extensive subjects of spectroscopy and polarized light are excluded by the limitations which the writer has seen fit to impose upon himself. The phenomena of color sensations are treated rather briefly, because of the excellent available popular treatments of these phenomena. The chapter on atmospheric optics has been made quite extensive, in view of the fact that so little has been contributed to this subject during the last half century.

No branch of applied science, says Professor Hastings, has experienced a more remarkable revolution in recent times than that which the advance of theoretical optics has impressed upon the manufacturing optician. In the chapters on the telescope and on the microscope, Professor Hastings has explained the character and meaning of these recent improvements.

Professor Hastings has given us a book which is highly interesting and instructive, a book which must meet with the unqualified approval of every one who is interested in the wide dissemination of knowledge.

W. S. FRANKLIN.

Lessons in Practical Electricity, Principles, Experiments and Arithmetical Problems. C. WALTON SWOOPE. Pp. 462. New York, D. Van Nostrand Company, 1901.

Institute, Philadelphia. It contains much material, covering a wide field, being intended as a student's book to be used not only as a general treatise on the principles of electricity and their experimental demonstration, but also as a manual in making practical measurements and calculations.

The book contains much that is concrete and has more in it of practical value than is found in most books of its class. Although generally correct, the statements are not invariably precise; as illustration, we may note the following:

"Self-induction is defined as the cutting of a wire by the lines of force of the current flowing through it. * * * With a steady current the induction is only momentary" (§ 294).

"On account of the reactance, a larger size of conductor must be used in alternating current circuits than when direct currents are used" (§ 297).

"(The reactance) is equal to the product of its inductance, the number of times the current flow is reversed per second and a constant 6.28" (§ 297).

In the last definition instead of number of reversals we should use number of cycles per second (one cycle consisting of two reversals). It seems unfortunate that the simple factor " 2π " should be disguised as "a constant 6.28."

F. BEDELL.

An Elementary Book on Electricity and Magnetism and their Applications. By DUGALD C. JACKSON and JOHN PRICE JACKSON. Pp. 482. New York, The Macmillan Company, 1902.

This book is inspiring. It is attractive in treatment and mechanical make-up and proves interesting reading not only for the beginner for whom it is intended but also for one already familiar with the subject matter. In the preface we are told "Every effort has been exerted to make it clear, forceful, and of strict scientific accuracy, though it is written in reasonably colloquial language." It is a pleasure to note the success of the authors in these particulars. To the reviewer it has been interesting to see how various subjects are now made clear in an elementary text-book, which a few years ago were matters for advanced students only; then these were matters for research; to-day they are every-day knowledge.

F. BEDELL.

Standard Polyphase Apparatus and Systems. (Third Edition.) By MAURICE A. OUDIN. Pp. 282. New York, D. Van Nostrand Co. 1902. (Received.)

Electric Power Transmission. (Third Edition.) By LOUIS BELL. Pp. 625. New York, Electrical World and Engineer. 1901. (Received.)

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Number 3.

THE
PHYSICAL REVIEW.

THE ULTRA-VIOLET REFLECTING POWER OF
SELENIUM, CYANINE AND GLASS.

BY P. G. NUTTING.

WITHIN the past two years the reflecting power of substances for the shorter wave-length radiation has been attacked by several observers. Glatzel¹ investigated the reflecting power of silver, iron, nickel, copper and some alloys. His method was to reflect two beams of light, one from a known standard mirror and one from the mirror being investigated, on the two halves of a double slit and then vary the width of one half the slit until the light from the two halves produced the same photographic impression. His standard mirror was of Brandes-Schuneman alloy and its reflecting power, interpolated from observed values in the visible spectrum was considered constant in the ultra-violet. The author² observed the reflecting power of a number of metals and organic liquids by finding the time of exposure necessary to give the spectrum reflected from these substances the same photographic intensity as that from quartz, the reflecting power of quartz being calculated from its known refractive indices. All of the above-mentioned work was however of but little quantitative value and it

THE PHOTOGRAPHIC POLARIZATION PHOTOMETER.

Three improvements in the accuracy of photometric methods have been made in recent years. Wild¹ polarized the two beams whose intensities were to be made equal in directions normal to each other and then brought them through a quartz wedge polariscope to overlapping. In this way the two photometric fields are divided into narrow bands lying alternately and the eye is able to detect much smaller differences than in the case of ordinary photometers. Simon² allowed the two beams whose intensities were to be compared to fall on the two halves of a double slit and drew a photographic plate slowly past the slit which was gradually varied in width. The two bands thus produced on the plate would show at some point equal intensity and from the relative widths of the two halves of the slit at this point, the relative intensities of the two beams of light could be calculated. Thus by the application of photography, a means of interpolation is obtained which is impossible in direct eye observations. In the present investigation photography was applied to a polarization method somewhat similar to that of Wild.

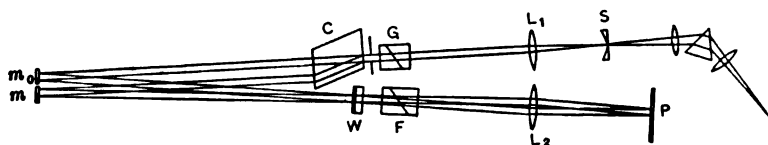


Fig. 1.

A lens L_1 gives an image of the slit S at the point W , monochromatic light from a quartz spectrometer being used. The large calcite crystal C divides the beam of light into two, polarized in and normal to the (horizontal) plane of the apparatus. These two rays are reflected on the two mirrors m_0 and m at nearly normal incidence. These two mirrors are slightly inclined to each other so that the two rays cross at the center of the wedge pair W and continuing divergent, pass through the nicol F and are brought together a second time by the lens L_2 upon the photographic plate P . On the plate then are two superposed images of the slit, each

¹ Pogg. Ann., 118, 193, 1863.² Wied. Ann., 59, 91, 1896.

crossed by interference bands produced by the wedge pair W . A perforated screen just in front of the crystal C makes sure that the two beams do not overlap on the mirrors. The distance SL_1 is made small and L_1W great so that the light between L_1 and W is a bundle of very nearly parallel rays. The whole distance L_1mW is about 200 cm. The large crystal C is about 60 mm. thick, so that for yellow light the separation of the two rays is about 7 mm. The angle of incidence on the mirrors is less than three degrees, so that the rotation produced by even metallic mirrors is quite negligible. The two mirrors are inclined to each other at an angle of about thirty minutes. The slit opening used is about 0.2 by 1.0 millimeter and the beams of light at the mirrors only about 3 by 10 millimeters so that very small mirrors may be used. These mirrors must, however, have good optical surfaces, else the images at W and P will not be uniformly bright. The phase wedges W were specially cut by Messrs. Steeg and Reuter with the wedge edges making angles of 45° with the optical axis of the quartz. Such wedges may be placed with either side or end parallel to the plane of polarization of the light used and still give bands parallel to the wedge edge. The wedges were put together with glycerine instead of Canada balsam. The mirrors were mounted like Fresnel mirrors on a specially designed brass table. This table was so constructed that the two mirrors could be easily interchanged by inverting the whole support without disturbing the adjustments.

Now each of the two rays gives at P an image filled with horizontal light and dark bands, but since their planes of polarization differ in azimuth by 90° as the rays pass into the wedge pair, the light bands in the image given by one ray exactly overlap the dark bands in the image from the other ray. When the two rays are of equal intensity, the bands disappear entirely from the resultant image. Their relative intensity is varied by rotating a Glan nicol G in the path of the ray before it enters the large crystal. If now the reflecting power of one of the mirrors is known, the reflecting power of the other may be calculated from the rotation of the nicol G from the zero position. The mirror of known reflecting power was of quartz cut with parallel faces parallel to the optic axis so that its reflecting power could be calculated from its refractive indices by

means of the formulas of Franz Neumann. The refractive indices have been very accurately determined by Sarasin, Simon and Trommsdorff. In practice an exposure is made with rotatable nicol nearly in the position for which the bands disappear, then the plate displaced a few millimeters, the nicol turned, say one degree, a second exposure made and the process repeated until perhaps six exposures have been made side by side on the same plate. Among these six will be one perhaps which shows no trace of bands, or an intermediate position may be estimated for which none would appear. The ordinary working sensitiveness was usually about one part in a thousand for the difference in the relative intensity of the two rays from unity. If all parts of the instrument were specially constructed for this particular use, this form of photometer ought easily to give results to a hundredth of one per cent. so that it would be by far the most accurate known.

From various causes, chiefly reflection on the various crystal surfaces, the two rays suffer other unequal losses than that at the mirrors. Let K be the ratio of the connection factors for the two rays. Then we have for calculating the unknown reflecting power

$$R = R_0 K \tan^2 \theta$$

where θ is the angle through which the balancing nicol is turned from its zero position. Exchange the two mirrors and take a new reading and we have

$$R_0 = RK \tan^2 \theta'$$

and by eliminating the unknown correction factor R ,

$$R = R_0 \tan \theta \cot \theta'$$

This method of exchanging the mirrors to eliminate experimentally all corrections was used throughout all the work.

As a source of radiation, the Nernst lamp was used in the visible

very gradually, not abruptly as in the case of glass. The time of exposure necessary varied from ten seconds to ten minutes according to the intensity of the spectrum line used and the reflecting power of the mirror being investigated.

MEASUREMENTS.

The first determinations of reflecting power were made on a steel mirror prepared by Zeiss. The results are shown in the table of data and the accompanying curve. There is a decided shoulder in the curve between 309 and 334 $\mu\mu$, otherwise the reflecting power falls off quite regularly from 56 per cent. in the yellow to about 33 per cent. at 257 $\mu\mu$. Hagen and Rubens have determined the reflecting power of steel and a number of other metals in the ultra-violet and infra-red¹ as well as in the visible spectrum,² and their work in the ultra-violet appearing about the time the above determinations on steel were completed, the author turned his attention to those non-metals which show metallic absorption at certain wave-lengths.

Reflecting Power of Steel.

Wave-length,	589	534	470	410	361	334	309	274	257
Ref. power,	55.9	55.4	54.0	52.0	48.8	43.7	43.0	35.3	33.3

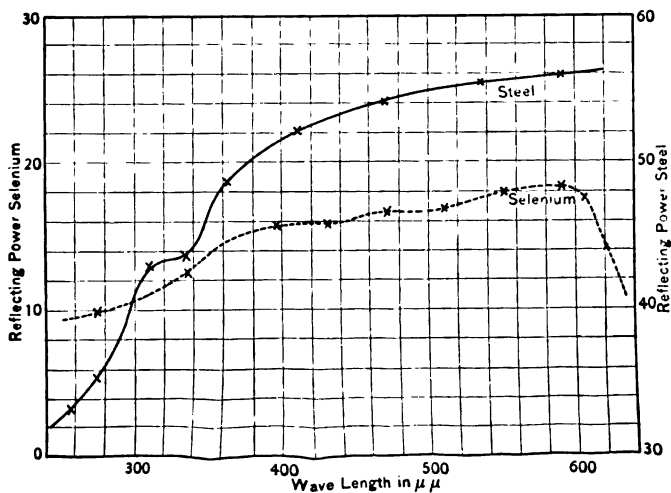


Fig. 2.

Selenium.—In a great many of its physical properties, selenium occupies a position midway between the metals and the non-metals. In its optical properties it is particularly interesting. Its strong and slowly varying absorbing power in the visible and ultra-violet, as well as its large reflecting power, would lead to its classification with the metals, while its transparency over so wide a region in the infra-red is just as strongly characteristic of the non-metals. The reflective and absorptive indices have been determined in the visible spectrum by Quinke¹ and in the visible and ultra-violet by Cornu² and by Wood.³ Excellent mirrors of selenium were obtained by fusing it on ground glass, covering with plate glass and, when cold, removing this plate. The observed reflecting power increases abruptly from a low value in the red to a maximum in the yellow, falls off slightly toward the violet and then more rapidly in the ultra-violet. These results are not concordant with those of Wood. If we calculate the reflecting power from his refractive and absorptive indices by means of Drude's reflection formula

$$R = \frac{n^2(1 + \kappa^2) + 1 - 2n}{n^2(1 + \kappa^2) + 1 + 2n},$$

we obtain a curve rising rapidly from the yellow to the violet and then rising more slowly in the ultra-violet. Cornu observed at but three wave-lengths : 721, 439 and 280 $\mu\mu$; but the reflecting powers calculated from his values of the principal incidence and ellipticity agree roughly with those observed.

Reflecting Power of Selenium.

Wave-length,	620	605	589	550	510	470	430	395	334	274
Ref. power,	14.34	17.63	18.39	18.01	16.85	16.67	15.83	15.14	12.65	9.98

Only the last figures of the reflecting powers quoted are considered uncertain as regards error in measurement. This is the most accurate work done with the spectrometer and shows what work it will do when carefully adjusted, working with substances of sufficient reflecting power.

¹ Wied. Ann., Jubelband, p. 336, 1874.

² Compt. Rend., 108, 917 and 1211, 1889.

³ Phil. Mag. (6), 3, 607, June, 1902.

Cyanine. — With its strong, narrow absorption band in the yellow, cyanine is an ideal substance for studying the anomalous dispersion of solids. Its refracting and absorbing power have been investigated by Pflüger¹ and its refracting power by Wood and Magnusson.² The cyanine used was the di-amyl iodide, $C_{29}H_{33}N_2I$, furnished by Kahlbaum, easily soluble in alcohol, but only very slightly soluble in water. Excellent mirrors are easily obtained by casting, great care being exercised in guarding against burning and keeping free from impurities. Fused cyanine appears to be an excellent solvent for grease and many other organic substances and requires the greatest care in handling if it is to be kept pure. The reflecting power rises in the red to a maximum in the yellow, falls to 1.26 per cent. in the green and then gradually increases to a nearly constant value of 6.8 per cent. in the ultra-violet. The reflecting power curve calculated from Wood and Magnusson's refractive indices and Pflüger's absorptive indices by Drude's formula agrees closely in form with the observed. Using the observed reflecting powers and the refractive indices determined by Wood and Magnusson, the absorptive indices were calculated and are given in the table.

Optical Constants of Cyanine.

Wave-length.	Reflecting Power.		(W. & M.)	(calc.)
	Fresh.	Exposed.		
630	13.46		2.325	
610	14.22		2.120	0.192
589	13.31	6.25	1.660	0.555
560	10.50		1.250	0.954
540	7.81		1.125	0.538
520	2.21		1.130	0.257
500	1.26	6.14	1.255	0.070
475	2.04		1.400	0.00
450	3.42	6.57	1.490	
425		7.48	1.538	
395	5.68	7.8	1.578	
358	6.60			
334	6.94	6.9		
309	6.77			
274	6.77	6.5		

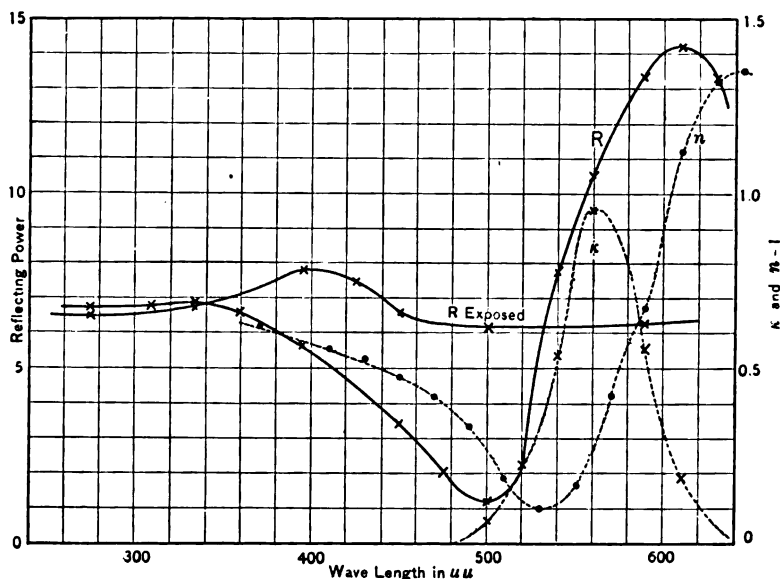


Fig. 3.

Exposure to light produces great changes in the reflecting power and other optical constants of cyanine in the visible spectrum. Freshly fused cyanine is of a deep brassy yellow color, but exposure for a few minutes to direct sunlight or for an hour to the dull light of a cloudy day turns it to a decided plum color. Exposed ten times as long in the same manner, it turns a steely blue-black and has the appearance of selenium. The change is not due to the heating effect of the absorbed radiation, for a piece of fresh cyanine kept warm in the dark shows no trace of change. On the other hand, a thin wash of cyanine on ground glass, exposed to light until well colored, recovers its brassy yellow color on refusion. An attempt to photograph the spectrum of a Nernst lamp on a cyanine mirror (exposure twelve hours), showed the effect to be strong in the yellow and but slight in the adjacent red and green. No trace of change could be detected in the remainder of the visible or ultra-violet spectra. The reflecting power curve for exposed cyanine shows a remarkable change to have taken place. Instead

through both regions and this is almost exactly the same as the constant reflecting power in the ultra-violet. The strong, characteristic anomalous dispersion of the cyanine has entirely disappeared. If we regard the strong absorption for yellow light as due to resonance, as is indicated by the anomalous dispersion, the effect of the light is to fatigue or destroy this resonance, warming restores it. The effect of this loss of resonating power on the absorptive and refractive indices is enormous. For example at $500\ \mu\mu$, where the absorption is so slight we may calculate the refractive index directly from the reflecting power, exposure to light increases the refractive index from 1.25 to 1.66. Evidently no reliable work on the dispersion of cyanine can be carried on where the prism or film is exposed to daylight. Again, in the yellow at about $580\ \mu\mu$ the refractive index of the exposed is the same as that of fresh cyanine. Here then the absorptive index has dropped from 0.70 to less than 0.7. This change in the absorption is easily seen by viewing a sodium flame through a piece of ground glass washed with fused cyanine. Through the fresh cyanine the flame is invisible but it is plainly seen through the cyanine exposed to light.

Glass. — The absorption of ordinary crown and flint glass is very strong in the ultra-violet and begins very abruptly, so that we might expect traces of anomalous dispersion in this region. To test whether the reflecting power showed any abrupt increase and to measure the absorptive index in this region, samples of standard Jena glass were kindly furnished by Zeiss through the kindness of Dr. Pulfrich and Prof. Voigt. These samples were 02388 telescope crown, 02546 baryt-flint and s228, a new glass of remarkably high refractive index (1.9) and low dispersion. The refractive indices of the first two varieties have been determined out to the region of absorption by H. Trommsdorff.¹

Of each kind were a thin plate (1.5 mm.), a thick plate (5 mm.) and a wedge of sufficient angle so that only the reflection from the first surface was observed.

For a parallel-sided plate, the whole reflection from front and back surfaces sums up to be

$$R \frac{1 + T^2(1 - 2R)}{1 - R^2T^2}$$

¹ Inaug. Diss., Jena, 1901.

where T is the transmitting power of the plate or the ratio of the transmitted to the incident light, corrected for losses by reflection. It bears the relation

$$T = e^{-4\pi \frac{d}{\lambda} \kappa}$$

to the absorptive index κ , where d is the thickness of the plate and λ the wave-length of the light used. In the above expression for the reflection, R is the reflecting power of a single face and experimentally is the observed reflecting power of the wedge. From the reflecting powers of the wedges and the plate then, the absorptive index may be calculated. By this method the absorptive indices of the glass *s* 228 were determined and are given in the table. For the crown and flint glass, the absorption begins so abruptly and it was so difficult to obtain a source giving a sufficiently intense continuous spectrum in that region that no reliable results were obtained.

Reflecting Power of Glass.							
Wave-length,	589	475	395	334	309	274	250
<i>o</i> 2388	4.16	4.22	4.31	4.48	4.52	4.67	
<i>o</i> 2546	5.37	5.63	5.86	6.58	6.94	7.48	7.83

Constants of <i>s</i> 228.							
Wave-length,	589	550	500	450	400	334	274
Reflection wedge,	9.39	9.30	8.96	8.88	9.37	9.48	9.55
Reflection plate,	17.18	17.12	16.81	15.63	12.20	9.50	
T	1.00	1.00	1.00	0.954	0.604	0.031	
κ	0.00	0.00	0.00	$117.10 \cdot 10^{-8}$	$107.10 \cdot 10^{-7}$	$615.10 \cdot 10^{-7}$	

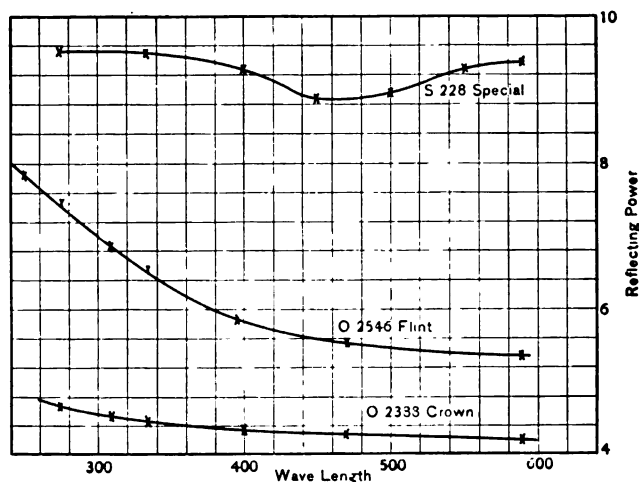


Fig 4

REFLECTING POWER CURVES.

None of the reflecting power curves shows any abrupt increase in the reflecting power where the absorption begins. The steep part of the absorption curve for σ 2388 lies between 298 and 304 $\mu\mu$, for σ 2546 between 318 and 323 $\mu\mu$. There is no indication of any anomalous dispersion. From Drude's reflection formula we find that x must be less than 0.001 in order not to affect the reflecting power by an observable amount. On the other hand, a plate as thin as one millimeter absorbs 99 per cent. of the incident light for x as small as 0.0002. For all three kinds of glass then, x lies between these limits in the ultra-violet in the region of the beginning of the absorption.

This work was done at Göttingen during the past year. In closing I wish to express my warmest thanks and appreciation for the unfailing courtesies extended by Professor Voigt, in whose laboratory the work was done.

AN ELECTRIC MICROMETER FOR LABORATORY USE.

BY PHILIP E. SHAW.

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I. DESCRIPTION OF THE INSTRUMENT.

This instrument is designed to measure any small dimension or movement. The above index shows that it has many and varied applications. It acts better (in some cases incomparably so) than the instruments at present in use for the same purposes.

Further applications will appear in Part II. of this paper.

The instrument has a unit of $\frac{1}{1000}$ mm. or micron (μ), the error being, under favorable conditions, not more than $\mu/2$. But in some readings given below the unit is only mm./200. The scale is very long, the range being from 1 μ to 40,000 μ or more.

The principle of a very sensitive instrument, the electric micrometer, has been explained in various papers.¹ It measures movements of the small amount $\frac{1}{50000}$ mm. ($\frac{1}{5}$ $\mu\mu$). For obvious reasons this instrument would be much too delicate for ordinary laboratory measurements; but, by discarding the train of levers (or

¹Shaw, Phil. Mag., Dec., 1900, and Phil. Mag., March, 1901. Shaw and Laws, Electrician, Feb. and March, 1901. Shaw and Laws, Electrician, March, 1902.

using only one lever in a subsidiary way), and by other simplifications, we obtain the instrument described below.

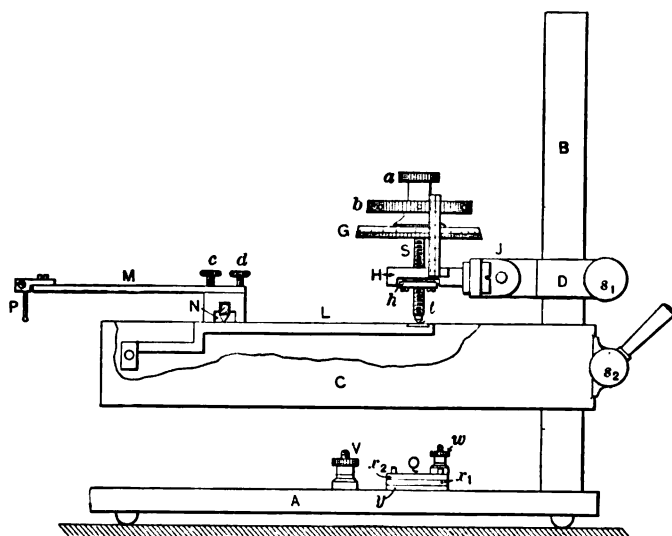


Fig. 1.

A horizontal base-plate *A*, in Fig. 1, has a vertical pillar *B* screwed to it. On this, and clamped by the clamping screw s_1 , runs the sleeve *D*, carrying the measuring screw *S* and the nut *H*, there being between *H* and *D* a universal joint at *J*. By this means the screw *S* can be raised or lowered, rotated round *B*, and pointed in any desired direction; but the most common case is when, as in the figure, *S* points downward. The screw has a pitch of $\frac{1}{2}$ mm. and there is a graduated disc *G*, having 500 divisions on it, so that each graduation corresponds to a movement of the screw point *t* of $\frac{1}{1000}$ mm. Above the disc *G* are two milled heads, *a* and *b*, for coarse or fine adjustment of the screw. A lock-nut *h* is shown for rendering the screw firm in the nut *H*.

For most measurements the simple mechanism so far described suffices; but in some cases it is necessary to have a lever in conjunction with the screw. If so, we require the cradle *C* which slides up and down *B* and is clamped to it by the clamping screw s_2 .

On hard steel plates let into the cradle rests the fulcrum *N* of the lever *LM*. By means of the counterpoise weight *O* the lever

is kept always pressing up against the screw at t , a plate of agate as bearing surface, being let into the lever.

It is evident that, provided S points vertically down, the lever can be adjusted horizontally under S , however the latter is raised or lowered, or rotated round the pillar. There is an adjustable measuring pin P on the lever; then if the arms L and M of the lever are equal, any movement of the screw is reproduced in amount at P .

The lever is so made that the point t , the fulcrum bearing, and the end of the measuring pin P , are in one horizontal plane. By this arrangement we avoid any tangential movement or sliding at the surfaces at t and P when the lever moves. Apart from other advantages of this truly normal displacement at t and P , we insure that bad levelling or irregularity at either of the surfaces will not produce errors in the measurements.

As the object of this paper is merely to establish a new method of measuring length, nothing will be set down but very brief descriptions of methods, and only such experimental results as are required to establish the accuracy of the new method.

The processes described are meant for laboratory practice and can all be done by any student of fair experimental skill. Some of the measurements given below have been taken by students and by no means represent the best possible results.

II. A MEASURING MACHINE.

To measure thin plates and wires we use the part marked Q in Fig. 1.

Above the base-plate comes an ebonite plate y , then a steel plate x_1 , and above this another steel plate x_2 . x_1 and y are clamped firmly to the base-plate by screws countersunk in x_1 and passing through an ebonite insulating bush.

But x_2 is free and can be lifted off, there being ebonite pins to guide it if desired. The three plates y , x_1 , x_2 , are all ground true — the opposing faces of x_1 and x_2 being specially surfaced up.

Remove the cradle C and the lever, and let the measuring screw S down to nearly touch the top of x_2 .

Connect up the following in series: The binding screw V , a cell, a resistance box, a telephone, the binding screw w .

Place the telephone on the head, and each time that t touches or leaves x_2 the telephone will sound.

Suppose we wish to obtain the thickness of a thin sheet of metal ; we find the position of contact, (1) when the sheet is between x_1 and x_2 , (2) when it is removed. The difference between (1) and (2) gives the thickness required. If the sheet is of metal and has any buckle in it, it is necessary to press x_2 down while taking the measurement.

If we wish to ascertain the diameter of a fine hair or wire we place the pieces far apart and parallel between x_1 and x_2 , and proceed as before.

In order to show the possibilities and sensitiveness of the instrument a few measurements are given.

Object.	Various Measures of Thickness in Terms of μ .		
Steel band.	87,	87,	86.
Silk fiber (coarse).	62,	63.	
Human hair (fine).	63,	59.	
Tissue paper.	58,	60.	
Rice paper.	50,	52.	
Mica.	30,	28,	30.
Dutch metal.	10,	11,	10.
Gold leaf (dark blue by transmitted light).	3,	5.	

These measurements are taken at various places on the substance and indicate how irregular these bodies are. Reading taken at any one place agree to $\frac{1}{2} \mu$.

One special purpose for which the instrument has been used is to obtain a mica sheet of correct thickness for $\frac{1}{4}$ wave plate for the H line in the spectrum. The thickness required is about 20μ . It is very easy to continue splitting and measuring till the required thickness is obtained.

Only thin substances are given here, though any body up to the limit of the screw (say 2 inches) can be measured with the same accuracy.

In the case of a metal block the plate x_2 can be removed, but it will be observed that for a soft or non-conducting body the upper plate is necessary.

It is a great advantage that in the electric micrometer method *pressure* is not necessary and there is no danger of crushing any of the above soft substances.

The fact that no appreciable pressure is necessary is an advantage which other measuring instruments, *e. g.*, the micrometer gauge, and Whitworth measuring machine, do not possess.

In the electric micrometer we can tighten the measuring screw in its nut, and so avoid loose working and back-lash and still retain perfect ease in taking readings. But in the other machines which depend on the *feeling of touch* this tightening is impossible.

The ordinary Whitworth machine measures $\frac{1}{100000}$ inch or about $\frac{1}{4000}$ mm. This can also be done with the simple electric micrometer if the divisions are divided by eye into four parts.

III. YOUNG'S MODULUS BY STRETCHING, USING THE MICROMETER.

The time-honored method of having two wires, say 10 meters long, stretching up through various floors in the laboratory building and measuring the stretch by the relative movement of a scale on one and a vernier on the other, is very crude.

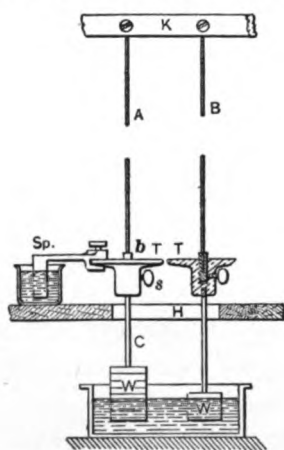


Fig. 2.

The following plan is adopted here.

The wire *A*, say of steel (about $2\frac{1}{2}$ meters long) is soldered firmly into the bolt *b* and this is held firm by the screw *S* to the brass platform *T*. The bottom of the platform holds the rod *C* and large weight *W*. A rod *sp* is clamped to *T* and dips into thick oil in a vessel on the table *H*, also *W* is nearly immersed in a thick oil (castor oil is used).

Place the simple electric micrometer on the table *H*. If now the micrometer screw is brought direct (or with the lever) on to the top of *T*, then as weights are added above *W*, we can measure the drop in the platform *T* and so

The wire *B*, say of copper, has fittings identical with those of *A* and the splasher *sp* can be transferred to *B* when required. The tops of both *A* and *B* are attached rigidly to a strong cross-bar *K* on the same bracket fixed to the wall. If the bracket moves as a whole, or the cross-bar yields by any amount, when *A* is loaded, the platform of *B* will drop by this amount.

Hence by measuring on the platform of *B* when *A* is loaded we can discover the movement in question and deduct it in the calculations. To the usual precautions of having few kinks in the wires and starting with a large initial load must be added that the weights should be put on with their centers of gravity in a vertical line, or there will be a side movement of the platform *T*. It is preferable to have for *C* a flexible wire to minimize any tilting of *T*.

The wire *B* can be next dealt with, using *A* as the fixed wire. Thus we can find the modulus for two different wires on this apparatus.

It may be mentioned that the above is the last of several methods tried. It is by no means easy to obtain high accuracy, but this apparatus has proved quite satisfactory. When *A* was loaded and

Steel Wire. Start with 8 Kilos Load.

Down.			Up.		
Extra Weight.	Micrometer Reading.	Stretching in Terms of μ .	Extra Weight.	Micrometer Reading.	Stretching in Terms of μ .
0	950		0	951	
2	654	> 296	2	652	> 299
4	354	> 300	4	354	> 298

Repeating.

0	951	> 297	0	955	> 300
2	654	> 299	2	655	> 300
4	355		4	355	

The mean for 2 kilos = 299 μ .

III a. YOUNG'S MODULUS BY STRETCHING, USING THE OPTICAL LEVER.

There is another delicate way of measuring the stretch, viz., by the optical lever. My method of doing this is to rest two legs of the lever on the platform of *A* and the third leg on the platform of *B*. The lever carries a concave mirror which throws an image of an incandescent lamp on to a scale. The results by this means are as follows :

Steel Wire. Start with 8 Kilos Load.

Down.			Up.		
Extra Weight.	Scale Reading.	Difference in Terms of μ .	Extra Weight.	Scale Reading.	Difference in Terms of μ .
0	257, 257	> 110, 110	0	255, 256	> 108, 109
2	147, 147	> 106, 105	2	147, 147	> 106, 105
4	41, 42		4	41, 42	

Mean for 2 kilos = 107 divs.

The optical lever must be calibrated. This can be done by inserting a piece of glass of known thickness beneath the third leg of the lever. But this method of calibrating has objections, one of which is the correction for obliquity of incidence on the scale. The method I find accurate and easy is to remove the platform of *B* and to rest the third leg of the optical lever on the lever *LM* just over *P* (see Fig. 1), then by working the measuring screw *S* the optical lever is tipped and the spot runs along the scale to any desired amount.

We then find what the units on the scale are in terms of the divisions of the graduated disc *G*. Having once calibrated the optical lever we can use it subsequently with ease. On calibrating thus, the movements on the scale correspond to 298 μ and 297 μ for 2 kilos.

Thus there is very good agreement between the measurements by the simple electric micrometer and by the optical lever (see the result in section III.). But it is obvious that the former method is much more accurate than the latter while being three times as sensitive.

IIIb. AN EXTENSOMETER.

It is easy to use the instrument on an engineering testing machine to determine any small movement made during a test.

IV. YOUNG'S MODULUS BY FLEXURE.

Let a metal rod be placed horizontally with its ends on two rounded knife edges, the usual care being taken to have the supports and the table specially rigid. Place the micrometer on the table bearing the supports, bring the measuring screw of the micrometer above the middle of the bar and take measurements of the position of the bar as successive kilograms are hung (through a hole in the table) from the bar.

The electric circuit is to be completed as follows: A cell, resistance box, telephone, the micrometer, the metal rod under measurement, a binding screw on one knife edge, joined to the cell.

Brass Bar.

Down.			Up.		
Extra Weight.	Micrometer Reading.	Difference in Terms of μ .	Extra Weight.	Micrometer Reading	Difference in Terms of μ .
0	248	> 875	0	247	> 874
1	1123	> 875	1	1121	> 878
2	1998	> 883	2	2001	> 880
3	2881	> 883	3	2882	> 881
4	3764	> 880	4	3764	> 876
5	4644	> 880	5	4742	> 879
6	5524		6	5524	

Mean movement for 1 kilo = 878μ .

If the supports and knife-edges have been compressed by the loading, the above value will be too great. To investigate this point, place the measuring screw above the bar just over each knife-edge in succession and find if there is any depression there as the center of the bar is loaded.

In this way I found movement at the two ends to be 1 and $3\frac{1}{2}$. Taking 2 for the mean depression and deducting we obtain net movement 876μ .

It will be found very useful to have the bottom permanent weight immersed in thick oil (as in section II.) to prevent up and down vibration. The depth and breadth of the rod should be measured by the micrometer. Great accuracy is needed especially for the depth, as this enters the calculation in the third power.

V. TORSION OF A ROD.

Fig. 3 is a plan in which is shown a cylindrical rod *A* having square ends. One end is gripped firmly in jaws at *B*, the lower

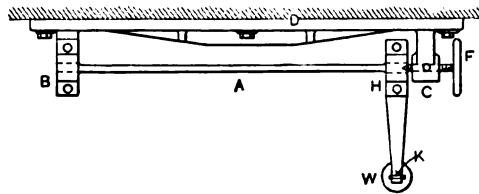


Fig. 3.

one of which is a bracket of the massive casting *D* and the upper one can be unscrewed for removal of the rod. Casting *D* is screwed to the wall.

Torsion of Steel Bar.

Down.			Up.		
Extra Load.	Micrometer Readings.	Difference in Terms of μ .	Extra Load.	Micrometer Readings.	Difference in Terms of μ .
0	70	> 645	0	75	> 640
200	715	> 640	200	715	> 640
400	1355	> 640	400	1355	> 640
600	1995	> 640	600	1995	> 640
800	2635	> 630	800	2635	> 630
1000	3265		1000	3265	

At the other end, the rod is clamped in jaws, the lower of which is the strong arm *HK* quite free from the bracket. The end of the rod is drilled to receive the coned bearing which can be screwed into it by the hand-wheel *F*. The screw is carried by the bracket *C* which is part of the main casting. Thus the left end is held and the rod will be twisted when a load is applied at *K* the end of the

when the load is applied there, place the micrometer on a table (not shown) and bring the end of the measuring screw to contact at *K*. As we are looking down on the apparatus *K* moves down when the load is applied to it. Take readings for every extra load of 200 gms. applied.

There is a slight increase in the numbers as the arm becomes oblique. This can be easily corrected for. To ascertain if the end of the rod is depressed when loads are added, I placed the measuring screw over the rod and observed a small movement, viz., 4μ for each load added. Then as usual we have

$$\text{couple} = \frac{\pi n r^4 \theta}{2l},$$

where n = rigidity, r = radius of bar, θ = twist of bar, l = length of bar.

It is obviously easy to arrive at each of the terms and so obtain n .

VI. THERMAL EXPANSION OF A TUBE.

A tube *A* (see Fig. 4) of steel, 1 meter long is fitted with entrance and exit tubes *a* and *b*, also with tubes for the insertion of thermometers *c* and *d*. One end of the tube is pressed against a rigid rod *B* which is capped with ivory, the other end of the tube is free to move.

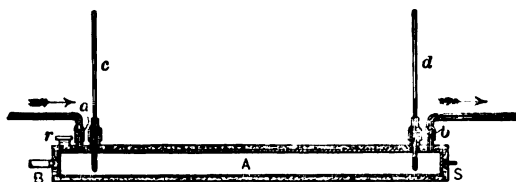


Fig. 4.

The expansion of the tube, when temperature rises, is shown by the micrometer screw *S*. The tube can be supported or suspended by cords, care being taken to have in either case a considerable thrust towards *B*.

The electric circuit for measuring is arranged as in previous sections. Commence by passing steam through the tube. Plenty of steam is required for good results, so it is produced from a small boiler. After the thermometer readings have been stationary for ten minutes read the thermometers and the micrometer. Now run water from the tap, taking care not to disturb *A* in changing tubes at the inlet and outlet *a* and *b*, and when uniform temperature is guaranteed, take new readings with the thermometer and the micrometer.

Results for Steel Tube.

Mean Temp. of <i>c</i> & <i>d</i> .	Diff. in Temp.	Microm. Readings	Diff. in Read. in Terms of mm/100.	Coefficient of Expansions.
98.9	> 92.8	338	> 236	0.00001272
6.1		102		
99.4	> 93.3	271	> 236	0.00001265
6.1		35		

The process adopted is easy to work—thorough stirring is effected and the results are as good as can be expected with rough thermometers. The errors of temperature measurement are much greater than those of length measurement in this case. When a bar is used in place of a tube and heating is done by an

electric current in a coil of wire surrounding it, we have less uniformity in temperature, but as a lecture experiment it is simple and direct.

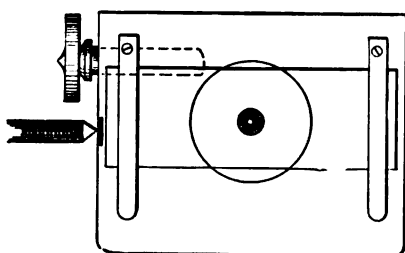


Fig. 5.

VII. MICROSCOPIC MEASUREMENT.¹

This is one of the most simple and reliable applications of the method. Suppose we wish to measure the number of lines per centimeter in a grating. Place the grating on the stage of the microscope and focus on the lines, using a high power. Set the cross wire of the eye-piece on any line, then bring up the micrometer

¹ More detail on this point will be found in a paper by myself in *The Royal Microsc. Soc. Journal*, December, 1902.

screw, which in this case works horizontally, to the side of the stage (see Fig. 5), and measure its position as explained in section I. Now turn the tangent screw of the stage and count the lines as they traverse the field of view. After counting 50 lines measure again the position of the side of the stage. We then obtain the difference between 50 lines.

The Lines in a Grating.

Micrometer Readings for every 50 Lines.	Difference in Terms of μ .	Micrometer Readings for every 50 Lines.	Difference in Terms of μ .
225	> 454	2040	> 458
679	> 455	2498	> 455
1134	> 459	2953	> 457
1593	> 447	3410	

Mean 455μ for 50 lines.

Another example is the internal diameter of a capillary tube. This is shown mounted on a microscope slide on the stage in Fig. 5, so as to be viewed end-on. Then proceed as before. Two diameters at 90° to one another were taken. The bore of the tube is very elliptical in section.

Two Diameters of a Capillary Tube.

	Micrometer Readings	Difference in Terms of μ .	
1.	634		
	1142	> 508	
	640		
	1147	> 507	
	642		
	1148	> 506	
2.	398		
	15	> 383	
	381		
	0	> 381	
	385		
	2	> 383	

mean 507μ .

mean 382μ .

Sometimes it is very important to be able to measure a capillary tube to this degree of accuracy. Thus, in the determination of viscosity of a liquid by the rate of flow in a fine tube, the diameter of the tube enters the calculation in the fourth power.

The measurements obtained by this simple process are much more accurate than by any other available to me. In microscopes which have no tangent screw and moving stage, it is easy to arrange a parallel movement of the body and to push the body before the screw of the micrometer.

Micrometer eye-piece measurement can never be very sensitive because the micrometer scale is only magnified by the eye-piece. But the stage micrometer has the advantage of magnification by both eye-piece and objective. In the process of using Nobert's tests, two sets of readings must be taken, which involves extra trouble and error. Or if, as is commonly done, the eye-piece micrometer is calibrated by the Nobert's test the sensitiveness of the instrument is reduced, as then the measurement is taken with the eye-piece.

VIII. DIRECT MEASUREMENT OF WAVE-LENGTH.

It seemed to me that a direct measure of wave-length involving no gratings, bi-prisms, etc., would be useful and instructive. To achieve this, a change is made in the arrangement shown in Fig. 1. The arm *M* is removed. Then insert a stalk *st* (Fig. 6), carrying a convex lens in the left arm of the lever.

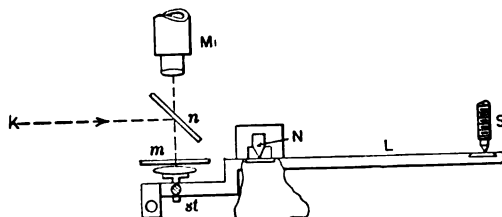


Fig. 6.

The lens is just below a microscope slide *m* and the distance between them can be made any small amount by working screw *S*. The cradle carries *m*, also another slide *n*, which can be rotated on a pin in a hole to an angle of 45° to the vertical. Allow sodium light from *K* to proceed to *n*. Then we shall be able to obtain Newton's rings between the lens and slide *m* and we can observe them by a microscope, *Mi* having a low power objective. By turning screw *S* one way, Newton's rings are produced from their

center; by reversing the screw the rings are absorbed into their center.

Then, knowing the leverage l , it is obviously quite easy to measure the wave-length λ for, suppose n microns is the movement of S required to produce 150 rings,

$$\text{the movement at the lens is } 150 \cdot \frac{\lambda}{2}$$

$$\text{the movement at the screw is } n \cdot \mu$$

and we have

$$150 \cdot \frac{\lambda}{2} \cdot l = n \cdot \mu.$$

To measure the leverage exactly is not easy. I found the following method reliable.

Place a microscope having a long horizontal traversing scale in exact line with the fulcrum edge N , then (1) traverse to a line through S and find the long arm of the lever, (2) traverse to a line through the center of the Newton's rings, hence find the short arm. In order to get the exact position of the center of the rings, place a strip of paper having a clean, straight edge, so that it appears to cut through the center of the rings. Then set the straight edge in the line of sight of the microscope and proceed to take the reading of the straight edge.

There are two special difficulties to be noticed in working by this method. (1) The stalk carrying the lens can be adjusted up and down, but if precautions are not taken in this adjustment, the center of curvature of the lens will not move normal to the surface of the slide m . If the lens is too high, then, as the lens rises, the center of curvature moves outwards—if too low it moves inwards and in either case the results are variable. But by trial an intermediate position is found when the readings are steady. In the table below a small progressive increase is seen, showing that exactly the best position was not used in this case.

(2) The milled head of the measuring screw must be turned very steadily so that the rings may be accurately counted. * A long pin is thrust into the capstan holes in b (see Fig. 1). If this long

leverage is used and the hand is steadied on a rest, the process is quite easy.

Newton's Rings. Readings for Every 50 Rings.

Down.		Up.	
Microm. Reading in Terms of μ .	Difference for 150 Rings.	Microm. Reading in Terms of μ .	Difference for 150 Rings
365.2		365.2	
324.2		324.0	
283.0	> 123.9	283.0	> 123.5
241.3	> 124.4	241.8	> 124.7
199.8	> 125.8	199.3	> 125.8
157.2		157.2	

Mean for 150 rings 124.7 μ .

Measurement of Leverage.

	Short Arm.		Long Arm.	
Right end reading.	7.57	7.57	13.72	13.72
Left end reading.	3.36	3.37	1.75	1.74
Difference.	4.21	4.20	11.97	11.98
	Mean 4.205		Mean 11.975	

Using the expression

$$\lambda = \frac{2 \cdot n \cdot \mu}{150 \cdot l},$$

we find

$$\lambda = 58.61 \times 10^{-6} \text{ cm.}$$

This is fairly near the true value (58.95×10^{-6}) for the mean of the D lines.

IX. PRECAUTIONS.

In the paper on the electric micrometer¹ directions are given whereby the fine measurements there mentioned are rendered possible. The present instrument, though so much less sensitive, yet is delicate and requires care in the following points:

1. The measuring screw point and the surface opposed to it must be smooth and should be kept clean by the finest emery paper. A little oil, dirt, or varnish renders the readings uncertain.

¹ *Loc. cit.*

2. Firm tables attached to the walls or on the firm ground floor are desirable. Vibrations from outside causes are thus avoided. Moreover, it will be very often found impossible to lean on the table without effecting the readings considerably.

3. The lock-nut of the measuring screw must be adjusted nicely so that the screw turns firmly in its nut or back-lash may occur.

4. The circuit of, a cell, telephone, resistance box, measuring-screw and the contacts is easy to arrange. The resistance is put as high as will give distinct but faint sounds. Thus I commonly use a current of only $\frac{1}{8000}$ ampère. Such a current will not be likely to heat the contacts much, or cause them to expand appreciably.

5. The best form of telephone is a "watch-receiver" provided with an attachment which fits on the head and leaves both hands free.

6. Damping should be attended to, notably in Young's modulus by stretching, in which case it is essential.

7. The measuring screw must always be set perpendicular to the surface on which it works.

X. SUMMARY.

All the processes described have been actually made to work. The scantiest experimental results are given; if time allowed, far better results could be obtained, but the object of this paper is to establish the method and not to obtain special results.

The instruments which could be used for taking such measurements as are mentioned in this paper and which have a great range of application are (1) the microscope, (2) the optical lever, and (3) the simple electric micrometer.

A good instrument should have: (1) delicacy; (2) accuracy; (3) length of scale. Let us compare the rival instruments in these respects.

Microscope.—A micrometer eye-piece has, say fifty lines, ruled to a mm.; to go further than this is not much use unless the light is very good. Thus the instrument is not very sensitive. If a micrometer screw eye-piece is used we obtain no more delicacy (although some instruments pretend to give $\frac{1}{800}$ mm.).

If the eye-piece has 500 divisions to 1 mm. the cross-wire will have to be moved 10 or 12 divisions to make any movement perceptible to the eye. This means that the unit is $\frac{1.2}{500} = \frac{1}{400}$ q. p. as before.

The unaided eye can resolve 4 lines to 1 mm. at 10 inches distance; with the help of an eye-piece, of magnifying power 10, the eye resolves 40 lines to 1 mm. When an eye-piece micrometer is used, this is all that is obtainable, the unit of measurement being $\frac{1}{40}$ mm. about. If however a stage micrometer of some sort is adopted the full resolving power of the microscope is used on the scale of measurement and as a limit 4,000 lines to the mm. can be obtained for the best possible microscope. But as pointed out in Section VII., these stage micrometer measurements are indirect and hence liable to error. The simple electric micrometer gives the same fineness of measurement but has the many advantages of being direct-acting.

The range of the microscope is small except by a step-by-step process which is objectionable. Hence the microscope as generally used, is accurate, but not delicate or long in scale.

Optical Lever.—Using no levers, but merely a three-legged stand carrying a mirror, we can easily get as unit $\frac{1}{500}$ mm., or possibly $\frac{1}{1000}$ mm. But there are two grave objections, (1) we have to correct for changing obliquity of the spot on the scale, (2) the calibration of the instrument is difficult to perform accurately and itself involves errors.

The scale is not long; it is necessarily limited, except by the awkward step-by-step process.

Hence the optical lever may be said to be delicate, but not accurate or long in scale.

Simple Electric Micrometer.—As to delicacy, $\frac{1}{1000}$ mm. can be obtained easily. Dividing by eye $\frac{1}{10000}$ mm. is obtainable. This is done in section VIII. The accuracy is great, successive readings being in excellent agreement. The sources of inaccuracy in the optical lever do not here exist. As to length of scale, the instrument works from 1 μ to 40,000 μ or more if desired.

Besides having delicacy, accuracy, and great length of scale, the instrument is adaptable to any sort of experiment. There is no

kind of small measurement, as far as I know, which the apparatus cannot make.

The arrangement shown in Fig. 1 need not be costly, and the accessories, cell, telephone and resistance-box, are in every laboratory.

A unit of much less than $\frac{1}{1000}$ mm., say $\frac{1}{10000}$ mm., could be obtained by having a larger graduated disc or by using levers, but if any greater accuracy is required it would be necessary to avoid vibrations by hanging the apparatus on rubber suspensions, and thus the present simplicity would be lost.

There is one simple improvement on the instrument as described above. A pulley groove can be worked on the large milled head (Fig. 1) and the screw rotated by a pulley string and outside hand pulley. The hand if used directly on the screw may be found to be too unsteady for the finest work.

UNIVERSITY COLLEGE,
NOTTINGHAM.

THE DAMPED BALLISTIC GALVANOMETER.

BY O. M. STEWART.

THE theory of the ballistic galvanometer as usually presented assumes either that there is no damping or that it is small. Consequently there is a general impression that a well-damped ballistic galvanometer cannot be used to measure or compare quantities of electricity. Yet it has been shown experimentally that such a galvanometer can be used. When used in determinations of magnetic induction¹ no appreciable errors have been detected. However this has not exempted such use from some criticism.²

Theoretically, damped ballistic galvanometers may be divided into two classes: one where the motion is vibratory or periodic, the other where the motion is aperiodic. As the theory of the latter does not seem to be available it has been thought advisable to present here a brief discussion of the subject.

The general equation of the motion of a vibrating system is

$$K \frac{d^2\theta}{dt^2} + \alpha \frac{d\theta}{dt} + \beta\theta = 0 \quad (1)$$

where K is the moment of inertia, $\alpha \frac{d\theta}{dt}$ the moment of the retarding force due to the damping and $\beta\theta$ the moment of the force of restitution. If the galvanometer is of the D'Arsonval type β is the moment of torsion of the suspension. If of the other or Kelvin type

$$\beta = MH$$

where M is the magnetic moment of the needle and H is the horizontal component of the earth's field.

¹ Sheldon and Cocks, *Electrical World*, 31, p. 735, 1898.

² See Discussion in *Proc. Am. Inst. E. E.*, Vol. 17, pp. 15 and 18, 1900.

The solution of equation (1) is

$$\theta = Ae^{\mu_1 t} + Be^{\mu_2 t}$$

where

$$\mu_1 = -\frac{a}{2K} + \lambda$$

and

$$\mu_2 = -\frac{a}{2K} - \lambda$$

and

$$\lambda = \sqrt{\frac{a^2}{4K^2} - \frac{\beta}{K}}.$$

If an initial velocity ω_0 be imparted to the suspended system the initial conditions are when $t = 0$,

$$\theta = 0, \quad \frac{d\theta}{dt} = \omega_0.$$

If this initial velocity is given by the discharge of a quantity Q through a D'Arsonval galvanometer

$$\omega_0 = \frac{ANHQ}{10K},$$

where A is the mean area of the coil, N the number of turns, H the intensity of the magnetic field and K the moment of inertia.

This may be written

$$\omega_0 = cQ.$$

For the Kelvin type

$$\omega_0 = \frac{MGQ}{10K} = cQ$$

where G is the "coil constant" of the galvanometer and M and K have the same meaning as before.

Applying these initial conditions the solution of equation (1) becomes

$$\theta = \frac{cQ}{2\lambda} (e^{\mu_1 t} - e^{\mu_2 t}) e^{-\frac{a}{2K}t}. \quad (2)$$

If λ is imaginary, that is if

$$\frac{a^2}{4K^2} < \frac{\beta}{K}$$

the motion is periodic and equation (2) becomes

$$\theta = \frac{cQ}{\sqrt{\beta - \frac{a^2}{4K^2}}} e^{-\frac{a}{2k}t} \sin t \sqrt{\beta - \frac{a^2}{4K^2}}. \quad (3)$$

This is the equation usually given. However, if, the damping is so great that λ is real the motion is aperiodic and equation (2) gives the value of θ .

From equation (2) it is seen that θ at first increases, reaches a maximum and returns to the zero position (theoretically) after an infinite time.

If the time it takes θ to reach its maximum value in equation (2), is independent of Q , that is, if this time is a constant, the maximum value of θ , or the throw, is directly proportional to Q . This can be shown as follows:

Let t_1 be the time when $\frac{d\theta}{dt} = 0$, that is when θ is a maximum.

Then from (2)

$$\frac{d\theta}{dt} = 0 = \frac{cQ}{2\lambda} e^{-\frac{a}{2K}t_1} \left[\lambda(e^{\lambda t_1} + e^{-\lambda t_1}) - \frac{a}{2K}(e^{\lambda t_1} - e^{-\lambda t_1}) \right]$$

or

$$\lambda(e^{\lambda t_1} + e^{-\lambda t_1}) = \frac{a}{2K}(e^{\lambda t_1} - e^{-\lambda t_1}).$$

This may be written

$$e^{2\lambda t_1} = \frac{\frac{a}{2K} + \lambda}{\frac{a}{2K} - \lambda}. \quad (4)$$

Hence t_1 is independent of Q , and is a constant. Equation (2) may be written

$$\theta_1 = \frac{cQ}{2\lambda}(e^{\lambda t_1} - e^{-\lambda t_1})e^{-\frac{a}{2K}t_1} \quad (5)$$

Or the throw in a galvanometer whose motion is aperiodic is directly proportional to the quantity and therefore conforms to the general rule of undamped ballistic galvanometers.

In the same way it can be shown that the time it takes θ to reach its maximum value in (3) where the motion is periodic is also a constant and the throw is directly proportional to Q . Hence, no matter how large the damping is, the throw is directly proportional to the quantity of electricity suddenly discharged through it.

The assumption has been made that the damping force is directly proportional to the velocity. If the damping is due to induced currents, no matter where, this is strictly true. In the case of air damping it is approximately true. It is also assumed that the force of restitution is proportional to the angular deflection. This for the movable needle type is only true for small deflections, but for the D'Arsonval galvanometer it is not so restricted. If the magnetic field of the D'Arsonval galvanometer is not uniform, it will affect this general law only as it may affect the damping factor α .

The quantity

$$\frac{\alpha^2}{4K} - \frac{\beta}{K} = \lambda^2$$

is negative for periodic and positive for aperiodic vibrations. When the damping is small or negligible, this quantity is not large as the period of vibration is $\frac{2\pi}{\sqrt{-\lambda^2}}$. As the damping is increased λ^2 increases, passes through zero and becomes positive. Hence for a considerable range of damping λ^2 is small.

If the exponential functions in equation (5) containing λ be expanded the equation becomes

$$\theta_1 = cQ t_1 \left(1 + \frac{\lambda^2 t_1^2}{3!} + \frac{\lambda^4 t_1^4}{5!} + \dots \right) e^{-\frac{\alpha}{2K} t_1}. \quad (6)$$

From this it is seen that excepting the effect the change has on t_1 , the value of the throw is approximately independent of λ^2 . From equation (4) it may be shown that changes in λ have but little effect on t_1 provided $\frac{\alpha}{2K}$ remains constant. Since $\lambda^2 = \sqrt{\frac{\alpha^2}{4K^2} - \frac{\beta}{K}}$

the changes in λ are confined to changes in β . Hence in an aperiodic galvanometer the throw is approximately independent of β . Or if the galvanometer is one where the magnet moves, the throw is approximately independent of changes in the earth's field. This was verified with an Edelmann galvanometer whose bell-shaped magnet is well damped by a copper damper. On reversing the controlling magnet so that the steady deflection for a certain current was about *four times* as great as before, it was found that the throw due to a discharge was increased only about 15 per cent. In the case of an aperiodic D'Arsonval galvanometer the throw is approximately independent of the moment of torsion of the suspension strips since β is here equal to the moment of torsion. This is of some practical importance, as it often happens that in low resistance galvanometers of this type the suspension strips have as great and sometimes greater resistance than the coil itself. When this galvanometer is intended for ballistic work with an earth inductor or in electromagnetic experiments where the damping is necessarily great, the galvanometer being used on closed circuit, a considerable gain may be had by not making the strips so small.

These facts may also be explained by saying that in a well-damped galvanometer the principal agent in overcoming the velocity imparted to the needle and thus limiting the throw is not the earth's field or the torsion of the suspension but the damping.

In the special case where $\lambda^2 = 0$, one has from (2)

$$\theta = cQte^{-\frac{a}{2K}t}. \quad (7)$$

By differentiating this with respect to t and remembering that $\frac{d\theta}{dt} = 0$ when $t = t_1$, one obtains

$$1 - \frac{at_1}{2K} = 0,$$

or

$$t_1 = \frac{2K}{a}.$$

Or

$$\theta_1 = \frac{2cQK}{a} e^{-1}.$$

Here θ_1 is independent of changes in the earth's field or suspension except as those changes affect the condition that λ is zero. That is the rate of change of θ_1 with respect to these changes is at this point zero.

Since

$$4\bar{K}^2 - \frac{\beta}{K} = \lambda^2 = 0$$

$$t_1^2 = \frac{K}{\beta}. \quad (8)$$

If there is no damping the period of vibration is

$$T = 2\pi \sqrt{\frac{K}{\beta}}. \quad (9)$$

From (8) and (9)

$$\frac{T}{t_1} = 2\pi. \quad (10)$$

Or the undamped period is 2π times the time of throw when the damping is just sufficient to make the needle aperiodic. Since the time of the throw when undamped is one fourth of the period the effect of this damping is to shorten the time of the throw by the factor $2/\pi$, that is to about two thirds of its undamped value.

In this special case the effect of the damping on the sensibility of the instrument is readily obtained. The value of the undamped throw or the maximum value of θ may be shown from (3) to be

$$\theta_1 = \frac{cQ}{2\pi} T \quad (11)$$

where T has the value given in (9).

Since when damped

$$\theta_1 = cQt_1e^{-1}$$

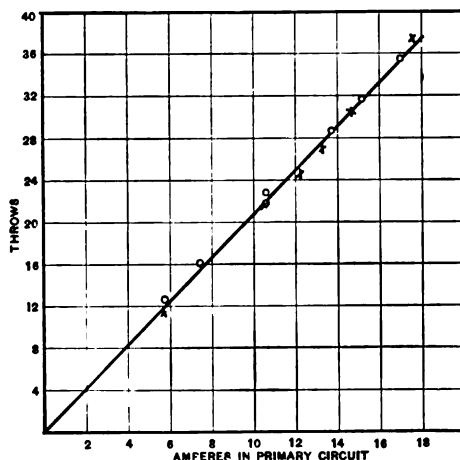
one has as the ratio of the damped throw to the undamped

$$\frac{2\pi t_1}{eT} = \frac{1}{e}$$

from (10).

Or the throw when the damping has been increased so that the motion is just aperiodic, that is does not swing past its zero position on the return, is about 37 per cent. of the undamped throw.

In Fig. 1 is given a curve showing the relation between the throws of a Weston milliammeter and the quantity discharged through it. The milliammeter was connected to the secondary of a



The Ballistic Calibration of a Weston Ammeter.

Fig. 1.

solenoid wound on a wooden core. The throw was obtained on making the current in the primary circuit. Since the quantity discharged through the secondary was proportional to the current in the primary circuit, currents have been plotted instead of quantities. It is seen from the curve that the throws are proportional to quantity. Since the throws of the index of the ammeter are very quick they are rather hard to read. The accuracy of reading may be judged from the curve, as two sets of observations taken by different persons are given. These persons had had but little previous practice in reading the throws of such an instrument.

In all the determinations of magnetization curves and hysteresis loops by the ballistic method a great deal of time may be saved by using as the ballistic galvanometer an ammeter without its shunt,¹ or a voltmeter without its series resistance.

¹ The Weston Type E, a portable instrument, and nearly all switch-board types are suitable, as they have external shunts.

There were two drawbacks to the use of the damped galvanometer or ammeter as a ballistic galvanometer:—first, its decreased sensibility, and second, that one cannot determine its constant from its period and figure of merit. In most work this method is not used, it being more convenient to use a solenoid. The decrease in the sensibility is in most cases not a serious drawback, for it rarely happens that this must be very great. Damped D'Arsonval galvanometers that are quite satisfactory for nearly all ballistic work are not only available, but far more convenient than the undamped galvanometers.

UNIVERSITY OF MISSOURI,
December 20, 1902.

REFLECTION FROM WALLS OF PHOTOMETER ROOM.

BY F. J. ROGERS.

AN experiment suitable for junior students which will demonstrate satisfactorily that the intensity of illumination varies inversely as the square of the distance from the source is not easy to arrange. The difficulty in varying the intensity of a source in a known ratio, the irregular variation of both sources, reflection from walls of photometer room, and lack of sensitiveness of the eye for small differences of illumination all conspire in producing results which are very unsatisfactory.

I had arranged an experiment of this sort in which the variable source consisted of one or more four-candle-power incandescent lamps. Instead of using a photometer room the lamps and photometer were enclosed in long photometer boxes, the head of the observer being covered with a black cloth to keep out stray light. The object of the experiment was to determine the relation between the intensity of illumination on one surface of the photometer screen and the distance to the source producing the illumination. The results obtained were worked up by plotting the logarithms of intensity and of distance. The slope of the resulting curve (which is a straight line) gives the power of the distance according to which the intensity of the illumination varies. The results obtained by my students gave, instead of the inverse square, inversely as the 1.8 to 1.9 power, or in symbols $I = kD^{-1.85}$. The fact that this power is less than 2 is readily explained by the irregular reflection which takes place from the walls of the photometer box. I was surprised not so much at the amount of reflection as at the deviation produced from the theoretical inverse square law, and determined to give the matter a further test in a photometer room.

Several series of observations were made in which the variable source consisted of one or more of eight four-candle-power lamps.

A Lummer-Brodhun photometer was placed at a *fixed* distance from the 4-c.-p. lamps while a 16-c.-p. lamp on the other side of the photometer was *moved* until the two sides of the photometer screen were equally illuminated. This was repeated for one, two, three, four, six and eight of the 4-c.-p. lamps. The 4-c.-p. lamps were approximately equal in illuminating power, but observations were so taken as to eliminate from the results the effect of the unavoidable inequalities in the different lamps. For example, when one lamp was used observations were made with each of the 4-c.-p. lamps separately and the mean taken; when six lamps were used observations were made successively with — all but Nos. 1 and 2, all but 3 and 4, all but 5 and 6, and all but 7 and 8.

Assuming the relation $I = kD^n$, the value of n was determined by the method of least squares from the observations given below and found to be — 1.92. Several series of preliminary observations gave for n values very nearly equal to the one just given. In order to be certain that the deviation from the inverse square relation was due to reflection from the wall of the photometer room the latter was draped — wherever any perceptible reflection to the photometer was probable — with black canton flannel plaited so as to reflect the least possible light.

Another series of observations were taken under exactly the same conditions. When n was computed from this series of observations it proved to be almost precisely equal — 2 as demanded by the inverse square law.

No. of 4-c.-p. Lamps at fixed Distance from Photometer.	Distance from Photometer to Movable Source.	
	With Drapery on Wall Bet. Photometer and Mov. Source.	Drapery "Pinned Up."
1 Lamp.	206.0 cm.	220.2 cm.
2	144.6	155.6
3	117.8	125.9
4	101.7	107.4
6	84.0	86.7
8	72.7	75.4

The table on page 173 gives the data from which was computed the value of n given above. Each "distance" given in the table is the mean of eight or sixteen different observations so taken as to eliminate the effect of inequalities among the 4-c.-p. lamps.

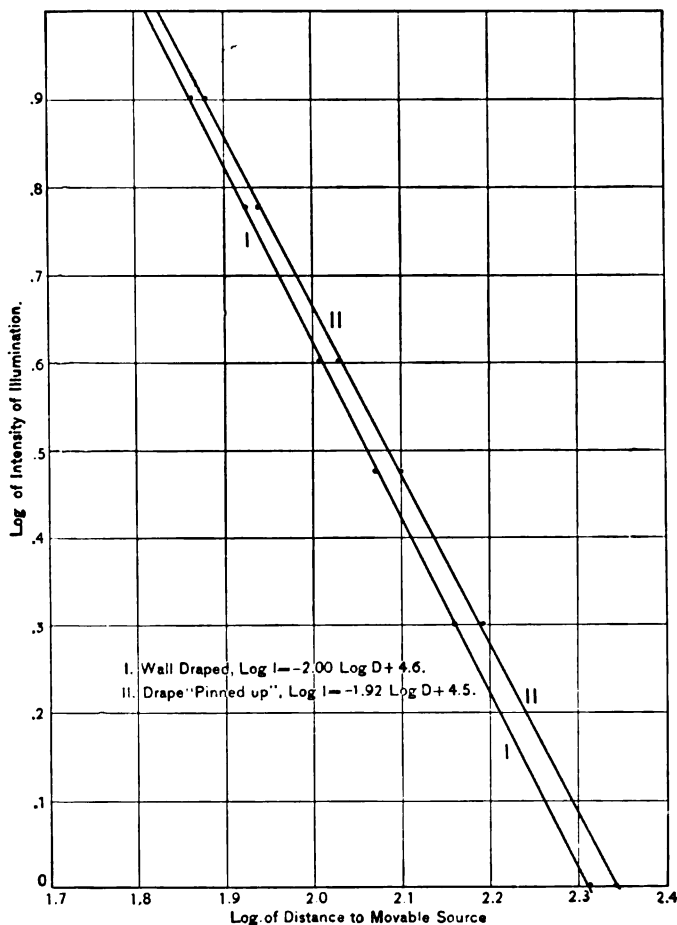


Fig. 1.

In the plot, Fig. 1, the logarithms of the data given in the table are plotted. The shifting of the line produced by removing the drapery from the wall illustrates the reflection from the walls of the photometer room. The change in slope illustrates the change in

the law which connects intensity of illumination with distance from source.

It is a little surprising that the use of drapery on the black wall of a photometer room should reduce the light reaching one side¹ of the photometer from 7% to 15% and at the same time change the relation of intensity to distance from the inverse 1.92ths power to the usual inverse 2d power. In the present case, however, the conditions were rather favorable for reflection, as the photometer bar was less than 30 cm. from the wall and the latter was painted with asphalt paint which did not give a "dead" black surface.

STANFORD UNIVERSITY, May, 1902.

¹ The drapery was placed so as to prevent reflected light from falling on one side of the photometer screen, the other side of the photometer was entirely unaffected by it.

WASHINGTON MEETING OF THE AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE.

THE experiment of holding a winter meeting of the American Association during Convocation Week, tried this year for the first time, proved eminently successful, the attendance being larger than at any meeting in recent years. The favorable location of Washington and the many attractions which this city offers to scientific visitors doubtless contributed much to the success of the meeting; yet it can hardly be doubted that the change in the time of the meeting was also a factor of importance.

Following the custom established at New York in 1900, a meeting of the American Physical Society was held in connection with the meeting of Section B of the American Association. On Wednesday, December 31, and Thursday, January 1, a joint session of the two societies was held. On the former day the program was made up of Physical Society papers only.

While the attendance in Section B, and in the joint session with the American Physical Society, was unusually large, the program was not so crowded as it has often been in the past. In addition to the address of the retiring chairman of Section B, Prof. W. S. Franklin, on "Popular Science," twenty-six papers were read before Section B. In the joint session with the Physical Society, nineteen papers were presented, making a total of forty-five. The hurried feeling that has often been felt at the meetings of the association, caused by the necessity of completing a too extended program, was less marked than usual, and a more extended discussion of the papers presented naturally resulted. It is to be hoped that this feature of the meetings will become even more prominent.

The following is a list of the papers presented to Section B:¹

The Semidiurnal Periods in the Earth's Atmosphere. Frank H. Bigelow.

The Construction of a Sensitive Galvanometer. C. G. Abbott.

The Conditions Governing the Coherence of Metals when there is an Electrical Discharge between Them. Carl Kinsley.

A Determination of the Frequency of Alternating Currents by the Automatic Adjustment of the Circuit to Resonance. Carl Kinsley.

¹ A list of Physical Society papers will be found on p. 173.

On Methods of Measuring Radiant Efficiency. E. L. Nichols and W. W. Coblentz.

The Infra-red Spectrum of the Mercury Arc. W. W. Coblentz and W. C. Geer.

Derivation of Equation of Decaying Sound in a Room; and Definition of Open Window Equivalent of Absorbing Power. W. S. Franklin.

On the Velocity of Light as Affected by Motion Through the Ether. Edward W. Morley and Dayton C. Miller.

Some Measures of the Speed of Photographic Shutters. Edward W. Morley and Dayton C. Miller.

On the Distribution of Pressure Around Spheres Moving with Constant Velocity in a Viscous Fluid. S. R. Cook.

A Portable Apparatus for the Measurement of Sound. A. G. Webster.

The Damped Ballistic Galvanometer. O. M. Stewart.

On the Electrical Conductivity of Solutions in Amyl Amine. Louis Kahlenberg.

On the Thermal Conductivity of Glass. H. W. Springsteen.

Some Relations Between Science and the Patent Office. Charles K. Wead.

Why the E.M.F. of the Daniell Cell Changes when the Densities of the Solutions Change. Henry S. Carhart.

Absolute Measurement of the E.M.F. of the Cadmium Cell. Henry S. Carhart and Karl E. Guthe.

The Characteristic Absorption Curves of the Permanganates. B. E. Moore.

The Magnetic Rotary Dispersion of Solutions of Anomalous Dispersive Substances. F. J. Bates.

Atmospheric Circulation Near the Equator. A. L. Rotch.

The Anomalous Dispersion and Selective Absorption of Fuchsin. Wm. B. Cartmel.

Sulphur Dioxide and the Binary Vapor Engines. R. H. Thurston.

The Coefficient of Expansion of Some Alloys of Nickel and Cast Iron. Theo. M. Focke.

A New Apparatus for Demonstrating Wave Motion. Fred. J. Hillig.

Demonstration of a Portable High Tension Coil. G. Lenox Curtis.

The next meeting of the Association will be held at St. Louis during Convocation week, 1903—*i. e.*, during the week beginning Monday, December 28, 1903. No meeting will be held during the coming summer. The omission of the summer meeting in 1903 is not to be regarded, however, as indicating the permanent policy of the association. Special reasons were advanced in the Council for omitting the summer meeting for this year; but a strong sentiment exists in the Association in favor of meetings during the summer, so that it seems not unlikely that in subsequent years two meetings may be held.

The officers of Section B for the coming year are E. H. Hall, chairman, and D. C. Miller, secretary. The Sectional Committee consists of Messrs. E. H. Hall, D. C. Miller, E. F. Nichols, G. F. Hull, A. G. Webster, D. B. Brace, Ernest Merritt, and C. E. Mendenhall.

PROCEEDINGS
OF THE
AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE EIGHTEENTH MEETING.¹

THE annual meeting of the American Physical Society was held at the Columbian University, Washington, D. C., on Wednesday, December 31, 1902, and Thursday, January 1, 1903, in conjunction with the meeting of Section B of the American Association for the Advancement of Science.

Vice-President Webster presided.

The following officers were elected for the year 1903 :

President, Arthur G. Webster ; *Vice-President*, Elihu Thomson ; *Secretary*, Ernest Merritt ; *Treasurer*, William Hallock ; *Members of the Council*, W. F. Magie and E. H. Hall.

The following papers were presented :

Results of Comparisons of Magnetic Instruments. L. A. BAUER.

The Pressure Due to Radiation (Final Results). E. F. NICHOLS and G. F. HULL.

Is there a Southerly Deviation of Falling Bodies? EDWIN H. HALL.

Elasticity of Copper and Steel at -186°C . J. R. BENTON.

Thermodynamic Formulæ for Isotropic Solids subject to Tension.
J. R. BENTON.

Experiments in Connection with Friction Between Solids and Liquids.
J. R. BENTON.

The Heat of Vaporization of Oxygen and Nitrogen. J. S. SHEARER.

The Magnetic and Electric Deviation of the Easily Absorbed Rays from Radium. E. RUTHERFORD.

A Penetrating Radiation from the Earth's Surface. E. RUTHERFORD and H. L. COOK.

Induced Radioactivity Excited at the Foot of Water Falls. J. C. MCLENNAN.

Some Experiments on the Electrical Conductivity of Air. J. C. McLENNAN and E. F. BURTON.

Radioactivity of Freshly Fallen Snow. S. J. ALLEN.

On the Double Refraction of Dielectrics in a Magnetic Field in a Direction at Right Angles to the Lines of Force. D. B. BRACE.

The Viscosity of Liquids at Low Rates of Shear. A. WILMER DUFF.

On the Mechanical Efficiency of Musical Instruments in Producing Sound. A. G. WEBSTER.

Experiments Concerning very brief Electrical Contacts. HERSCHEL C. PARKER.

A Comprehensive Boyle's Law Apparatus. W. J. HUMPHREYS.

Lecture Room Method of Analyzing Irregular Currents. W. J. HUMPHREYS.

The Critical Current Density and Drop of Potential at the Cathode in Vacuum Tubes. C. A. SKINNER.

Adjourned.

ERNEST MERRITT,
Secretary.

AN EXPERIMENT RELATING TO THE APPLICATION OF LAGRANGE'S EQUATIONS OF MOTION TO ELECTRIC CURRENTS.¹

BY WILLIAM S. DAY.

THE experiment described was analogous to one mentioned by Maxwell in his Treatise on Electricity and Magnetism, Section 574, Volume II. For the purpose of applying Lagrange's equations of motion to the phenomena of electric currents, Maxwell considered that the state of a system of electric currents could be given by two kinds of generalized coördinates, the mechanical coördinates which indicate the positions of the wires or conductors carrying the currents, and the electrical coördinates whose velocities represent the currents flowing. Following out the analogy of a purely mechanical system, the kinetic energy of an electrical system would in general be a homogeneous quadratic function of the velocities of the coördinates. For one linear circuit alone, having one degree of mechanical freedom, this would be of the form,

$$T = \frac{1}{2} \dot{x}^2 + K \dot{x} \dot{y} + \frac{1}{2} L \dot{y}^2$$

in which T = the kinetic energy, $\dot{x}$ = the velocity of the mechanical

¹ Abstract of a paper presented at the meeting of the Physical Society held on April 21, 1902.

coördinate, $\dot{x}$ = the current, and I , K and L are coefficients which may be constants, or functions of x , but as Maxwell shows, not of y . I of course corresponds to a mass or a moment of inertia, and L is the self-induction of the circuit. Maxwell's experiment referred to above was for the purpose of finding out whether or not any part of the kinetic energy of the circuit should be expressed by the term $K\dot{x}y$. In his experiment he takes as his mechanical coördinate one whose variation expresses motion of the wire in the direction of its length. If such a term existed, a wire would experience a sudden impulse in the direction of its length when a current was suddenly started, or suddenly stopped in the wire. Maxwell states that no evidence of the existence of such a term has ever been discovered. So far, however, as Lagrange's equations of motion themselves are concerned, and apart from any hypothesis as to the nature of an electric current, a different mechanical coördinate might have been considered, namely, one whose variation expresses rotation of the wire around its axis. If the term $K\dot{x}y$ had an existence, this being the meaning attached to the mechanical coördinate, the wire would experience an impulsive torque when a current was suddenly started or stopped in it. The present experiment was made for the purpose of discovering such an effect if it existed, or at least of fixing as small a maximum limit to its value as possible. For this purpose a straight aluminum wire 30 cm. long and 0.25 cm. in diameter was suspended by a quartz thread so that while it was free to rotate about its axis a current of 10 ampères could be passed through it at will by means of mercury cups, in either direction. The actual moment of inertia of the system was about 0.07 C.G.S. units, and the free period of vibration around the axis about 2 seconds. After reducing all disturbing forces to a minimum, no certain indication of such an effect could be detected. If the coefficient K supposed to be a constant as the simplest hypothesis, and referred to unit length of the wire, had had a value as great as about 0.00002, measured in C.G.S. electromagnetic units, it could have been detected. If K had this value it would mean that if a current of 10 ampères in a wire were made to decrease to zero at a uniform rate in exactly one second there would be during this second a constant torque acting on the wire on each centimeter of its length equal to a force of 0.00002 dyne with a lever arm of 1 centimeter.

This coefficient could also be tested for by observing the electromotive force, if any, produced when a rapidly rotating wire is suddenly brought to rest.

A NEW METHOD OF INTEGRATING ONE OF THE DIFFERENTIAL
EQUATIONS OF THE THEORY OF HEAT DIFFUSION.¹

BY R. S. WOODWARD.

MANY problems in the theory of heat diffusion require integration of the equation

$$\frac{\partial v}{\partial t} = a^2 \frac{\partial^2 v}{\partial x^2} \quad (1)$$

subject to given initial and boundary conditions. In this equation a^2 is a constant (diffusivity), t is time after some initial epoch, x is distance and v is temperature or the product of temperature by x .

To find an integral of (1), assume

$$2az\sqrt{t} = q \pm x, \quad (2)$$

where q is an arbitrary constant distance and z is a new variable. Thus v may be regarded as a function of z , and it follows that

$$\frac{\partial v}{\partial t} = \frac{\partial v}{\partial z} \frac{\partial z}{\partial t} = - \frac{\partial v}{\partial z} \frac{z}{2t},$$

$$\frac{\partial v}{\partial x} = \frac{\partial v}{\partial z} \frac{\partial z}{\partial x} = \pm \frac{\partial v}{\partial z} \cdot \frac{1}{2a\sqrt{t}},$$

$$\frac{\partial^2 v}{\partial x^2} = \frac{\partial^2 v}{\partial z^2} \cdot \frac{1}{4a^2 t};$$

whence

$$\frac{\partial^2 v}{\partial z^2} = - 2z \frac{\partial v}{\partial z}.$$

The integral of this is

$$\log \left(\frac{\partial v}{\partial z} \right) = - z^2 + \log C$$

or

$$\frac{\partial v}{\partial z} = Ce^{-z^2},$$

whence

$$v = C \int_{z_1}^{z_2} e^{-z^2} dz. \quad (3)$$

respectively. Or, since q is an arbitrary distance, let it be replaced by nx_0 , where n is zero or any integer and x_0 is an arbitrary distance. Write also for brevity

$$m_0 = \frac{x_0}{2a\sqrt{t}}, \quad m = \frac{x}{2a\sqrt{t}}. \quad (4)$$

Then (3) may be written

$$V = C \int_{nm_0-m}^{nm_0+m} e^{-z^2} dz. \quad (5)$$

Now since this satisfies (1) for every value of n , (1) will be satisfied also by the sum of any number of such expressions. Hence a general solution of (1) is

$$V = \sum_{n=0}^{\infty} C_n \int_{nm_0-m}^{nm_0+m} e^{-z^2} dz. \quad (6)$$

To illustrate the application of this, consider the case of a homogeneous sphere cooling from an initial uniform temperature u_0 above an assumed constant temperature of the surface of the sphere. Then if u be the temperature at a distance r from the center of the sphere at any time t after the epoch, v in (1) may be replaced by $r(u_0 - u)$, x by r , and x_0 by the radius of the surface of the sphere, r_0 , say. Thus

$$\frac{\partial r(u_0 - u)}{\partial t} = a^2 \frac{\partial^2 r(u_0 - u)}{\partial r^2}$$

must be integrated subject to the following conditions:

$$ru = ru_0 \quad \text{for} \quad t = 0 \quad \text{and} \quad r > r_0, \quad (a)$$

$$ru = 0 \quad \text{for} \quad r = r_0, \quad (b)$$

$$ru = 0 \quad \text{for} \quad r = 0, \quad (c)$$

$$ru = 0 \quad \text{for} \quad t = \infty. \quad (d)$$

Condition (a) shows that $C_0 = 0$, and condition (b) shows that

$$C_2 = C_4 = C_6 \cdots = 0,$$

$$C_1 = C_3 = C_5 \cdots = \frac{2r_0 u_0}{\sqrt{\pi}}.$$

With these values of the constants C_n , equation (6) becomes

$$r(u_0 - u) = \frac{2r_0 u_0}{\sqrt{\pi}} \left\{ \int_{m_0-m}^{m_0+m} e^{-z^2} dz + \int_{3m_0-m}^{3m_0+m} e^{-z^2} dz + \cdots \right\},$$

and this will be found to satisfy the conditions (c) and (d).

ROTARY POLARIZATION MECHANICALLY PRODUCED.¹

BY ARTHUR W. EWELL.

IN previous papers² the author has shown that when plane-polarized light traverses jelly, subjected to twist in a rubber tube, the plane of polarization is rotated in the opposite direction to the twist. Some such result had been looked for in vain by F. E. Neuman and Drion.³

The work to be very briefly reviewed in this abstract was performed during the two past years and had as its object a search for definite quantitative relations between the rotary polarization and the twist and twisting moment, and to determine what other conditions besides the twist influence the rotary polarization. Most of the work has been performed with jelly composed of the choicest calves' foot gelatine, water and glycerine; the proportion best adapted to most of this work was 1 g. of gelatine to 5 c.c. of water to 5 c.c. of glycerine. A simple form of biquartz polarimeter was used with mechanical attachments for applying and measuring twists and longitudinal compression and elongation. The polarizer and graduated circle of the analyzer could be rotated together by means of gearing, thus facilitating reading the rotation in different azimuths to eliminate double refraction.

A lateral envelope is not essential, but the rotatory polarization is greatly increased by an elastic envelope. For example:

Jelly cylinder 8 cm. long, 2 cm. diameter—

No Envelope.			Rubber Tube Slipped Over.	
Twist,	-170°	190	90	-90
Rotation, ⁴	1°	-1	-6	9

Longitudinal compression increases the rotation while longitudinal elongation decreases it. For example:

3.7 cm of jelly in a soft-rubber tube subjected to -90° twist, decrease of length is taken as positive—

Change of length (mm.),	0	1.4	2.5	-1.6	-3.5	-5.4
Rotation,	14	35	74	6	5	1

Upon plotting changes of length as abscissæ and rotations as ordinates the observations are found to lie quite closely upon exponential curves.

Hydrostatic pressure was applied to jelly twisted in rubber tubes up to 160 lbs. per sq. in. with no observable change in the rotation.

¹ Abstract of a paper presented at the meeting of the Physical Society held on October 25, 1902.

Relaxation effects are very conspicuous but may be eliminated by reading the actual zero of twist before and after each twist. There is a curious influence of the previous history of the jelly; a prolonged preceding twist increases the rotation for an immediately following less twist in either direction but has little influence upon a greater twist. A rapid succession of twists in opposite directions of short duration has no effect. The rotation for a given twist falls off with excessive rise of temperature.

If then, the longitudinal strain is kept constant, if relaxation effects are eliminated, if small twists precede greater, and if the temperature remains sensibly constant, under these conditions we should expect that the rotatory polarization of a given jelly cylinder inside a permanent rubber lateral envelope, for a given twist, would be constant and definite, and such is approximately the case. For example :

3 cm. Length of jelly in rubber tube—

July 21. A. M.

Twist,	-90	90	-110	110	-120	120	-130	130
Rotation,	1	-4	2	-13	2	-18	2	-26
Sum,		5		15		20		28

July 21. P. M. After Tube has been Removed from Instrument, Turned End for End, and Replaced.

Twist,	60	-60	90	-90	110	-110	120	-120	130	-130
Rotation,	-1	1	-6	0	-13	0	-21	1	-26	1
Sum,		2		6		13		22		27

July 22. A. M. After Tube has been Again Removed, Turned End for End, and Replaced.

Twist,	-60	60	-90	90	-110	110	-120	120	-130	130
Rotation,	0	0	2	-4		-11	3	-17	4	-22
Sum,		0		6		13		20		26

The dissymmetry in the two directions is largely due to dissymmetry in the rubber tube. The sum or average of the rotations is found to be quite closely proportional to the fourth power of the twist.

The mechanical properties of the jelly were carefully studied. The most interesting discovery was that the rigidity is greatly increased by longitudinal elongation. The method of vibration was used, timing the period of vibration of a clamp attached to the lower end of a jelly cylinder, loaded with different weights. The cross-section was determined by the displacement of a known length of jelly in benzine, correction being made for the change in cross-section when the jelly was stretched. The rigidity of a certain jelly cylinder was found to be 1.8×10^5 when the length was 15.7, 2.5×10^5 when the length was 16.2 and 7.6×10^5 when the length was 16.9.

The author has also formulated a theory to explain the general phenomena, a full statement of which would be too long for this abstract.

The points successively disturbed by two opposite circularly polarized rays, parallel to the axis of a twisted cylinder, lie along two opposite spirals. Since the stresses and strains increase as one leaves the axis of a twisted cylinder, conditions on the side of these spirals farther from the axis will have a determining influence. The twist will produce elongation along the outside of the spiral of the same direction of twist as the cylinder, which will belong to the circularly polarized ray of opposite sign, and compression along the outside of the other spiral. Since in general decrease in the velocity of light accompanies increase in density or longitudinal compression, we should expect that the ray circularly polarized in the opposite direction to the twist of the cylinder would be propagated faster or the plane of polarization would be rotated in the opposite direction to the twist. The positions of the axes of strain are such that longitudinal compression increases the compression along one spiral more than it decreases the elongation along the other and hence should increase the rotation.

WORCESTER POLYTECHNIC INSTITUTE,
November 10, 1902.

THE PRESSURE DUE TO RADIATION.¹

BY E. F. NICHOLS AND G. F. HULL.

AN account of the results of later measurements of radiation pressure in comparison with measurements of radiant energy was communicated. The principal change in the method of measurements was the substitution of a silver disc for the bolometer earlier used to measure the energy of the beam. Pressure measurements were carried through for a beam which had traversed a plate of red glass and one which had passed through a water cell in addition to a beam as employed in the preliminary work.² No difference in the relation of pressure to energy depending on the wave-length of the incident beam was discovered.

The later measurements were far more comprehensive and accurate than the earlier ones had been. Many corrections to the final results were made after a careful and exhaustive study of all the conditions affecting the experiments. The results are in agreement with the Maxwell-Bartoli theory to within about one per cent. A study of the degree of accuracy

in the forthcoming Proceedings of the American Academy of Arts and Sciences for 1902-3. More condensed accounts will appear elsewhere. A demonstration of the effect of radiation pressure upon the torsion balance used in the experiments was made before the society.

A vacuum tube in the form of an hour-glass containing calcined puff-ball spores mixed with emery sand, was exhibited to illustrate the application of light pressure to the repulsion of comets' tails by the sun. In the preparation of the tube the exhaustion by a good Geissler mercury pump was continued for two days and after sealing off the tube from the pump, mercury in a connected flask was frozen for an hour in solid carbon dioxide and ether. Finally the mercury flask was sealed off from the hour-glass. It was thought that the traces of mercury vapor in the system might thus be frozen out and a very high vacuum obtained. The hour-glass was heated to the point of softening during the entire pumping and freezing process. When the glass was placed in an upright position, and the concentrated beam from an arc light directed horizontally on the falling stream of particles just below the neck of the glass, the small spores were driven off to one side in the direction of the beam while the sand was not deflected from its vertical path.

A hasty computation of the magnitude of the deflecting angle of the spores, due to radiation pressure, was made from the energy of the beam, roughly known, and the size of the spores. The observed deflection was approximately equal to that predicted by computation and the writers believed, for a time, that the forces due to the action of the residual gas constituted only a fraction of the total deflecting force. A later review of the computation disclosed an error in the numerical work which had made the predicted deflection due to radiation pressure much too large. The conclusion, therefore, was that the action of the residual gas in the tube considerably outweighed the pressure due to radiation. The writers purpose to make further experiments in which it is hoped the gas effects may be greatly reduced and the radiation pressure remain alone evident.

THE MAGNETIC AND ELECTRIC DEVIATION OF THE EASILY ABSORBED RAYS FROM RADIUM.¹

BY E. RUTHERFORD.

THE greater portion of the energy radiated by the natural radioactive substance is in the form of α or easily absorbed rays. Two other types of rays are also emitted, viz.: the β rays, readily deviable by

By employing strong magnetic fields and arrangements to detect a minute curvature of the path of the rays, it was found possible to show that the α rays are, like the β rays, completely deviable by a magnetic field. The deviation is in the opposite sense to the cathode rays so that the rays consist of positively charged particles projected with great velocity.

The general method employed was to pass the rays from the active substance (radium chloride activity 19,000) through a number of narrow slits placed in parallel and to observe the effect of a magnetic or electric field on the ionization produced by the issuing rays by means of a special gold leaf electroscope.

A current of hydrogen gas passed through the electrometer and through the slits in order to prevent the diffusion of any emanation into the testing vessel.

The rays were also found to be partly deviated by passing through a strong electric field.

By combining the results of the magnetic and electric deviation in the usual way it was found that

$$V = 2.5 \cdot 10^9 \text{ cm. per sec.}$$

$$\frac{e}{m} = 6.10^8$$

where V is the velocity of projected particles, e the charge on the particle and m its mass. These results are only approximate and merely indicate the order of the results obtained.

The rays thus consist of positively charged bodies projected with a velocity about one tenth that of light and of mass of the same order as the hydrogen atom and large compared with the electron. The α rays are thus very analogous to the "Canal Strahlen," which Wien has shown to consist of projected particles, atomic in size, carrying with them a positive charge.

The projective nature of the α rays offers a satisfactory explanation of the characteristic properties of these rays. Since all the active bodies and also excited bodies give rise to α rays, it seems probable that they consist in all cases of heavy positive particles projected with great velocity.

The projection of α rays is probably the underlying cause of the series

A PENETRATING RADIATION FROM THE EARTH'S SURFACE.¹

BY E. RUTHERFORD AND H. L. COOKE.

IT has been shown by Rutherford (*Phys. Zeit.*, 1902) that the radiations from the naturally radioactive bodies and also the excited radiations include some rays of an extremely penetrating character, which are able to pass through great thicknesses of matter. Since the excited activity obtained from the atmosphere is very similar in character to the excited radiations from thorium and radium, it was thought possible that some penetrating rays might be given off from the surface of the earth and walls and rooms on which excited activity from the air is distributed.

In order to test this point, the amount of ionization was observed in testing vessels of about 1 liter capacity. The method used by C. T. R. Wilson in his experiments on the "spontaneous" ionization of air was employed. The rate of discharge of a well insulated gold-leaf system served as a measure of the ionization. The effect of placing metal screens outside the testing vessel was observed. Screens of thickness of 2 mm. of lead had very little effect on the rate of discharge. A thickness of 5 cm. of lead, however, was found to cut down the rate of discharge by 30 %. A greater increase of thickness had no effect, although 5 tons of pig lead was placed around the apparatus. A thickness of 5 cm. of iron and 70 cm. of water also cut down the rate of discharge about 30 %.

On removing the screens the discharge returned to the original value. These results show that about 30 % of the ionization inside a closed vessel is due to an external radiation of great penetrating power. This radiation appears to come equally from all directions and is probably due to excited activity on the surface of the room in which the observations were made.

These effects could not be due to the presence of thorium or radium in the laboratory, for similar results were observed in the library which was free from all possible contamination by radioactive substances.

A decrease was also observed with metal screens when the testing vessel was placed down on the frozen earth at some distance from the laboratory. Under the conditions of the experiment, the effect was not quite so marked as inside the laboratory.

In brass vessels which had been thoroughly cleaned the rate of dis-

SOME EXPERIMENTS ON THE ELECTRICAL CONDUCTIVITY OF
ATMOSPHERIC AIR.¹

BY J. C. McLENNAN AND E. F. BURTON.

I. INTRODUCTION.

IN a paper by H. Geitel² reference is made to a gradual increase observed in the conductivity of a mass of atmospheric air after being confined in an air-tight chamber. This effect was found to require from four to five days to reach its maximum value and was observed in localities where no thorium compounds or other known radioactive substances existed.

In a subsequent investigation Elster and Geitel³ found that the air which had been confined for some time in closed caves or house cellars possessed an abnormally high conductivity. This phenomenon, together with the observed increase in conductivity mentioned above, they concluded, could not be due to the presence of dust or water vapor. They traced it rather, in both cases, to the existence of some undetermined radioactivity in the confining walls.

More recently these physicists discovered that atmospheric air possessed the property of exciting induced radioactivity in bodies exposed under negative electrification. This phenomenon of induced or excited radioactivity had been previously observed by Rutherford in bodies exposed to air drawn from the neighborhood of thorium compounds and had been connected by him very directly with an emanation which these salts emit. This emanation he found possessed the property not only of exciting radioactivity in all solid substances in its neighborhood, but also of ionizing any gas with which it was in contact.

Since atmospheric air has been shown by Elster and Geitel,⁴ C. T. R. Wilson⁵ and others to be continually ionized by some agent and since it has also been shown to possess the property of exciting radioactivity one is forced to conclude there is present in the air an emanation possessing properties similar to that emitted by thorium compounds.

Hitherto the source of such an emanation has not been determined, but, as the phenomena of induced radioactivity and spontaneous ionization universally characterize atmospheric air, it seems evident, since thorium compounds are but sparsely distributed in nature, that sources other than these must exist.

¹ Abstract of a paper presented at the meeting of the Physical Society, held on December 31, 1902.

² *Phys. Zeit.*, 2, pp. 116-119, 1900.

³ *Phys. Zeit.*, 2, pp. 560-563, 1901.

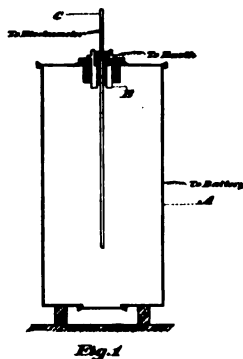
⁴ *Loc. cit.*

⁵ *Proc. Roy. Soc.*, March, 1901.

Recalling the experiments of Elster and Geitel, it seems probable that the earth's surface, and possibly too the materials used in the construction of their apparatus, are sources of this emanation. As but little evidence existed in favor of this conclusion, the writers recently made a series of observations upon atmospheric air confined in air-tight vessels of different metals. The result of the investigation showed that the conductivity of the enclosed air depended very largely upon the material of which the receiver was made, and the effects observed would seem to indicate that all metals in varying degree are the sources of a marked though feeble radioactive emanation.

II. APPARATUS.

In these observations the air whose conductivity was to be measured was confined in a cylinder 125 cm. in length and 25 cm. in diameter, similar to that shown in Fig. 1. In the first experiments it was made of sheet iron coated with zinc, and in the later experiments linings of various metals were inserted in order to examine their effect upon the conductivity of the enclosed air. The bottom and cover were removable, and, when in position, were made air-tight by means of cement. Into a flanged opening in the cover was fitted an ebonite plug about 5 cm. in diameter. A brass tube *B* was passed through this and into it a second ebonite plug was tightly fitted. This second plug carried a brass rod *C* which extended almost to the bottom of the cylinder. The brass tube *B*, which was earthed throughout the measurement, served as a guard ring and prevented any leak from the vessel to the rod *C* across the ebonite plugs.



The conductivity was measured by placing the cylinder upon an insulated platform, charging it by means of a set of small storage cells to a potential of 165 volts which sufficed for the saturation current, and observing the rise in potential of the electrode *D* which was joined to a quadrant electrometer in the usual manner. The sensibility of the electrometer was such as to produce a deflection of 1,000 mm. on a scale above meter distance for a potential difference of one volt between the quadrants.

III. CONDUCTIVITY MEASUREMENTS — TIME EFFECTS.

Before enclosing air for examination the cylinder was placed in an open window in the laboratory, with the ends removed and the air allowed to blow through it for some time. The top and bottom were then replaced,

cemented in position and the cylinder connected with the electrometer as quickly as possible.

Measurements on the conductivity were made at intervals of a few minutes at first, and it was invariably found that a rapid decrease in the ionization took place until a minimum value was reached. The conductivity then slowly increased and approached a limiting value in the course of two or three days.

TABLE I.

Current; Arbitrary Scale.

Time.	Current: Arbitrary Scale.	
	Pressure—501 cm.	Pressure—74.2 cm.
Column I.	Column II.	Column III.
10 minutes.	30.0	17.1
15 "		9.3
30 "	17.5	6.2
50 "	13.2	5.5
1 hour.	12.0	
1.25 "		5.13
1.5 "	9.9	
1.75 "	8.9	5.1
2.00 "	8.0	4.9
4.00 "	6.6	4.8
5.00 "	7.6	
6.00 "		5.0
10.00 "		5.8
22.00 "	19.2	
25.00 "		6.0
29.00 "		6.0
32.00 "	22.0	
34.00 "		6.3
44.00 "	24.0	
45.00 "		6.5

In repeated tests carried out in this manner with the zinc cylinder it was found that, while the initial conductivity varied from day to day, there was always observed a rapid decrease to a constant minimum followed by a gradual rise to a constant limiting value.

The ionization curve for these values, Fig. 2, shows that the minimum current, 4.8, was reached in about four hours after the air was enclosed.

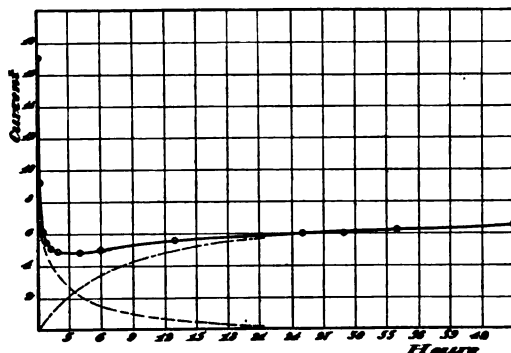


Fig. 2

After about eighteen hours the curve indicates that the conductivity was tending towards a limiting value, which the reading taken after forty-four hours showed to be about 6.5.

As a variation in the experiments a series of tests were made in air confined in a receiver at high pressures. The cylinder in this case was of heavy rolled iron coated with zinc and was of the same dimensions as that used in the first measurements.

The results of observations on the conductivity of air confined at a pressure of about seven atmospheres are given in column II., Table I., the scale used being the same as before. The curve representing these values is shown in Fig. 3, and exhibits the same characteristics as that for the lower pressure.

We have again the rapid decrease to a minimum followed by a gradual rise tending towards a limiting value. The minimum conductivity in this case was about 6.6, and was reached in about four hours after the required pressure in the cylinder had been obtained. The time occupied in pumping the air was about one hour.

In seeking for an explanation of the curves shown in Figs. 2 and 3, their twofold origin, as indicated by the dotted lines, is at once suggested, the conductivity in the initial stage being due to an agent subject to rapid decay, and that in the second to one whose power shows a gradual increase.

The first of these dotted curves is similar to that given by Rutherford for the conductivity of the air in a chamber which had been cut off from a second containing thorium oxide after the two had been in communication for some time, while the second is similar to that given by him for the increase in the conductivity of the air in one chamber when placed in communication with another containing thorium oxide.

It will thus be seen that the first portion of the curves in Figs. 2 and 3 can be explained upon the supposition that a radioactive emanation, probably having its origin in the earth's surface, was introduced into the cylinder with the air, the decay of this emanation being the cause of the decrease in the conductivity, and the second portion upon the supposition that a radioactive emanation is given off by the walls of the containing vessel. On this view the limiting value to which the conductivity curves tend would represent a condition of equilibrium, where the rate of decay of the emanation was equal to the rate at which it was produced.

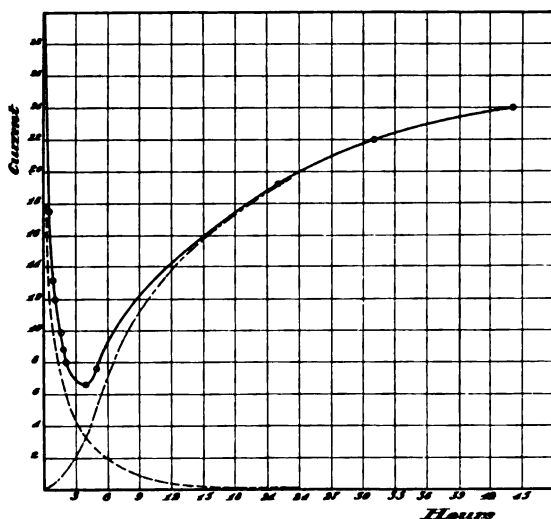


Fig. 5

As both the low and the high pressure cylinders were made of the same material and were of the same size, one would expect the same amount of the emanation to be present in both when the steady state was reached. With an easily absorbed radiation from this emanation, we should obtain a limiting conductivity independent of the pressure. But, since a very great difference was found in the limiting conductivities at the two pressures, it would appear that the radiation possesses considerable power of penetration and is not easily absorbed.

The difference in the initial conductivities given in columns II. and III., Table I. may also be readily explained by the difference in the air pressures. The time required to fill the high pressure cylinder and the decay taking place during that time in the emanation introduced with the air would account for the difference.

greater than that in the low-pressure cylinder, the difference in the initial conductivities is explained, while, if it were less, the greater density of the air in the high-pressure cylinder and the consequent greater absorption would still account for the higher conductivity.

IV. EFFECT OF DIFFERENT METALS.

To ascertain whether the conductivity of the air enclosed was affected by a change in the metal composing the walls of the receiver, linings of tin and lead were in turn fitted into the zinc cylinder used in the first experiment.

Tests of the conductivity were made both with and without the tin and lead linings. Before each test the cylinder was well aired and sealed in the manner already described. As soon as the air was enclosed measurements on the conductivity were begun and continued at stated intervals as before.

TABLE II.

Time.	Current; Arbitrary Scale.		
	Zinc.	Tin.	Lead.
10 minutes.	13.0	13.0	15.4
15 "	8.3	11.0	
30 "	5.4	7.6	13.4
45 "	5.3	7.3	12.1
1.0 hours.	5.0		11.55
1.5 "	4.85	7.2	10.8
2.0 "		7.3	11.5
2.5 "		7.3	
5.0 "	5.0		11.8
7.0 "	5.3		11.9
9.0 "			12.0
12.0 "		7.9	
18.0 "	6.0	8.0	

The values obtained for the conductivity with each of the metals are given in Table II. and curves representing these values are shown in Fig. 4.

The curves for the different metals, it will be seen, have the same characteristics. In each there is a rapid drop to a minimum and a gradual rise towards an ultimate limiting value. It is interesting to note that a considerable difference was found in the minimum conductivities for the three metals and that the final limiting values also varied.

The decay of an emanation introduced into the cylinders with the air would again account for the first portion of the curves; a radioactive emanation from the metallic walls would explain the existence of the

second portion, while the differences in the minimum and limiting values may be considered to have their origin in variations in the rate at which the emanation is given off by the different metals.

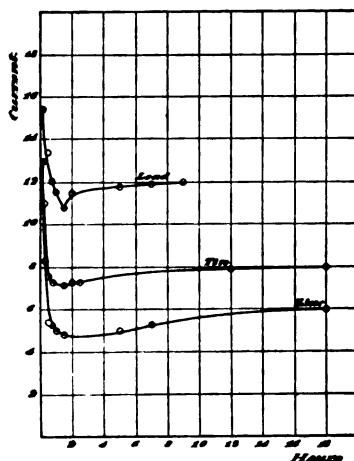


Fig. 4

In this connection it will be noted that the limiting values of the conductivities range according to the atomic weights of the metals, lead having the highest, tin the next and zinc the lowest.

V. EFFECT OF VARIATIONS IN PRESSURE.

In order to investigate the relation between the conductivity of air and the pressure at which it was confined, the heavy cylinder was filled to a pressure of about seven atmospheres and allowed to stand for some days until its conductivity assumed a constant value.

TABLE III.

Pressure in cm.	Current; Arbitrary Scale.	Pressure in cm.	Current; Arbitrary Scale.
501.0	23.5	69.3	5.8
481.0	22.7	62.0	5.4
420.0	21.2	53.0	4.7
384.0	20.3	44.2	4.2
272.0	15.8	35.0	3.5
227.0	14.1	22.4	2.7
176.0	11.7	18.2	2.2
125.0	9.3	14.0	1.8
74.2	6.5	4.4	1.1

The air was then allowed to escape gradually and the pressure reduced from 501.0 cm. to 4.4 cm. of mercury, the conductivity being measured at these and at various intermediate pressures. The results are given in Table III. and the conductivity curve in Fig. 5.

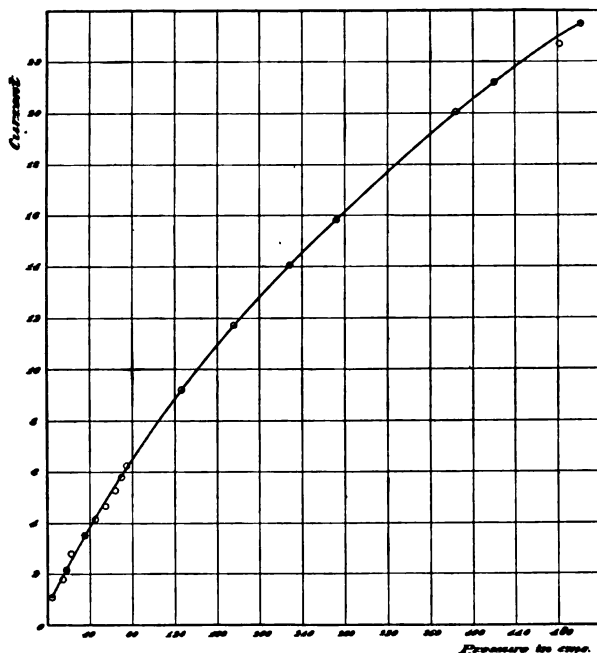


Fig. 5

The ionization curve so nearly approaches a straight line that we may almost conclude, in view of the wide range of the pressures examined, that the conductivity was proportional to the pressure. This result is what we should expect to obtain with an emanation maintained at a constant strength and emitting a radiation of a penetrating nature.

VI. PENETRATING RAYS FROM EXTERNAL SOURCES.

While the effects described up to the present may be explained by supposing the ionization to be caused by a radioactive emanation sent off from the metals, it has been found that part of the conductivity cannot be accounted for in this way, but must be attributed to an active agent external to the receiver.

The heavy cylinder referred to above was filled with air to a pressure of about 400 cm. of mercury and allowed to stand until its conductivity

had become steady. It was then placed in an insulated galvanized iron tank, 150 cm. in height and 75 cm. in diameter, which was gradually filled with water so as to surround the cylinder with a layer 25 cm. in thickness. The initial conductivity before the water was admitted was 21.1. As the water rose the conductivity decreased and fell to 13.3 when the tank had been filled. The values for the conductivity during the experiment are given in Table IV., and they show that the loss was almost directly proportional to the rise of the water. The total fall in conductivity, it will be seen, was about 37 per cent.

TABLE IV.

Depth in cm.	Current ; Arbitrary Scale.	Depth in cm.	Current ; Arbitrary Scale.
0.0	21.1	97.0	14.3
15.0	20.5	110.0	13.75
60.0	18.0	120.0	13.3
77.0	17.0		

The experiment was repeated with the cylinder placed in a tank 50 cm. in diameter. This tank permitted the cylinder to be surrounded by a layer of water 12.5 cm. in thickness, and it was found when the water was poured in that the conductivity fell off 17.5 per cent.

From these results it is evident that the ordinary air of a room is traversed by an exceedingly penetrating radiation such as that which Rutherford has shown to be emitted by thorium, radium and the excited radioactivity produced by thorium and radium.

In order, therefore, to reach definite conclusions regarding the extent and true character of the effect of various metals upon the conductivity of the air which they enclose, it will be necessary to cut off the enclosing vessel entirely from the action of this external radiation, and the writers have not yet carried their experiments to this point.

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THE PHYSICAL REVIEW.

THE NUCLEATION DURING COLD WEATHER.

BY C. BARUS.

1. The results shown in the accompanying figures were obtained with the same apparatus and by the same method described in *Science* in December, 1902, p. 948. No attempt has been made to reduce the nucleation rigorously to absolute numbers, as it seems ill advised to carry this reduction out at present. Further study of the apertures of coronas will first have to be made, and the apparatus thereafter changed to appropriate plate-glass troughs. Each cylindrical vessel in which coronas are produced is apt to have a coefficient of its own, and the measurement of aperture alone does not at once indicate the nucleation.

The numbers are, however, roughly absolute and reasonably satisfactory as relations; they, therefore, meet every demand which can be made of them at the present preliminary stage of progress.

2. It will be noticed that during the winter months and particularly during excessive cold, the air is very rich in nuclei. One naturally infers that this is to some degree due to the amount of coal burnt at the time. One must be careful not to overestimate the local effect. There are more nuclei per c.c. than can be produced by twenty needle jets playing at high pressure into a ten liter jar. The nucleation is variable with atmospheric conditions under like local conditions as to wind, etc. Moreover, the winds which bring the nuclei come from the highlands, and not from Pawtucket or the thickly inhabited parts of Providence. Indirectly, however, the local effect must be considered (see below § 12).

The present experiments furthermore show that ordinary terrestrial dust properly so-called, has no bearing on the nucleation; for this is so frequently a maximum immediately after a rain when the earth is blanketed with water, or after the earth has been covered with snow or with sleet when there can be no dissemination of dust as such.

3. I do not attempt in this paper to give a detailed discussion of the correlative atmospheric conditions, as this will have to be done with more data in hand. The temperatures and winds (for 8 A. M.) taken from the weather maps for Block Island or Nantucket will often be entered.¹ In some of the figures the changes of temperature δt and barometric pressure δp are also given, with the occurrence of clear weather ☉, cloudy ● and partly cloudy weather ☂, sunshine S, rain R, snow Sn, haze h, falling temperature — δ , c. w. cold wave, etc. References in Fahrenheit or centigrade are indicated.

In the paper in *Science*, the results were shown as far as November 3 (Fig. 1), and they are reproduced here on the first figure chiefly for comparison with those following. The scale is the same throughout.

4. The feature of the data from November 1–10 (Fig. 2), is the tendency to notched minima at mid-day or in the early afternoon. The maximum on the 11th coincides with northerly winds and a slight drop in temperature, but beyond this the temperature is so nearly constant until the 24th, that reasons for the variations of the figure are not forthcoming. The remarkable rise on the 21st may be noticed. On the 14th and 23d there are mid-day minima, but others are not apparent.

5. During the remainder of November (Fig. 3), rain minima occur on the 25th, 26th and on the end of the 29th. The maximum on the 29th is clearly associated with the drop in temperature accompanying northwesterly winds. There is here a mid-day minimum, but no others.

Throughout December, cold waves associated with northwesterly winds are productive of maxima. After the rain on December 1, the nucleation rapidly rises, accompanying a second drop in tem-

¹ As the work progressed, superfluous data were gradually dropped.

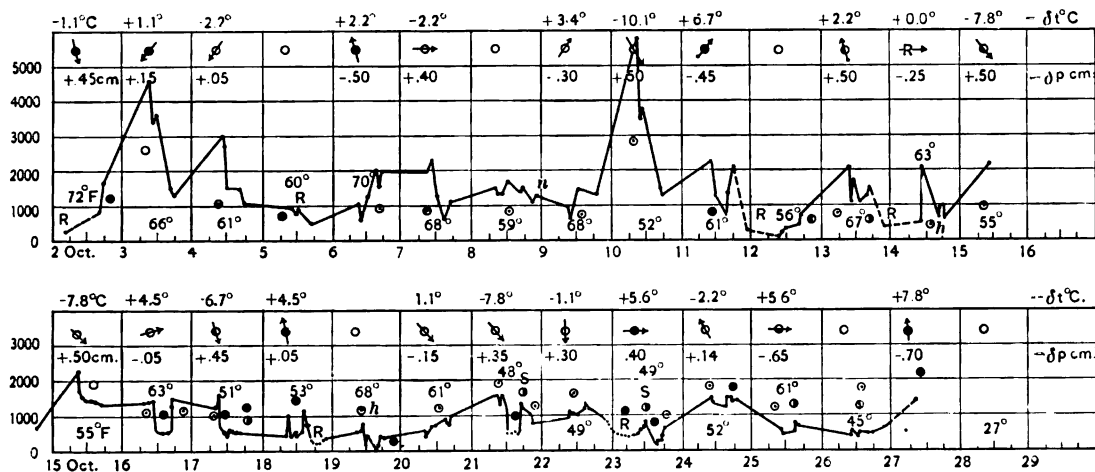


Fig. 1. Number of nuclei per cubic centimeter of atmospheric air or *nucleation* (ordinates) from October 2-28. Wide receiver (diameter 22 cm). Increments of pressure (cms. of mercury), temperature (degrees Centigrade) and winds given above the curves from U. S. W. B. (Block Is.) observations. Local weather data (temperatures in degrees F., at about noon) nearer the curve. Observations from October 2-15 without water-bath. All data inserted in figure at about the time of observation.

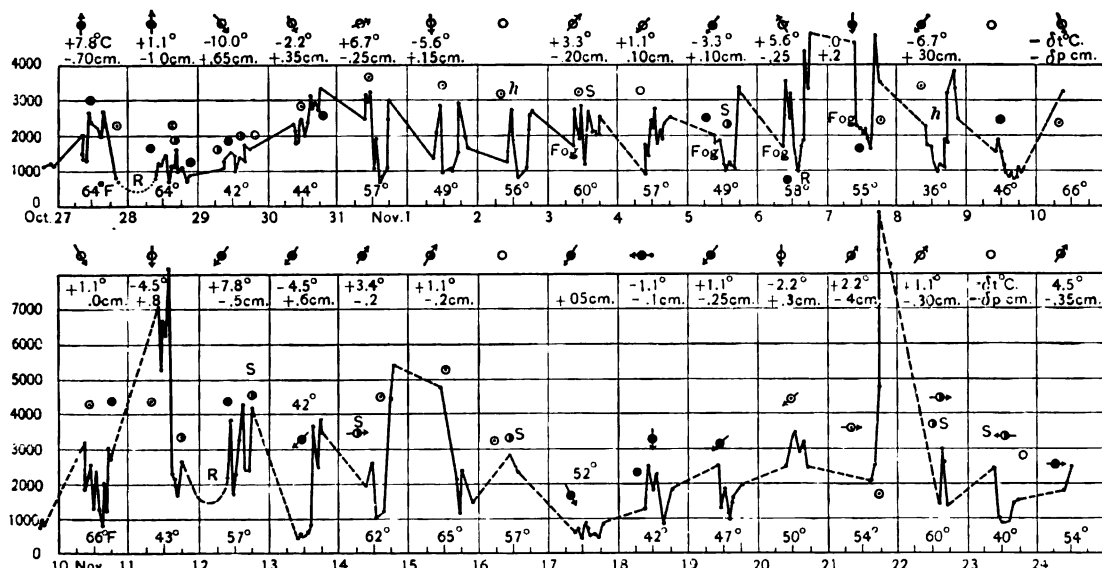


Fig. 2. Nucleation, October 27 to November 24. Tall cylindrical receiver (50 cm. long, 13 cm. diameter). Absolute values too large. Winds, temperature and pressure increments above curves, from U. S. W. B. Local data nearer the curve. Places of data here and elsewhere correspond to the time of observation.

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perature and northerly winds. The maximum is moderately sustained until the 3d, where there is a fall into the rain minimum, partial recuperation thereafter and then another drop into the rain on the 5th.

6. This rain soon changes into snow, and from here the nucleation (Fig. 4) of the cold wave begins, lasting from the 6th to the thaw on the 11th. One may note the reduction due to very cold snow on the 7th, and the curious minimum accompanying the gale on the evening of the 8th. The maxima on the 9th and 10th accompany northerly winds with a drop in temperature.

The very remarkable maximum from the 13th to 16th, follows the fall of temperature after the thaw on the 15th, introducing a rain minimum. This interesting region is no doubt referable to the drifting snow storm on the evening of the 13th, the winds remaining northerly. The ground was frozen on the 13th and 14th. It is not due to temperature.

The maxima on the 17th and 20th are less sustained, but they come with temperatures much above freezing and westerly winds. The absence of night observations is a dilemma in these cases. The low nucleation indicated on the 20th continued for several days (20th-23d) beyond the limits of Fig. 4, but is shown on Fig. 5.

6. One may regard it as a probable result of the first four figures that the nucleation in general increases when temperature suddenly decreases, but that temperature is *not* the sole factor involved. To see whether this view would work out in detail, I compared the observatory thermograph data for Providence during the three months, kindly placed at my disposal by my colleague, Professor Upton, with the data for nucleation given in the above figures. A limited degree of similarity was in this way made out, both in the general march of temperature throughout longer intervals and in its detailed variations. Thus the mid-day minima were obviously the counterparts of corresponding temperature maxima. There were, however, many outstanding discrepancies and it seemed probable that much of this would be referable to local differences as the observatory is differently situated and at a considerable distance from the laboratory. Beginning with December 28, therefore, I have been making

daily temperature observations simultaneously with the coronal observations, using a long mercury thermometer placed besides the influx pipe. These temperatures are given at the tops of Figs. 5 and 6, and *positive temperatures* in degrees F., are laid off *downwards* to insure an easier recognition of coincidences, remembering that nucleation maxima and temperature minima correspond.

7. In the fragmentary temperature observations before December 28, Fig. 5, some relations are seen; thus the maximum between December 23 and 25 has a temperature equivalent; the broad minimum on the 22d and mid-day minimum on the 24th correspond, etc.

Turning to the cases under detailed observation, the mid-day minimum on December 28, the fall on December 29, and to a smaller extent the data on December 30 and 31 (in the latter case other factors are also active), agree. The marked minimum on both cases on January 1 and to some extent that of January 2 are especially striking. The rain minimum on January 3 as also the low nucleation on January 4 correspond to slightly variable temperature, etc.

8. In Fig. 6 the general maxima marked c. w. (cold wave) may be noted. The details are similar; one may point out the marked fall on January 11 and the day minimum on January 14. On this figure there is however one good example of opposition; for on January 13 a pronounced minimum of temperature occurs simultaneously with maximum nucleation. Effects such as those on January 13 are not infrequent at certain times and one must guard against overestimating the temperature effect.

SUMMARY AND INFERENCES.

9. The experiments, as a whole, are in the first place to be looked at as a severe test amounting almost to a strain of the method employed. Since both maxima and minima are registered with equal facility at exceedingly low as well as at high atmospheric temperatures, the temperature error of the method itself is not menacing. Rain minima, snow minima and even gale minima have all been

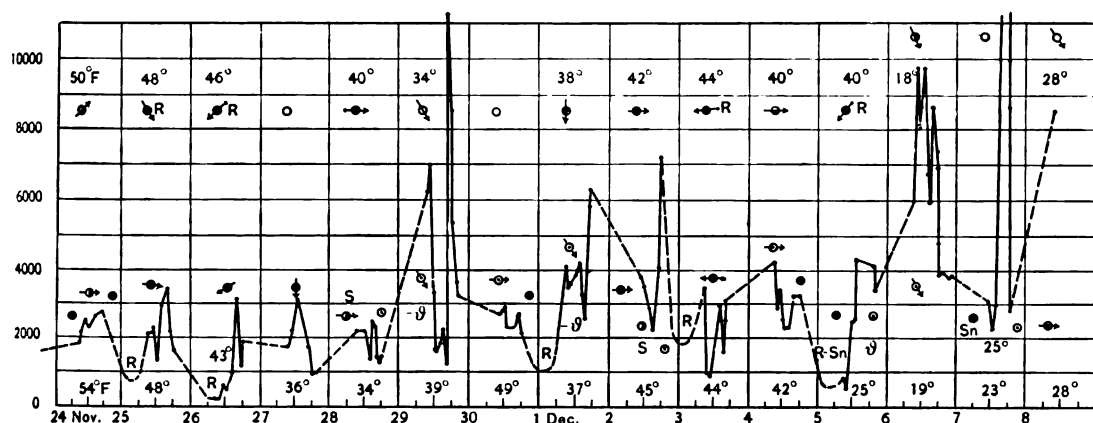


Fig. 3. Nucleation, November 24 to December 8. Temperature (degrees F.) and winds from U. S. W. B., above curve. Local data nearer the curve.

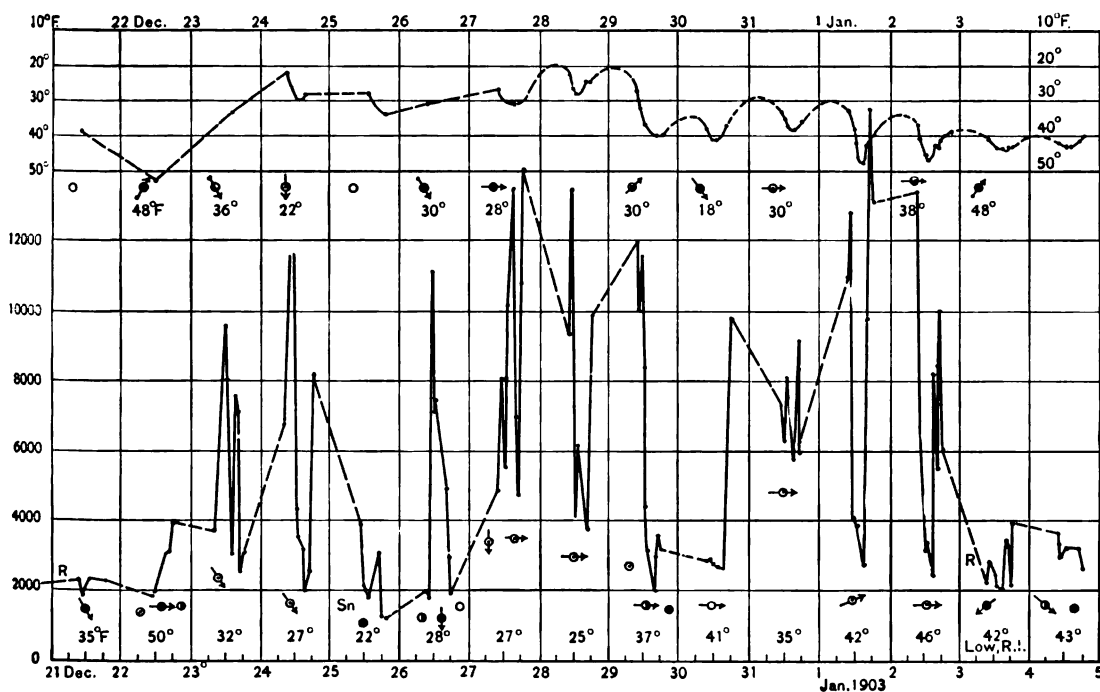


Fig. 5. Nucleation, December 21 to January 4, lower curve. Temperature (degrees F.) and winds (U. S. W. B.), above curve. Local data nearer curve.

Upper curve shows corresponding observed temperatures in degrees F., laid off *positively downward*.

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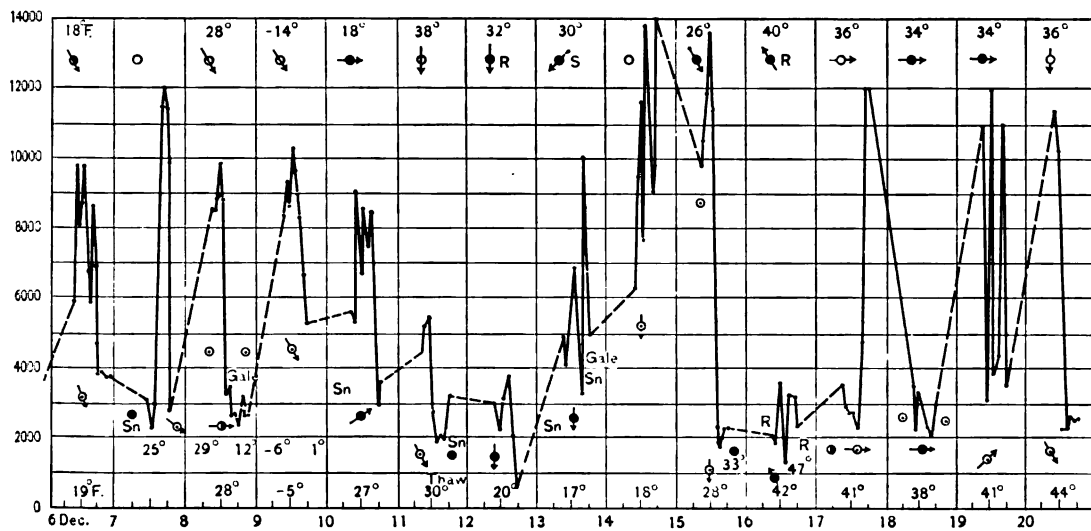


Fig. 4. Nucleation, December 6-20. Data inserted as in the former case.

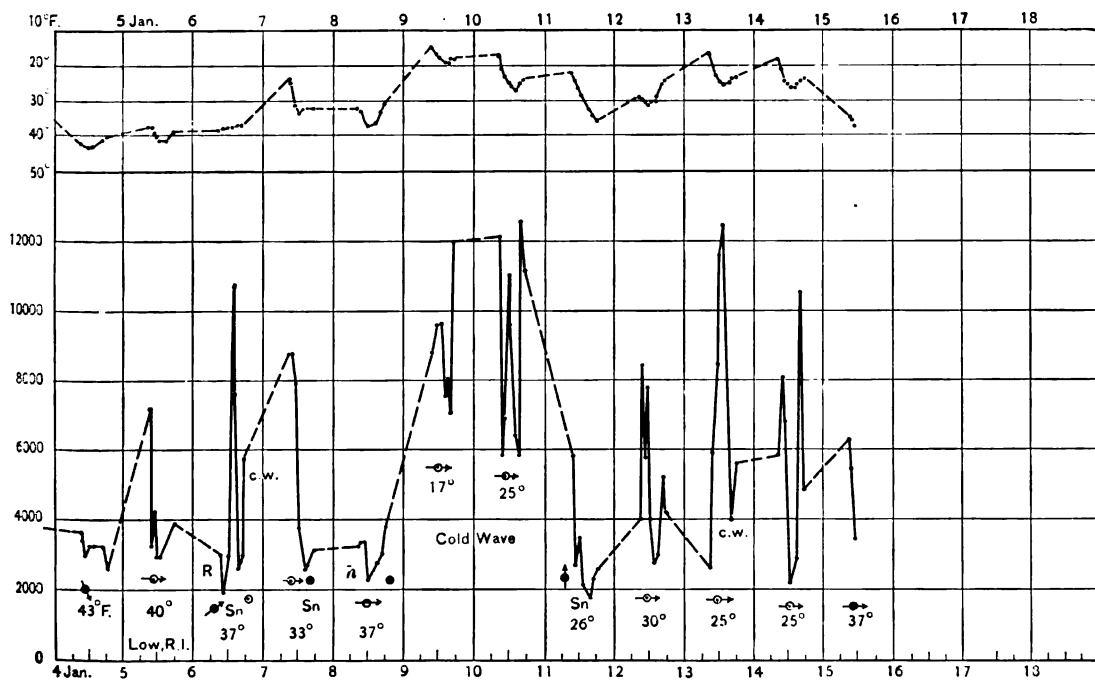


Fig. 6. Nucleation, January 4 to 16, lower curve. Local temperatures (degrees F., at about noon) accompany the curve. Upper curve, corresponding observed temperatures in degrees F.

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half a meter of the condensation chamber; but the maximum of atmospheric nucleation does not exhaust the limit of measureable coronas. Nevertheless there are some serious outstanding difficulties: Thus the influx tube must be long (coil and water-bath) to heat the cold air to the given temperature of observation, but if so there may be excessive absorption on the way. It is necessary to wait before observing to insure saturation of the dry winter air (and this is the most insidious of errors, for deficient saturation would be interpreted as high nucleation), but on waiting the time losses of nuclei are increasingly important. All the difficulties are accentuated for high nucleation. Observations carried through long intervals of time, systematically, are alone adapted to test the ultimate trustworthiness of the method and progress is thus necessarily slow.

10. The second general result is the frequent occurrence of maxima of nucleation contemporaneously with sudden fall of atmospheric temperature, often associated with northerly (perhaps overland and dry) winds. When the drop of temperature is alone sufficient, one would have the conditions for the formation and the growth of water nuclei, remembering that persistent nuclei may be made in an atmosphere not too far from saturation, from solutions or in the presence of a solute. Without actual *accessions* to their numbers, however, there could be no increment of the number of nuclei in the condensation chamber prior to the observations at about 20° C. In other words an actual rise and fall of the number of nuclei per c.c. must occur in the atmosphere at the place of observation.

11. As to the wind or convection effect, it is probable that a polar air current or a current from the upper atmosphere will be sometimes associated with the cold waves in question. In this case a part of the region where there is continual production of nuclei has been bodily transported to the place of observation. For the same reason cold air may be nearly free from nuclei.

12. Supposing that the nuclei of any kind are loaded with water by the fall of temperature, various inferences may be drawn to account for the observed increments of nucleation. In the first place, the result may be an accelerated precipitation of an atmospheric layer of excessive nucleation overlying cities and produced, particularly in winter, by combustion. Such conditions are producible

in the laboratory apparatus; for instance I have frequently obtained layers of nucleated air above, below, or even between strata of dust-free air. Before condensation these layers are equally clear; but they are differentiated nevertheless by density. Rise of temperature would retard the precipitation by partly removing the loads of the water nuclei. To test this question as a whole corroborative observations must be made in a place remote from civilization.

13. In the second place if water nuclei newly produced by fall of temperature are radio-active, these would in their turn produce fresh nuclei in succession, and an actual augmentation of nuclei should then be observable, such as has been found in the cold wave maxima or reciprocally in the day minima. In this case the conditions of high nucleation must be shown to correspond to conditions specially favorable to radio-activity, for which there is no evidence. All such questions, however, should find an answer in the direct experiments which I am now making.

BROWN UNIVERSITY, PROVIDENCE.

NOTES ON THE COHERER.

BY A. H. TAYLOR.

INTRODUCTORY.

THE present knowledge of the action of an electrical contact under varying currents and different distances between the parts of the contacts is limited, and the best coherer theories yet advanced fail to account for the action of this simple yet wonderfully sensitive little instrument as it is actually used in detecting electrical waves or impulses.

This article does not claim to add much to that which is already known on this subject, nor to put forth a complete theory of the coherer. Its object is to give the results of several years' intermittent work with the coherer, coupled with some practical experience with the instrument as used in wireless telegraphy, and to reconcile if possible, these results with the work of others preceding, notably that of Guthe and Trowbridge (*PHYSICAL REVIEW*, July, 1900). The latter authors, as far as I know, are the only ones to give anything like a regular and systematic set of results, represented by curves, or at least, capable of being so represented. Hence their results are the only ones with which it is possible to make much of a comparison.

The use of an ordinary relay as a wireless receiver, using the secondary contact as a coherer, was suggested by the work of Guthe and Trowbridge, but inasmuch as their theory did not explain the action of this instrument as I used it, several curves were taken from it, which were radically different from their curves, although obtained by somewhat similar means. (See *PHYSICAL REVIEW*, July, 1902.) It is also the object of this investigation to verify these new curves.

It appears from previous investigations that very few people have any idea of how sensitive a coherer can be. For instance, a single-contact coherer which I used to get signals from the college to

Lansing, a distance of nearly four miles, would respond at that distance without tuning or synchronizing of the circuits, to a spark of three-eighth inch. The same coherer in combination with a telephone (PHYSICAL REVIEW, July, 1902) would detect the spark from the ignition choke coil of the gasoline motor of an automobile over half a block away. This was no accident, but occurred every time a gasoline-driven machine passed in the street nearest. The effect is a faint but regular clicking in the telephone. The slightest mechanical jar will affect a sensitive coherer, and I hope to show further on in this paper that a displacement of a ten-millionth part of an inch, of the contacts, can not only be observed, but can be *measured* with considerable accuracy. Sparks from a street car trolley nearer than one quarter mile invariably affect the instrument from which most of the data here given were taken. In this laboratory there is a system of electrical clocks, operated from a relay in one master clock, which is about ten meters from the place where most of the observations were taken. It was found absolutely impossible to get any results with these clocks going. Thinking it was the spark from the secondary of the master clock relay, which was in series with the other clocks and four volts, I opened that circuit. The disturbance was less than at first, but still persisted. I then opened the primary circuit of the clock relay, which was in series with 1.5 volts. The disturbance ceased. To verify this, a relay and a gravity cell were set up with a key in series, the key at a distance of about six meters from the coherer. The effect was unmistakable, both on closing and opening the relay circuit. In fact, the coherer seems to be affected by *any* electrical displacement in its neighborhood, but more especially by sparks and electrical displacements in circuits containing self-induction.

Many of the following observations were taken by gas light, as the turning on or off of an electric light in the same room with the coherer often affected it. Lights on A. C. circuits did not affect it nearly as much, and hence were relied upon for light in most cases.

I have found it absolutely impossible to get any satisfactory results in the daytime, and the data to which I attach the most importance were obtained after midnight, when the electric cars had ceased running, and the campus was quiet. It was always found

necessary to stop the clocks and shut off the steam in order to avoid the "thumping" in the pipes. Many a set of observations has been abruptly terminated by the passage of a wagon on a road fully two blocks away. These precautions are not ultra-refinement; they are absolutely necessary if we are to investigate the coherer *in its most sensitive stage*.

From this brief review of some of the difficulties, it can be inferred that a complete solution of the coherer problem may not be forthcoming until the experimenter is able to isolate himself and his apparatus in some remote country where there are no street cars, no electrical storms or such disturbances, no sounds, and no mechanical jars of any kind. Even then he would be bothered by electrical displacements in his measuring apparatus.

For instance, I have found it necessary to put a high resistance in series with the condenser, which, according to the method of Guthe and Trowbridge, is put across the coherer terminals and then discharged through a galvanometer to obtain the terminal coherer potential. This high resistance prevents too sudden fluctuation of the potential at the coherer terminals. Sudden changes of the current through it are sure to affect the state of a coherer when in the sensitive condition.

Considering these difficulties, it is no wonder that it has so far been impossible to get a curve which shall represent the behavior of the coherer in all its stages. The best we can do is to collect a large amount of evidence, and to plot the curves separately for the different stages, and to determine under what conditions one type of curve will predominate, and under what conditions another form will always appear. I have enumerated these difficulties chiefly as a plea for leniency in judgment of results, some of which have undoubtedly experimental errors of as much as 15 per cent.

The tables and curves have been taken from my note-book just as they came, with very little regard to their bearing upon the points which I wish to emphasize. Some of the curves, such as nos. VIII., IX. and XVIII., I would have preferred to omit.

Considerable experience with single-contact coherers in wireless telegraphy practice has demonstrated that such a coherer may be adjusted so as to receive a large number of signals, extending over

even several hours' time, without the use of any decohering device. In other words, the so-called negative coherer action must take place very nearly as often as the positive, when the coherer is in this stage. It has always seemed to me that this stage is also the one of greatest sensibility, and for all wireless telegraphy tests exceeding a half mile in distance, without synchronized circuits, I have found it necessary to adjust the coherer to this stage in order to get the signals. This sensitive stage corresponds to a rather high resistance, a small applied E.M.F and a current varying from 5 to 100 ten-thousandths of an ampère.

This, then, is the stage in which the coherer must be investigated in order to have results applicable to wireless telegraphy practice, and it is in this field that, so far as I am aware, no previous observations have been made.

Three coherers were used during the investigation. The first was the one described in the *PHYSICAL REVIEW* of July, 1902, and was discarded on account of the difficulty of adjustment for experimental purposes, although for practical work it is the best. The second was the "spherometer coherer." A small piece of nickel was stepped into the foot of the spherometer, which was mounted firmly on a metal base, with an insulated terminal in the center, under the foot of the spherometer screen. This terminal was also of nickel. This coherer had a contact adjustment of .001 mm. and gave quite satisfactory results, except the contact adjustment was not nearly fine enough.

The third coherer was the one shown in Fig. 2. The dimensions given in Fig. 3¹ give a nominal contact adjustment of $\frac{1}{88000}$ inch, but the real adjustment is less on account of some unavoidable play in the micrometer screen.

This coherer has the slot, point and plane support for the part carrying the movable contact, and the whole is very heavy and massive to give greater stability. I am greatly indebted to Mr. F.

Not realizing at the outset that changes in distance of the order of the hundred-millionth of an inch between contacts would noticeably affect the coherer, many of the earlier results probably contain very large temperature errors. In later observations the coherer was protected from temperature changes as far as possible.

The ammeter used in the experiments was a Jewel mil-ammeter, calibrated to ten-thousandths of an ampère, having a resistance of 27 ohms and reading to .0150 A. When used with a shunt it read to 1.50 A. and had a resistance of about 0.27 ohm. This instrument was compared with a standard Weston voltmeter and I am certain that the readings are accurate within 2 per cent.

The resistance box was a 10,000-ohm Willyoung box reading to 0.5 ohm and certified to one half of 1 per cent. The galvanometer was a Rowland and was used ballistically with a condenser, and also shunted and in series with 36,000 ohms, with which latter arrangement the coherer terminal potential could be measured directly without drawing enough current to affect the ammeter. This latter arrangement has the advantage of not causing sudden electric displacements in the measuring circuit, as the galvanometer key may be kept closed.

It is logical to suppose that the apparent resistance of a coherer depends upon the distance between the contacts, but it might not be immediately inferred that this apparent resistance depended upon the value of the current passing through it. Inasmuch as no one has, as far as I am aware, obtained a straight line curve of terminal voltage and current, except when the contacts are pressed tight together and have a very low resistance, it is certain that the apparent resistance is variable with the current. The term "apparent" resistance is used here because it is possible that a coherer may show something similar to the back E.M.F. of an electrolyte.

When a coherer is placed in series with a low resistance ammeter and a fixed applied voltage such as a storage cell, a condition of unstable equilibrium results, unless the coherer is *not* in the sensitive stage. I have never been able to adjust a coherer to this extremely sensitive stage with an applied voltage exceeding a few volts, probably not with over four, even with a large "ballasting" resistance in series, and have found it best in wireless telegraphy

practice as well as in these tests to use pressures under two volts. Even with low voltage, but no series resistance, it is impossible to obtain the sensitive adjustment and keep it long enough to take any observations. The current "creeps" up with varying rapidity, and when the creeping has ceased the current has reached a value such that the coherer is no longer extremely sensitive.

For the sensitive adjustment it is necessary to have a low applied voltage, and for the current to be very small in proportion to the coherer terminal potential. There must also be a series resistance, or "ballast." These facts have an important bearing on the theory of the coherer.

When in the sensitive state, the current through the coherer should fluctuate up or down by at least 10 per cent. or 15 per cent., upon the breaking of an inductive circuit such as I used for testing for sensitiveness. This consisted of a 180-ohm relay in series with two volts, the break in the circuit being at least two meters away.

Many attempts were made to shield the coherer from such disturbances as the above-mentioned, but with little success. With the coherer resting on a great block of iron, an old engine base, and completely covered with copper, the disturbing effect of the clock relay was very noticeable. This is undoubtedly due to electrical impulses collected by the lead wires from the testing circuits. Of course the observer must use the utmost precaution against mechanical jars. His chair must not squeak, and if it is necessary to repeat the observations to a second person, it had best be done in a whisper, as the rough guttural tones often affect the coherer if the speaker is close to it. I have found it best to set the coherer on a solid support with a pad of cotton under it. The leads to the coherer can be made of very light wire and fastened firmly to some points on the support before reaching the coherer. This checks the transmission of vibration along the lead wires to the coherer, an effect very difficult to get rid of at times. Many of the observations with the spherometer coherer were made with the instrument in a sling constructed of rubber bands. These absorb the vibrations to a large extent, but naturally it is difficult to adjust the instrument in this dangling position.

Fig. 1 shows the method of connection as used for most of the observations. B is a gravity cell or a chloride accumulator, in series with a small resistance R and a Wheatstone's rheostat, an arrangement allowing a smooth variation of the applied voltage within considerable limits, say from .05 V. to 1.8 V. A is the milliammeter, in series with the coherer, CO , and the resistance box, R . The condenser C is connected by means of the charge and discharge key K to the coherer terminals or to the galvanometer G .

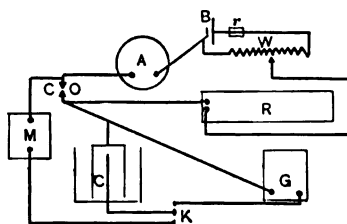


Fig. 1.

The large resistance M (36,000 ohms) is put in series with the condenser and the coherer in order to avoid sudden fluctuations of potential. The current is varied by varying the resistance R and sometimes by varying the applied potential. An ordinary resistance box in the place of R has the great disadvantage of not allowing a smooth change of resistance. On this account several liquid resistances in long tubes, several tubes of varying diameters being in parallel, were tried. This was not, however, as successful as it might have been had the electrodes been larger. The accumulation of gases around the electrodes, and varying polarization made it very difficult to maintain a constant current long enough to take measurements. A large "ballasting" resistance in R and a variable applied voltage seemed to give the best results, the Wheatstone's rheostat giving quite a smooth variation of applied voltage, after the wire and contacts in the rheostat had been polished.

The contacts of the coherer were always nickel. They were cleaned by scraping with a clean steel knife-blade, no attempt being made to preserve any particular contour of the surfaces. Previous experience with the coherer seemed to show that the contact distances were almost of molecular dimensions; hence it is useless to attempt to work with spherical or otherwise particularly shaped surfaces. The most accurately made spherical surfaces which I could have obtained would certainly not have been "smooth" if distances of an order less than the ten-millionth part of an inch are considered. Subsequent investigation showed that the coherer

shown in Fig. 3 was sensitive to such distances. Hence it seems useless to attempt to get smooth contacts. This point is perhaps open to some question, however, and it is possible that polished metal may behave somewhat differently. But contacts prepared simply by scraping with a steel knife-edge are well suited for practical use in wireless telegraphy and it is therefore proper that such contacts should be used here.

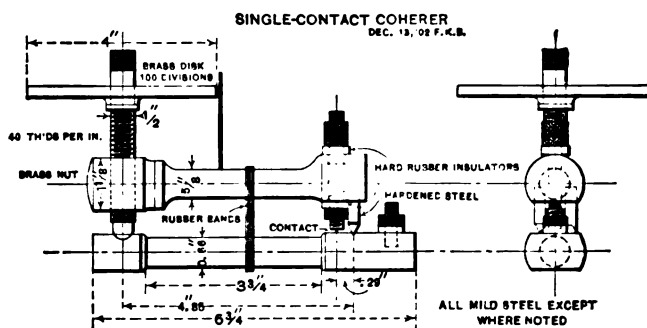


Fig. 2.

The coherer was adjusted to the sensitive condition by changing the distance between the contacts by turning the micrometer head

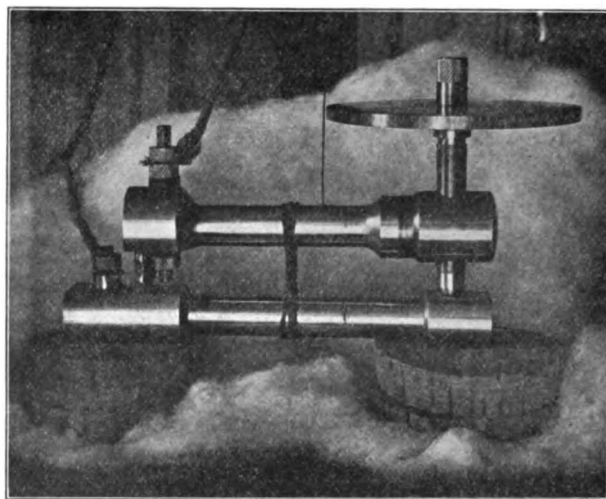


Fig. 3.

(see Fig. 3). One division on the micrometer head should give a change in contact distances of $\frac{1}{6800}$ inch according to the dimen-

sions given in Fig. 2. This adjustment was very difficult to make, and was best accomplished by regulating the applied voltage and "ballasting" resistance R , until the coherer on short circuit carried from .0007 to .01 ampère. The micrometer head was then adjusted until the current was about 30 per cent. of this value.

This current value was noted, and the coherer tested by a spark not exceeding one millimeter in length, at a distance of about ten meters, or by the opening of the relay circuit previously mentioned. If the current plainly showed fluctuations on thus testing, the coherer was considered to have been in a sensitive condition, and was re-adjusted to the current value just before the test. The best general rule to follow is that the current should be a small part of that which the coherer showed on short-circuiting it.

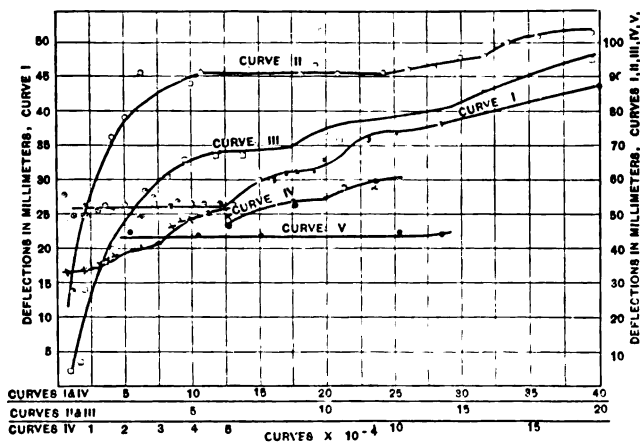


Fig. 5.

In Table I. are given the results of a set of observations for clean contacts in air. The current is expressed in ten-thousandths of an ampère, and the deflections, given in centimeters, are proportional to terminal coherer potential. The constant K was determined by use of a standard cell with the condenser. Curve I. is shown in Fig. 5, and has some striking peculiarities. In the first place there is a long interval, from $C = 1.2$ to $C = 13.0$, during which the potential at the coherer terminals is fairly constant. After this

constant for an interval. It was thought at first that these steps were due to errors in observation, but inasmuch as they appear in every curve of this type, that is in every curve having intervals of nearly constant potential with increasing current, it was finally concluded that the "steps" were really part of the true curve.

TABLE I.

Capacity 0.5 mf. $K = 4.54 \frac{\text{cm.}}{\text{volt.}}$ Clean contacts in air.

$C \times 10^{-4}$	Galv.	Defl.	$C \times 10^{-4}$	Galv.	Defl.
0.5	27.8	2.8	11.0	27.5	2.5
1.2	27.5	2.5	12.2	27.7	2.7
1.5	27.5	2.5	13.0	27.7	2.7
2.0	27.5	2.5	15.0	28.0	3.0
3.0	27.55	2.55	16	28.0	3.0
3.5	27.6	2.6	17	28.1	3.1
4.0	27.55	2.55	18	28.15	3.15
5.0	27.65	2.65	19	28.1	3.1
5.5	28.0	3.0	20	28.3	3.3
6.5	27.8	2.8	21	28.55	3.55
7.0	27.75	2.75	23	28.55	3.55
8.5	27.6	2.6	25	28.65	3.65
9.0	27.65	2.65	28	28.8	3.8
10.0	27.6	2.6	41	29.4	4.4
10.8	27.6	2.6	78	27.5	2.5

TABLE II.

Capacity 0.5. Clean contacts in Kerosene. $K = 4.54 \frac{\text{cm.}}{\text{volt.}}$

$C \times 10^{-4}$	Galv.	Defl.	$C \times 10^{-4}$	Galv.	Defl.
0.5	27.7	2.7	12	34.0	9.0
0.8	30.0	5.0	13	34.2	9.2
1.0	30.2	5.2	14	34.4	9.4
2.0	32.1	7.1	15	34.5	9.5
2.5	32.7	7.7	16	34.6	9.6
2.7	34.0	9.0	16.5	35.0	10.0
5.0	33.8	8.8	18	35.1	10.1
5.5	34.0	9.0	19	35.0	10.0
7.0	34.1	9.1	20	35.23	10.23
9.0	34.0	9.0	21	35.3	10.3
9.5	34.15	9.15	20	36.0	11.0
10.5	34.0	9.0			

In Table II. are given the results for clean contacts in kerosene. Curve II. is similar to Curve I. except that there is at the beginning, between $C = 0.5$ and $C = 5.5$, a gradual rise in voltage. Curve I. is very nearly parallel to the axis of current between the same limits. This preliminary rise of potential before the constant potential and "steps" is shown by other curves to be characteristic of contacts not cleaned before using. Hence it may be that the impurities in the kerosene caused this effect.

Curve II. however, also shows the "steps," there being one more "step" than is shown on the plot. At $C = 21$ occurred a mechanical jar which sent the current back to 20 and raised the potential to 11.0 from 10.3. Such accidents are sure to interrupt the observer before many observations are taken. Curve III. is the only one for clean contacts in air which shows the gradual rise in potential. It differs from curves of the type obtained by Guthe and Trowbridge in that the potential does not reach a finite maximum. If more observations had been taken between $C = 7$ and $C = 20$, this curve might also have shown the steps. This set of observations was terminated by some mechanical disturbance.

Value of K for each Curve.

No. Curve.	K in cm. volt.	No. Curve.	K in cm. volt.
III	8.17	XII	1.41
IV	1.816	XIII	11.28
V	.908	XIV	1.41
VI	9.09	XV	9.09
VII	9.09	XVI	1.41
VIII	9.09	XVII	4.54
IX	9.09	XVIII	4.54
X	9.09	XIX	2.08
XI	1.41		

Curve.	$K \times 10^{-8}$ inches. second.	Curve.	$K \times 10^{-8}$ inches. second.
XXI	2.2	XXIV	2.2
XXII	2.2	XXV	2.2
XXIII	2.2	XXVI	2.2

The general form of Curve III. is similar to that of Curve II., and in view of subsequent results it seems more than likely that the con-

tacts were not thoroughly cleaned. Curve IV. is plotted in two separate parts, as a disturbance caused the current to fluctuate uncertainly around the value $C = 5$, and a readjustment was necessary. Both parts of the curve show the steps. Curve V. is rather remarkable in showing an almost constant potential from $C = 5$ to $C = 28$, after which the current suddenly rose to 71 and there was a corresponding drop in potential.

None of these curves is drawn through the origin, as it is by no means certain that they should not intersect the axis of potential difference at a point corresponding to the voltage necessary to start ions across the narrowest part of the gap of the coherer.

Some experiments were undertaken with a view to determining this point. The coherer was placed in series with a Rowland galvanometer sensitive to 1×10^{-7} ampère and a variable source of E.M.F. As the contacts were approached, a gradual creeping of the galvanometer occurred, which corresponded to a leakage current due very likely to the presence of water vapor. Then came a sudden deflection of considerable magnitude, indicating a current of

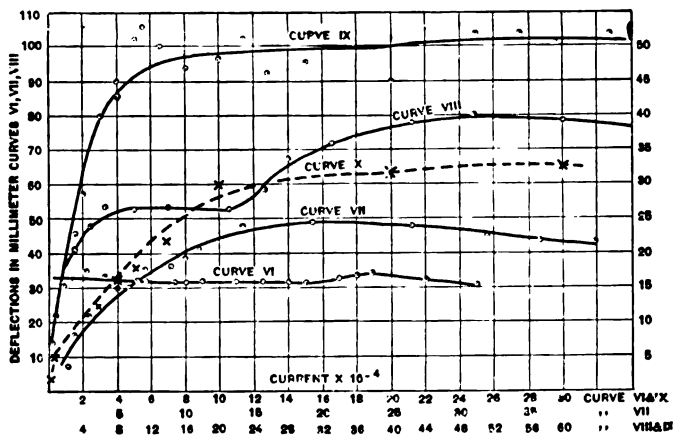


Fig. 6.

several millionths of an ampère. This latter would seem to indicate that the curves should intersect the potential axis. These tests

coherer as described in the latter part of this paper, using an actual adjustment of less than $\frac{1}{1000000}$ inch.

Curve VI. is for clean contacts in kerosene, and shows one step. The potential is nearly constant from $C = 1.2$ to $C = 15$. Compare with this Curve VII. which is for contacts in kerosene after two days' standing in air. Curve VII. is plainly of the same type as those of Guthe and Trowbridge. Curve IX. is also for contacts in kerosene, after three days in air, and is of the same type as Curve VII. Curve X. is also of this type, being for contacts in kerosene after four days in air. The idea in using kerosene was to determine whether the nature of the dielectric affected the type of curve obtained. As far as I am able to judge from these results, the same types of curves are obtained in kerosene as in air.

Curve VIII. is either a mixture of the two types of curves, due to partial oxidation, or it is a magnified "step." I think the first hypothesis the more probable.

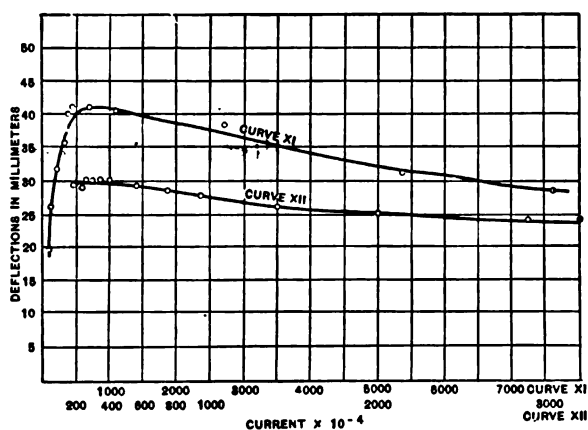


Fig. 7.

Curves XI. and XII. are taken for vastly larger current values than any preceding but plainly show the two types. Curve XII. is for clean contacts in air. Curve XI. is also for clean contacts, but is taken after passing a current of 0.5 ampère through the contacts. The potential drops off very decidedly after reaching a maximum. This is probably a heat effect. The large currents here used develop heat enough to expand the contact terminals and bring them closer

together. Hence the drop in potential. It is not to be understood that curves of the type of Curve XII. are simply parts of curves of the other type. For points on the flat part of curves of the type of Curve XI. the coherer is not sensitive; but is very sensitive for any part of curves like Curve XII. Moreover, repeated trials failed to detect any gradual rest in potential for curves of the latter type. It must be admitted, however, that unless great precautions are taken, the result of the observations will be a curve of the type of Curves XI., VIII., IX. and X. This is due to the fact that the coherer is thrown out of the sensitive state by unclean contacts, by mechanical or electrical disturbances, or by a sudden rush of current, as for Curve XI.

I have never had to use any great precaution in obtaining a curve of the Curve XI. variety, as the behavior of the coherer is not nearly so erratic as when in the very sensitive condition.

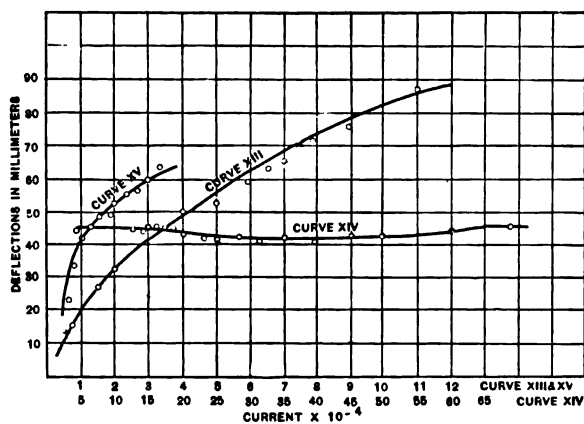


Fig. 8.

Curve XIV. is for clean contacts in kerosene, and shows very nearly constant potential throughout, from $C = 7$ to $C = 68$. There is a slight droop in the middle which may be a temperature effect. Curve XIII. is for contacts which stood for two days in kerosene, while for Curve XV. the coherer was thrown out of the

Curves XVI. and XVII. are also representative of the two types. Curve XVI. is for clean contacts in air; Curve XVII. is for the same, but affected by 0.5-mm. spark at 10 meters distance.

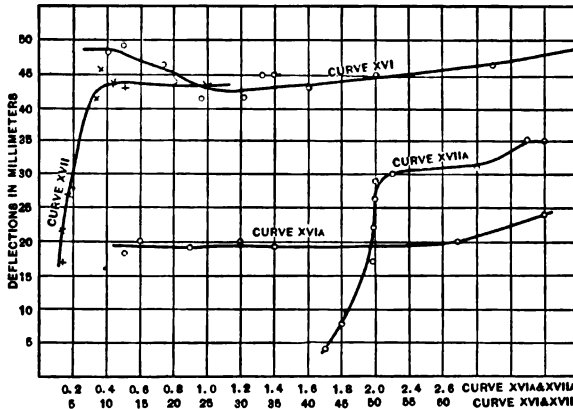


Fig. 9.

Curves XVI_a and XVII_a are similar curves taken for exceedingly small currents. They are not very reliable on account of the chances for error in estimating the tenths of a division on the ammeter.

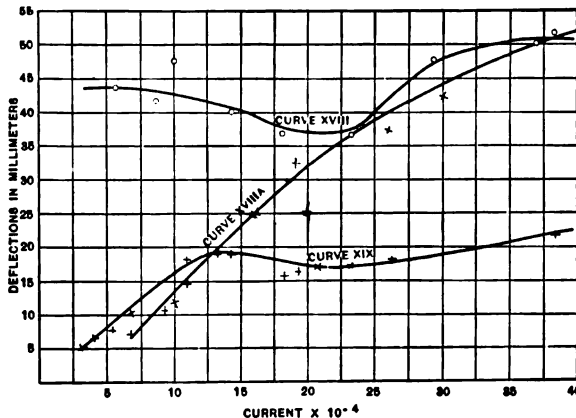


Fig. 10.

Curve XVIII. was taken from the relay coherer, and is rather irregular, showing, however, two steps and a droop in the middle. Curve XVIII_a was taken from the same instrument after being in-

fluenced by a spark at 10 meters. Curve XIX. is taken from the heavy coherer illustrated in this article, and after influence by a spark. It is not very regular, as it was taken in the daytime under rather unfavorable conditions. But irregular as they are, Curves XVIII. and XIX. belong plainly to two different types. Let us call the curves of gradual rise in potential, type *A*, and the curves beginning with periods of constant potential type *B*. The results may be summarized as follows: Type *A* curves are obtained invariably when a high applied potential is used, or when the ballasting resistance *R* (see Fig. 1) is not used, and also when the coherer has been previously influenced by a comparatively large current. Type *A* curves are generally obtained after oxidation, more especially after oxidation for several days, and after influence by a spark from an inductive circuit. Type *A* curves will be obtained whenever the current is large in proportion to the potential at the coherer terminals, which means whenever the contacts are too close together, if we suppose the terminal potential to increase with the distance.

Type *B* curves will be obtained only under the most favorable conditions, with clean contacts, absence of outside disturbances, smoothly varying current and small current in proportion to the total at short circuit. They vary in the extent and number of "steps." This is hardly a fair distinction, however, as no one of the curves is complete, having been cut short by unavoidable disturbances.

Curves of type *A* differ in the slope of the latter part. For large currents the potential decreases (due to heat effect, probably); for medium currents the potential may be slightly rising or falling; for small currents the general tendency is toward a rising potential.

Curves of type *A* would appear to pass through the origin, while those of type *B* would appear to cut the axis of potential, although the evidence for the latter point is weak.

While in the sensitive condition it appears that the conductivity of the coherer varies directly as the current for comparatively great ranges of current. It can be inferred that a curve of type *B* would ultimately end in one of type *A*, on account of the increased applied E.M.F. and increased current. Curves of type *A* are almost always found when the current exceeds a few hundredths of an

ampère. In fact, I have obtained a curve of type *A*, starting from the sensitive condition and a type *B* curve, but owing to unavoidable disturbances in so long a series of observations, there are many interruptions in the curve. Hence I have not given the results. Curve XI. illustrates the point well enough.

These results lead to the conclusion that the coherer phenomena may be divided into three classes corresponding to the three stages in the condition of the contacts. First, there is a feeble leakage current across the contacts due to vapor conductivity. Second, the potential rises high enough to cause conduction by metallic ions. During considerable intervals of current the potential remains nearly constant, then rising by steps until the coherer reaches the third stage corresponding to curves of the *A* type. The coherer can, it seems, be brought directly to the third stage by a strong electrical impulse, or by making the initial contact under comparatively high applied potential. Oxidation and weak electrical impulses cause the curves to become like type *A* even for small currents. But in this case the coherer is often still sensitive, and not in the third stage, for the potential may continue constant for intervals, rise, show steps, and finally develop a second branch of the *A* type. It is then in the third stage. I have seen this actually happen, though not in steps free from interruption.

It appears that many of the coherer phenomena can be better understood, if not explained, on the simple assumption that the contact surfaces contain in the limit many pairs of elements of surface approximately parallel and differing in the distance between them. If this assumption be allowed, and the contacts are clean, it follows that as the potential gradient is increased, there will be gaseous or leakage conduction across the contacts. If the applied potential be such as to cause even a very feeble spark, or a comparatively sudden rush of current, the coherer passes at once to the third stage. A comparatively strong electrical impulse has the same effect, causing surgings of potential if the coherer resistance is infinite, and surgings of both current and potential if it is finite.

after oxidation, but their equations can by no means be applied to a coherer in its extremely sensitive condition; *e. g.*, in the second stage. Not that a coherer does not respond to some electrical impulses even when in the third stage, such as sparks from an induction coil or a Holtz machine. But it will hardly detect such impulses as those caused by the breaking of a 1.5-volt inductive circuit at a distance of several meters.

Upon further increase of potential gradient, conduction by metallic ions will take place, and hence the resistance will be much lowered. The potential difference is then that necessary to ionize at the nearest pair of contact elements.

If there is no resistance in series, the current will at once rush to such a value that the space between the first pair of elements becomes saturated with ions, and the momentum acquired may raise the potential high enough to ionize at the next pairs of surface elements of nearly the same distance apart. The result will be an unstable condition and an undue increase in current which carries the coherer immediately to stage three.

But with a series resistance, a sudden rush of current would cause a sudden drop of potential which would check ionization. Hence the term "ballast" in speaking of the resistance R in Fig. 1. The potential will then remain very nearly constant until the first pair of surface elements is saturated with ions. Increased current can now be brought about only by increased ionic velocity, hence there comes a rise in potential until that necessary to ionize at the next pair of elements is reached. Then will ensue another interval of nearly constant potential, and the same thing will be repeated again. This hypothesis is based on the "steps" in the B type curve. They always start out with a constant potential, but sooner or later come the "steps," unless the observations are terminated by some untimely disturbance.

When all the spaces between elementary surfaces are saturated

This condition corresponds to the top branch of Curve XI., but not to the similar branches of all the other curves of the type *A*. For in many cases the coherer is still somewhat sensitive, and the curves if continued show rising potential and sometimes "steps," as in Curves VIII. and II. It may be that the influences of oxidation, of electrical impulses, or of comparatively large currents, extends to only a few of the many pairs of surface elements. For instance, suppose only one contact pair of elements to be brought into actual contact by the influence of a small spark at some distance. The resistance of the coherer would remain nearly constant, though still large, for considerable increase of current, and the voltage would rise gradually until ionization took place at the next pair of contact elements, after which the voltage would remain nearly constant for increasing current until that elementary space was saturated with ions.

But the curve would finally have a second branch of the *A* type when the coherer reached the third stage proper, *e. g.*, when all the contact element's spaces were saturated. Curve VIII. illustrates this point best, although other curves of the *A* type show indications of rising potential at the end, indicating that the coherer has not reached the third stage.

Oxidation of the contacts probably would produce little effect until the film of oxide approached in thickness the distance between the two nearest pairs of contact elements which would normally (with clean contacts) be carrying current. When the oxide attained this thickness, it would require actual contact to cause a current to flow, and the coherer would thus be brought at once to a state giving a curve of the *A* type.

The action of a comparatively large current may be such as to form a welding of the contacts, or a metallic chain across an elementary gap may be produced by a sudden rush of current. This would result in a type *A* curve even before the third stage is reached.

nearly constant voltage. This corresponds to an unsaturated condition of one or more elementary contact spaces, and the slightest change in voltage will cause a large change in the current. Hence the coherer will show either increase or decrease of resistance for similar impulses, the electrical surgings set up at the contacts being just as liable to cause positive as negative coherer action. The coherer in the second stage, in series with a small E.M.F. and a ballasting resistance, makes an exceedingly sensitive detector, and shows the positive coherer action to occur very nearly as often as the negative. The slight preponderance of the positive action may be due to the fact that some of the impulses especially in continued action, cause the welding effect, or the formation of a metallic bridge as previously referred to. The fact is that the stronger the impulses the more the positive action predominates. Thus the positive action often has a permanent effect. Even the feeblest impulses may have a permanent effect, as the positive or negative action necessarily means a change in the distribution of the ions and of the electrostatic attraction over the surfaces.

If these suggestions have any value, it follows that the more nearly the coherer curve approaches type *B* the more sensitive is the instrument. Hence when used with large currents (see Curves XI. and XII.), or with partially oxidized contacts (see Curve VIII.), the coherer is not so sensitive as when used with small currents and clean contacts. Such has always been my experience.

If curves of the *B* type really represent the behavior of the coherer in the second stage, and if the conclusions drawn from this investigation, with the assumptions and explanations given, are of any value, it is to be inferred that a curve of *distance* and terminal voltage would show different slopes in different parts.

That is, with a constant current, the rate of change of terminal potential with distance would not be constant, but would vary with the number and contour of the pairs of contact elements carrying current.

Experiments with the coherer shown in Figs. 2 and 3 gave some data from which partial curves of distance and terminal poten-

the coherer beam. Under the ends of the beam were placed two pieces of No. 22 copper wire which flattened out on the application of a heavy load to the beam, giving an even bearing taking up any irregularities in the glass and bearing points of the beam. The coherer was mounted as shown in Fig. 4.

A hole was bored in the top of a stout box and two pieces of plate glass, *PP*, placed on the box. At *WW* are the two copper wires, upon which rests the coherer beam *B*. A piece of steel piano wire, *S*, passes over the middle of the beam and supports the load, *L*.

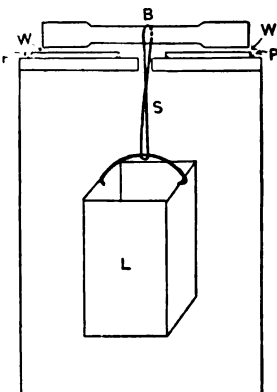


Fig. 4.

If we assume that the beam bends only in the part of circular cross-section, that is, if we assume the large flat ends of the beam to be rigid, then the dimensions given in Fig. 2 show that the change in the distance of the contacts should be 1×10^{-6} inches per pound load applied at the center of the beam.

In calibrating the beam, the load *L* was a block and tackle with a large spring dynamometer reading to two hundred pounds in 5-pound divisions. The apparatus was connected as in Fig. 1 and the current kept constant at 60×10^{-4} A while the load was varied. The current fell off on increasing the load, and was brought to the desired value by adjusting the micrometer head (see Fig. 3). The calibration was made in terms of divisions on the micrometer head. The curve of calibration is shown in Fig. 2 and from the dimensions given in Fig. 2 and the slope of this curve, we find that one pound applied in the middle of the bar causes a change in contact distance of 0.986×10^{-6} inch, agreeing within 1.4 per cent. with the theoretical constant.

The curve of calibration was very irregular until the copper wires were put in at *WW*, Fig. 4, to take up the unevenness of the bearing surface. Now the coherer is easily sensitive to a load of $\frac{1}{50}$ pound when in the sensitive condition. That is, the ammeter will show a very easily readable deflection on changing the contact distance by two hundred-millionths of an inch, or 2×10^{-8} inch. The ammeter

needle frequently goes clear off the scale for a load of a tenth of a pound ; that is a deflection of a ten-millionth of an inch. Breathing on the contacts, or holding the warm hand near them will cause a large deflection, due to different expansion of the nickel contacts and the steel. Placing the finger on the lower beam near the contacts will invariably cause a lowering of the current (due to increased load) and then a rise in the current (due to expansion from the heat of the finger). This shows better than anything else, the wonderful sensitiveness of the single-contact coherer.

It is important that the variation in load be made very uniformly, as otherwise the coherer is liable to be thrown out of the sensitive condition. In this investigation the load was changed by having a syphon empty into or out of the rubber box at *L*, Fig. 4. This gave a flow of water proportional to time, since the lower arm of the syphon was very long (15 feet) in comparison with the changes in level of the source. The syphon was timed and found to give a fairly regular rate of flow, sufficient to cause a deflection of the contacts of 11×10^{-8} inch for every five seconds of flow. The longest series of observations so far taken, extended over a distance change of only 44×10^{-7} inch, and yet there was a large change of terminal potential within this range.

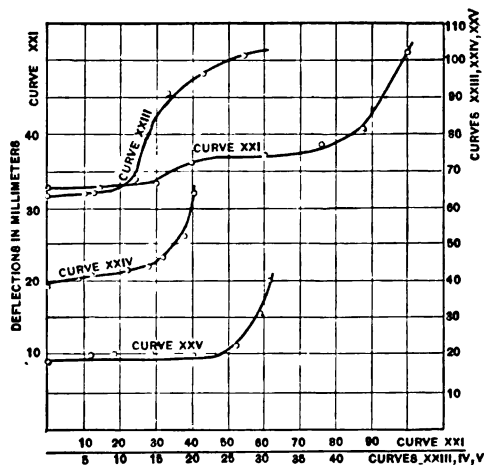


Fig. 11.

The current was kept constant by means of the Wheatstone's rheostat at *W*, Fig. 1. Of course with the arrangement shown in

Fig. 4 it is very difficult to protect the apparatus from mechanical disturbances, and the work has to be done after 11:00 P. M. when the electric cars stop running. Under such conditions data are not accumulated very rapidly, and so far I have been able to investigate the variation of terminal potential only with increasing distance. Tables XXI. to XXVI. give the observations taken for distances and terminal potential. The current was kept constant, but at different values for different trials, always being such as to have the coherer in the sensitive condition.

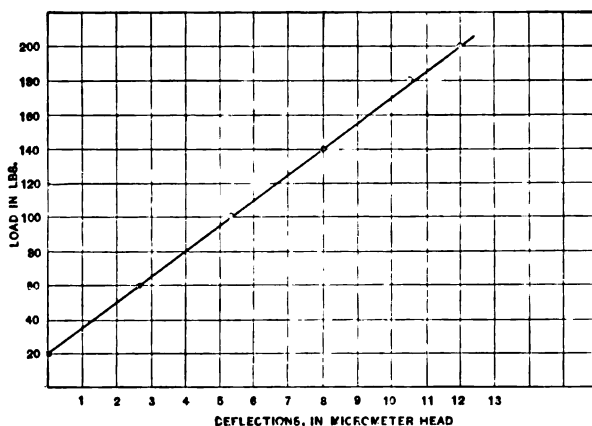


Fig. 12.

Figs. 12 and 13 show the curves of deflection (proportional to terminal voltage) and time (proportional to increase of distance between contacts). By reference to the constant of the syphon K , giving the deflection of the contacts for every second of flow, it is seen that Curves XXII., XXIV. and XXV. cover a range of change of distance of the contacts of only 0.680×10^{-6} inch, and yet show very large changes of terminal potential. Curve XXI. covers a range of 2.2×10^{-6} inch, while Curves XXII. and XXVI. curve about 5.72×10^{-6} inch. I place little reliance upon these curves so far, as they represent observations subject to many experimental errors. However, they are fairly similar, and are in harmony with the conclusions and suggestions already advanced. These observations were terminated in each case by some slight mechanical or electrical disturbances. The coherer seems to be in more and

more unstable equilibrium as the distance between the contacts and the applied voltage increase.

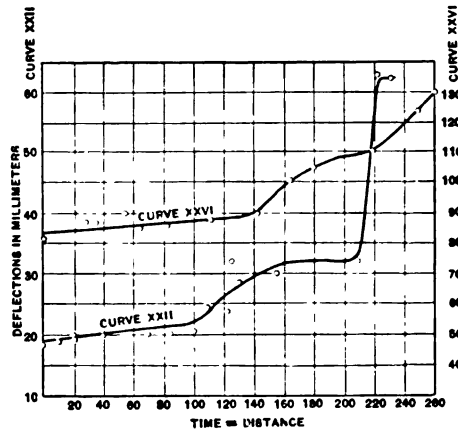


Fig. 13.

GENERAL CONCLUSIONS.

1. The coherer may give either of two distinct types of curves, depending on the applied voltage, the amount of ballasting resistance, the strength of the current, the amount of oxidation of the contacts, and the influence of electrical disturbances.

2. When in a condition such as to give a curve of type B, the coherer is most sensitive, and will show either positive or negative coherer action to very nearly the same degree.

3. The coherer is easily sensitive to changes of contact distance of the order of a ten-millionth of an inch, as well as to electrical and mechanical disturbance of exceedingly small magnitude.

4. The order of distance to which the coherer is quite sensitive shows that the *single-contact* coherer really has no practical existence. What we have called a single-contact coherer is in reality a coherer of many contacts.

5. On account of the variable potential, and consequent variable electrostatic attraction, it is not likely that the distance and voltage curves represent very closely the true relations between those two quantities. Still, these curves show considerable similarity, and, in accordance with the assumption of many parallel pairs of contact elements, they show a variable rate of increase of voltage with

distance, the curve getting steeper as the contacts are pulled apart, and fewer contacts come into play.

An interesting method of determining Young's modulus by the use of this apparatus could be devised, and the sample of metal required for such a determination would not need to be very large.

6. The theory of the coherer is far from complete. There are theories which may account for the action of the coherer under special conditions, but there is no theory, as far as I am aware, which satisfactorily explains all the different phases of coherer action. In this paper I have offered some new suggestions, and outlined the explanation of many of the coherer phenomena on the basis of the assumption of parallel elements of contact surface, but I make no attempt to incorporate these ideas into a theory until the problem of the variation of the terminal potential with distance has been further investigated.

In conclusion, let me express my sincere thanks to Professor M. D. Atkins for many valuable suggestions and criticisms made during the progress of the work, and to Mr. F. K. Brainard for assistance in taking observations and making distance tests of the coherers used as wireless telegraphy receivers, as well as for the design and construction of the coherer used for most of these experiments.

PHYSICAL LABORATORY, MICHIGAN AGRICULTURAL COLLEGE,
December 10, 1902.

SOME EXPERIMENTS IN MAGNETIC MOMENT.

BY W. P. BECK.

AT the suggestion of Professor James S. Stevens, and under his direction I have undertaken during the past year a study of the effects of certain mechanical changes upon the magnetic moment of a bar magnet. These changes were comprised under four heads: bending, twisting, concussion, change in temperature. The experiments were carried on in the constant temperature room on the first floor of the Shannon physical laboratory; the conditions for taking observations were favorable, and the liability to errors due to external disturbances was small.

EFFECTS OF BENDING.

A bar of tool steel $1' \times 1'' \times \frac{3}{16}''$ was magnetized by being placed in a helix through which was passing a strong current. It was then placed on two brass blocks as shown in Fig. 1. The edges

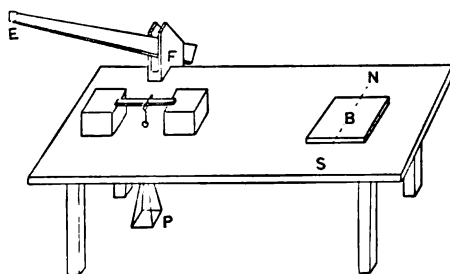


Fig. 1.

of the blocks served as knife-edges and were kept at a constant distance of 29.5 cm. apart. The magnetometer was placed on the block *B* and the "end on" position of the magnet bar *L*. Lead weights were placed upon the wooden scale-pan *P*, and resulting

changes in deflections of the magnetometer needle were read by a telescope and scale. By means of a micrometer microscope it was seen that the bar returned to its original position when the weights were removed, and therefore the limit of elasticity was not reached in any case.

TABLE I.

No.	Scale Reading.	Microscope Reading.	Weights on Pan.	Temp. C.
1	46.45	0-2.1	500 g.	17.7°
2	46.54	5-4.2	4,500	17.7
3	46.30	11-9.1	9,650	17.7
4	46.22	14-8.5	12,025	17.7
5	46.24	12-1.2	9,650	17.8
6	46.01	5-8.1	4,500	18.0
7	46.09	0-1.9	500	18.0

For zero deflection, scale reading = 9.84.

Tables II. and III. give representative readings. These were carefully taken but the results are not very satisfactory. In Table IV. a greater bending was effected by use of the lever *EF*. A force due to a weight of 16,000 grams was thus alternately applied and released, with the results indicated in Table IV.

TABLE II.

No.	Total Weight on Bar	Scale Reading.	Temp. C.
1	0g.	46.51	19.3°
2	590	46.32	19.4
3	2,590	46.14	19.4
4	5,455	46.11	19.4
5	8,500	46.13	19.4
6	10,700	46.08	19.4
7	12,200	45.92	19.4
8	12,200	45.82	19.4
9	15,100	45.52	
10	17,000	44.78	19.0

For zero deflection, scale reading = 9.84.

Distance of S. pole from needle = 76.30 cm.

Curve No. 2 represents the relation between the magnetic moment and the weights applied. The time of taking the observations is noted in the tables, since considerable variations of *H* occur during 24 hours, and this work assumes *H* to be constant. The changes in *H* should be noted here as a source of error, but the general result remains, that bending a magnet tends to decrease its moment.

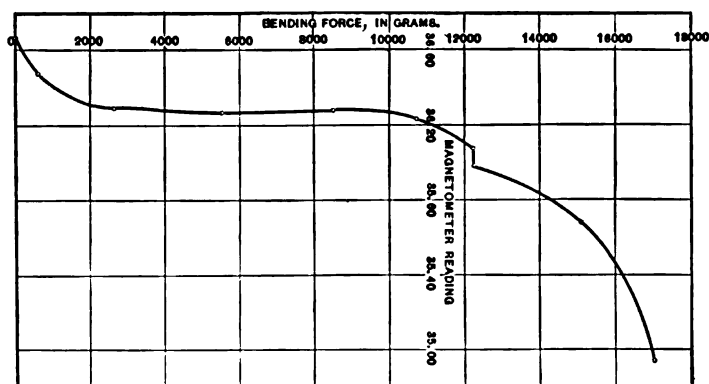


Fig. 2.

TABLE III.

No.	Force, in Grams.	Scale Reading.	Temp. C.
1	0	42.09	18.8°
2	0	42.02	"
3	16,000	41.91	"
4	0	41.81	"
5	16,000	41.77	"
6	0	41.68	"
7	16,000	41.73	"
8	0	41.66	"
9	16,000	41.66	"
10	0	41.58	18.8
11	16,000	41.53	"
12	0	41.45	"
13	16,000	41.51	"
14	0	41.34	18.8
15	16,000	41.40	"
16	0	41.30	18.8
17	0	41.22	"
18	0	41.18	"
19	0	41.22	"
20	16,000	41.28	"
21	16,000	41.34	"

EFFECTS OF TWISTING.

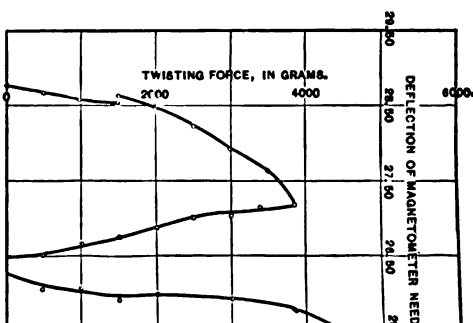
In order to twist the magnet each end was rigidly fixed in slots in brass blocks, and the center attached to a bar which served as a lever. To the other end of the lever was fastened a cord which in turn was attached to one arm of a balance. By applying weights to the other arm, the desired torsion could be given to the magnet.

TABLE IV.

No.	Force.	Deflection.	Temp. C.
1	0	28.77	17.0°
2	500 g.	28.68	"
3	1,000	28.59	"
4	1,500	28.53	"
5	1,500	28.63	"
6	2,000	28.46	"
7	2,500	28.23	
8	3,000	27.91	17.0
9	3,370	27.66	"
10	3,870	27.19	17.2
11	3,370	27.18	
12	3,000	27.05	17.3
13	2,500	27.04	"
14	2,000	26.90	"
15	1,500	26.76	17.4
16	1,000	26.68	19.2
17	500	26.52	19.1
18	0	26.48	"
19	0	26.25	"
20	500	26.08	
21	1,000	26.06	
22	1,500	25.93	19.9
23	2,000	26.00	"
24	3,000	25.93	
25	3,870	25.78	
26	4,800	25.30	19.9
27	5,670	24.68	

Table IV. expresses the results obtained, and they are shown graphically in plot No. 3.

The results show that there is a loss in magnetic moment produced by twisting; and that the loss is



magnet was twisted in the opposite direction, when similar results were obtained.

EFFECTS OF CONCUSSION.

The object of this part of the experiment was to discover if any law governed the loss of magnetization when a magnet was severely jarred. The deflection of the magnetometer needle was noted when the magnet was in its normal condition ; the magnet was then removed, held vertically five cm. from a stone pier and allowed to drop, striking squarely on its N. pole. It was caught on the rebound and placed in its former position with reference to the magnetometer, and the deflection again read. This was repeated for several heights with results which follow :

TABLE V.

No.	Pole Struck.	Height of Fall.	Scale Reading.	Deflection.	Decrease of Deflection.
1		0 cm.	35.09	20.26	
2	N.	5	34.76	19.93	.33
3	S.	5	34.19	19.36	.57
4	N.	10	33.84	19.01	.35
5	N.	10	33.75	18.92	.09
6	N.	20	33.46	18.63	.29
7	N.	30	31.92	17.09	1.54
8	N.	30	31.81	16.98	.11
9	N.	30	31.67	16.84	.14
10	N.	40	31.78	16.95	-.11
11	N.	50	31.88	17.05	-.10
12	N.	100	28.70	13.87	3.18

No regularity is observed in the falling off of the deflections ; but it may be noted that blows of a certain intensity have a greater proportional effect than others, and that if the magnet be struck a series of blows of about the same intensity, the first will produce by far the greatest effect. This would suggest that if one is experimenting with a magnet which he wishes to remain at a constant intensity, but which must undergo some jar in the experiment, it would be well to subject the magnet at the outset to a jar similar to those it is likely to receive.

Conclusion.—In general it may be said that the sources of error which were likely to exist in these measurements were the following: Uncertainty as to values of H at different times; instability of the magnetometer needle; the presence of other bodies in the field which might affect the needle. The first of these has already been discussed. The second was eliminated to a considerable extent, but not entirely. Regarding the third, great pains was taken to make the apparatus free from iron.

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SPECIAL CASES OF THE "VELOCITY OF ENERGY."

BY F. J. ROGERS.

IT is probably true that we cannot, in all cases, individualize a portion of energy — give it "personal identity" — and follow it as it is being transmitted from one place to another. In all cases of wave-motion we naturally assume that the velocity of the energy is the same as the velocity of the waves. In such cases we do individualize a portion of energy and follow it on its journey, namely, the energy associated with a given wave or train of waves.

In 1854 Lord Kelvin¹ made an indirect application of this idea. The final object of the paper referred to was to make an estimate of the density and elasticity of the ether. The immediate object was the determination of the energy of a "cubic mile of sunlight." The method was briefly to assume that all the energy in 186,000 (miles)³ of sunlight is received in one second by a surface of one square mile taken perpendicular to the sun's rays, and then to compute the energy in one cubic mile from the energy received in one second. This latter can be readily computed from the value of the "solar constant" of approximately 3 calories per cm.² per minute.

The relationship of energy per unit volume, power transmitted and velocity may be expressed in the form of the following equation

$$(1) \quad \frac{E}{V} = \frac{P/a}{v},$$

in which P/a is the power transmitted per unit area taken at right

Now there is nothing to prevent our transforming equation (1) and using it to compute the velocity of energy when the other quantities are known.

$$(2) \quad v = \frac{P/a}{E/V}$$

Let us apply this equation to the computation of the velocity of energy in a few special cases.

VELOCITY OF ENERGY ALONG AN ELECTRICAL CIRCUIT.

For this purpose we shall take the special case of concentric tubular conductors in which the resistance is so small that the fall of potential thereby produced may be neglected. In this case it will be more convenient to put (2) into the form

$$(3) \quad v = \frac{P}{E/l}$$

in which P is the whole power transmitted and E/l is the energy per unit *length* stored in the dielectric between the two conductors.

Using symbols with their usual signification equation (3) becomes for the electrical case

$$(4) \quad v = \frac{(V_1 - V_2)I}{\int \frac{KE^2}{8\pi} dA + \int \frac{\mu H^2}{8\pi} dA}.$$

The two terms in the denominator of (4) represent the electrostatic and the electromagnetic energy respectively and are to be integrated over the cross-section of the dielectric between the conductors.

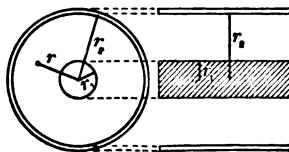


Fig. 1.

In the case of tubular conductors (see Fig. 1) of radii r_1 and r_2 the values of E and H at distance r from the common center may

Substituting these values in equation (4), integrating over the cross-section of the dielectric we obtain, when K and b are constant,

$$(6) \quad v = \frac{(V_1 - V_2)I}{\frac{K(V_1 - V_2)^2}{4 \log \epsilon \frac{r_2}{r_1}} + \mu I^2 \log \epsilon \frac{r_2}{r_1}}.$$

Using A and B as constants we may write (6) as follows :

$$(7) \quad v = \frac{(V_1 - V_2)I}{A(V_1 - V_2)^2 + BI^2}.$$

From the inspection of (7) it is obvious that the velocity is zero when either I or $(V_1 - V_2)$ is zero and that it is a maximum when

$$(8) \quad A(V_1 - V_2)^2 = BI^2 \text{ or } \frac{V_1 - V_2}{I} = \sqrt{\frac{B}{A}}.$$

This means that the velocity will be a maximum when the electrostatic energy is equal to the magnetic energy. Substituting in (7) for $(V_1 - V_2)/I$ its value obtained from (8) and we have

$$(9) \quad v_{\max.} = \frac{1}{2} \sqrt{\frac{1}{AB}} = \frac{1}{\sqrt{K\mu}}.$$

We may arrive at the same result as above by making use of Poynting's theorem. Assume an electromagnetic field in which the electric and magnetic lines of force are mutually perpendicular. For such a case Poynting's theorem gives

$$\frac{P}{a} = \frac{HE}{4\pi}.$$

The energy per unit volume is as before

$$\frac{KE^2}{8\pi} + \frac{\mu H^2}{8\pi}.$$

The maximum value of this expression is

$$(10) \quad v_{\max.} = \frac{1}{\sqrt{K\mu}}.$$

It may be impossible to form a mental picture of the motion of energy through an electromagnetic field when the latter is perfectly steady and unchanging at every point. Nevertheless, it is interesting to find that the above method gives for the *maximum* velocity with which energy may be transmitted by an electromagnetic field the same value as the velocity of electromagnetic waves.

VELOCITY OF ENERGY ALONG A ROTATING SHAFT.

Let the shaft be of uniform circular section of radius r , moving with the constant angular velocity ω and sustaining a uniform angular twist of φ/l radians per unit length. If G_0 is the torque required to twist one end of the shaft through unit angle when the other end is clamped the power transmitted by the shaft and absorbed at the distant end will be

$$(11) \quad P = G_0 \varphi \omega.$$

The energy, both kinetic and elastic, stored in unit length of the shaft is

$$(12) \quad E/l = \frac{1}{2} \frac{K\omega^2}{l} + \frac{1}{2} \frac{G_0\varphi^2}{l}.$$

Substituting these values in (3) we have

$$(13) \quad v = \frac{2l G_0 \varphi \omega}{K\omega^2 + G_0\varphi^2}.$$

This shows that the velocity with which energy is transmitted along a rotating shaft varies with the relative values of the angular velocity and the angular strain; also that this velocity is a maximum when the kinetic and elastic energies are equal. This maximum velocity is

$$(14) \quad v_{\max.} = l \sqrt{\frac{G_0}{K}}.$$

If G_0 is expressed in terms of the modulus of rigidity N , and dimensions and K in terms of the density ρ , (14) becomes

$$v_{\max.} = \sqrt{\frac{N}{\rho}}.$$

But this is just the velocity with which torsional waves would be transmitted by the shaft.

VELOCITY OF ENERGY TRANSMITTED BY A FLUID UNDER PRESSURE.

In the case of a liquid in which stress is proportional to strain the maximum velocity of energy deduced in a manner similar to the above comes out the same as the velocity of compressional waves.

STANFORD UNIVERSITY,
May, 1902.

PROCEEDINGS
OF THE
AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE NINETEENTH MEETING.

A REGULAR meeting of the American Physical Society was held in Fayerweather Hall, Columbia University, New York City, on Saturday, February 28, 1903, President Webster presiding.

The following papers were presented :

The Nucleation of the Atmosphere During Cold Weather. CARL BARUS.

An Experiment with the Holtz Machine. CARL BARUS.

The Nucleation and Ionization of Phosphorus. CARL BARUS.

Diffusion and Supersaturation. G. W. PIERCE and H. W. MORSE.

The Rôle of Thermal Electromotive Forces in a Voltaic Cell. HENRY S. CARHART.

A Method of Determining Internal Resistance of Cells, especially applicable to those which polarize rapidly. J. R. BENTON.

A Simple Geometrical Principle and its possible Relation to a General Physical Theory. JOHN MILLIS.

Adjourned.

ERNEST MERRITT,
Secretary.

RESULTS OF RECENT COMPARISONS OF MAGNETIC INSTRUMENTS.¹

BY L. A. BAUER.

THE Coast and Geodetic Survey has been engaged during the past three years in making elaborate comparisons of its various magnetometers and dip circles, almost every known variety of make being represented. The dip instrument selected as standard is a large earth-inductor of the Wild-Edelmann pattern, mounted at the Cheltenham Magnetic Observatory. This superb instrument has been found to agree

¹ Abstract of a paper presented at the meeting of the Physical Society held on December 31, 1902.

with two other inductors, one of Eschenhagen-Schulze design and the other the portable Wild-Edelmann pattern, within 0.1 of a minute of arc. The various dip circles were found to have corrections on this standard amounting in some instances to as much as 6 and 8 minutes of arc; these large corrections are in general traceable to impurities in the brass of which the instruments were constructed. The magnetometers, with the exception of three cases, were found, in general, to be in good accord. The standard is a large Wild-Edelmann observatory magnetometer at the Cheltenham Magnetic Observatory.¹

Next a *brief statement* was given of the results of the international magnetic observations made during the total solar eclipse of May 18, 1901. A small magnetic variation again revealed itself, as during the eclipse in the United States, May 28, 1900, this variation being detected at stations within the belt of totality and at the Batavia magnetic observatory, 350 miles south of the central line of the shadow and at the Manila observatory, three times the distance north of the central line that Batavia was south of it. It was shown that the nature of this variation is precisely similar to that of the solar diurnal variation. We have thus had revealed to us a phenomenon that will play an important part in unfolding the causes of the solar diurnal variation of the earth's magnetism.²

NOTE ON A SUBSTITUTE FOR A SMOKE FILM ON GLASS.³

BY ERNEST BLAKER.

OVER a year ago it seemed advisable to find some substitute for smoke for coating glass, on which to get tracings of a stylus attached to a prong of a freely falling tuning fork. The fork and its carriage are a part of a piece of apparatus used by students in the Junior Physical Laboratory, in the study of uniformly accelerated motion and in the determination of g . The glass plate on which the tracings are made is about 140 cm. long, 15 cm. wide and 1 cm. thick and to coat it uniformly with smoke film is a matter of some difficulty. Among the substances tried was a soap used to clean windows and of general use in scouring. This soap, Bon Ami, gave the best results and has been used ever since in the experiments above noted.

If the Bon Ami be put on the glass in a thin layer, by using a wet piece of cloth or waste which has been rubbed over the soap and allowed

to dry before using, it will be found that very fine, clean tracings may be obtained. If intersecting lines are desired the intersections will be found to be very sharp, there not being the liability of checking as with smoke when not properly put on.

It is necessary that the stylus be a little stiffer when used with Bon Ami than with a fine smoke film, and when Bon Ami is used in the manner suitable for the above noted experiments the damping may be a little greater than if a stylus suited for smoke be used, as the film is perhaps a little tougher. However, in tests made in which the glass was covered with parallel vertical stripes of soap film and smoke film, the average wavelength under the same conditions of position and distance from the origin was found to be a little greater on the Bon Ami film than on that part of the plate covered by smoke. This would indicate either a greater damping or a greater acceleration on the Bon-Ami-covered portion of the plate.

When tracings are made with a stylus driven by levers or a mechanism not affected by damping the smoke film has no advantage over the Bon Ami.

The Bon Ami tracings on films that are quite thick yield to fixing with shellac as do those on smoke, but such tracings are not good for lantern projection, the film being white and allowing much light to pass through it. For such work it cannot then take the place of smoke films. This is the indication of the observations here.

The Bon Ami film is so tough that a scale may be placed directly on it for measurements without damage, and it does not stick to the scale.

The advantages in its favor for such work as has been noted, over smoke, are its ease of application and consequently lessening of the danger of breaking the glass plates due to handling or to unequal heating; the clearness of the lines traced and the sharpness of intersections; and the ease of removal by simply rubbing off with a dry cloth, which leaves a clean glass surface without scratches.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY.

RADIOACTIVITY FROM FRESHLY FALLEN SNOW.¹

BY S. J. ALLAN.

C. T. R. WILSON has shown that freshly fallen rain, after being evaporated down to dryness, leaves behind it a radioactivity which decays with time. The author was led to believe that freshly fallen snow would also show this property. This on examination proved to be the case. Freshly fallen snow was gathered from a thin layer on the surface

¹ Abstract of a paper presented at the meeting of the Physical Society held on December 31, 1902.

during a snow storm. This on being evaporated down to dryness in a metal dish, and tested was found to ionize the air in the immediate vicinity. From about .1 liter of snow during a heavy snow storm could be obtained about the same effect as from about .1 gram of uranium. Decay curves were taken and the activity was found to fall off in a regular geometrical progression with time, falling to half value in about 30 minutes. This is quite unlike the excited radioactivity from air as obtained on a negatively charged wire, which decays to half value in about 48 minutes. The penetrating power was found to be about the same as that for the excited activity on a charged wire. Experiments over a wide range showed that the amount of radioactivity in a given quantity of snow varied as the amount of snow falling per second, and was constant so long as the fall of snow was constant. At the end of 24 hours after the snow had ceased to fall only a very minute fraction of the radioactivity remained. The conclusions to be drawn from these results are, that this radioactivity is different from the excited, or that the rate of decay for the excited is more complicated than at first supposed and may have different values in different portions of the curve. These points are at present under investigation.

INDUCED RADIOACTIVITY EXCITED IN AIR AT THE FOOT OF WATERFALLS.¹

By J. C. McLENNAN.

A LITTLE over a year ago Elster and Geitel² found that if a negatively electrified wire were exposed for some hours in the open air or in a very large room, it became temporarily radioactive. Since then a number of observations have been made upon this effect, and the consensus of opinion appears to be that it is due to the presence in atmosphere of some peculiar constituent similar to the emanation from thorium, which has been shown by Rutherford³ to induce radioactivity in any body with which it comes in contact, especially when that body is negatively electrified.

The difficulty of determining and of regulating the atmospheric conditions for observations upon this excited or induced radioactivity suggested the desirability of resorting, for purposes of experiment, to a locality where exceptional electrical conditions were known to exist

results, is preëminently such a locality, as the fine drops of spray into which water is broken on passing over waterfalls were found by him to communicate, on striking the wet rocks at the foot of the falls, a negative charge of electricity to the surrounding air and a positive charge to the water. Through the kindness of the Hon. Thomas Walsh, Superintendent of the Niagara Falls National Park Reservation, the author was enabled in September last to carry out a short series of observations upon excited radioactivity at the foot of the falls. The general result of the investigation was that, during the course of the experiments, the amount of radioactivity induced in a wire exposed at the foot of the falls was found to be very much less than that in a wire exposed in the same manner in Toronto.

Apparatus.—In these observations the measurements were made with a quadrant electrometer of the Mascart type as constructed by Carpentier. The silk suspension in the original apparatus was replaced by a phosphor bronze strip less than 0.025 mm. in thickness, which was attached at its upper end to an ebonite rod to secure insulation. The needle was kept charged by a battery of small storage cells similar to those installed in the Reichsanstalt. The deflections were measured by the movement of the image of an incandescent lamp filament upon a transparent scale placed at a distance of one meter from the electrometer.

With a potential of 480 volts applied to the needle, the sensibility of the instrument was such as to produce a deflection of 1,000 mm. on the scale for a potential difference of one volt between the quadrants. In measuring the induced radioactivity bare copper wire, No. 24, was exposed in the open air by means of specially constructed insulating supports, Fig. 1, attached to a series of bamboo poles erected at convenient distances. The wire was charged by a small Toepler-Holtz electrical machine driven by a water motor, which maintained a potential of from eight to ten thousand volts.

The insulator shown in Fig. 1 consisted of a brass tube about 20 centimeters long, closed at one end and having a bell-shaped opening at the other. Into this tube an ebonite rod was screwed, which could be easily removed when it was necessary to renew its surface. Hooks fastened to the tube and to the ebonite rod provided for the support of the insulator and the suspension of the wire.

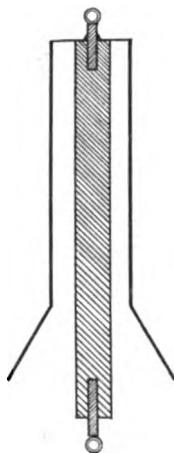


Fig. 1.

In the experiments at the foot of the Falls it was found that, with these insulators, a wire could easily be maintained at a potential of 10,000 volts for hours even in a drenching spray.

The exposed wire was tested for induced radioactivity by means of the apparatus shown in Fig. 2. *A* is a galvanized-iron cylinder about 30 cm. high and 20 cm. in diameter, resting on an insulated platform and having a movable cover provided with a flanged opening into which was fitted an ebonite plug about 5 cm. in diameter. A brass tube *C* was passed through this plug and into it a second ebonite plug was tightly fitted. This second plug carried a brass rod *D* from which *B*, a brass reel, was suspended, and on this reel the wire to be tested was wound. The brass tube *C*, which was earthed throughout the measurements, served as a guard ring and prevented any leak from the vessel *A* to the rod *D* across the ebonite plugs.

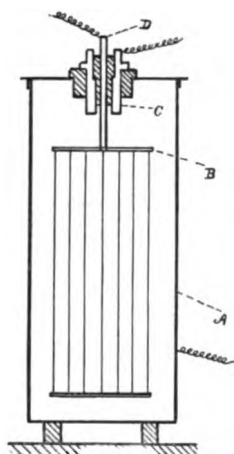


Fig. 2.

The saturation current, due to spontaneous ionization of the air in the vessel, having been first determined by connecting the supporting rod *D* to the electrometer in the usual way, and applying a potential of 100 volts to *A*, the exposed wire was then wound on the reel, inserted in the vessel, and the saturation current again ascertained. Any increase observed in the ionization current was taken as a measure of the radioactivity induced in the exposed wire.

As it was impossible, in setting up the apparatus in two different localities, to be certain that it was of exactly the same sensibility, a standard of ionization was deemed necessary for purposes of comparison. A radioactive substance which the writer had in his laboratory at the time of making the experiments was chosen, and a small quantity in a glass phial being found to give a constant ionization current when placed in a given position in the chamber *A*, this current was adopted as a standard and the ionization currents measured in the various tests were expressed in terms of it.

Experiments. — Both before proceeding to Niagara and after returning a series of exposures was made in the quadrangle of the University of Toronto. Copper wires, approximately 30 m. long, were exposed on a number of days for periods of two hours at a potential of 8,000 to 10,000 volts. After a wire had been exposed it was placed in the ionization chamber, the saturation current measured and its radioactivity, thus ascertained, was expressed in terms of the standard radioactive substance.

The resulting values, taking the standard saturation current as unity, showed considerable variation in the amount of radioactivity excited, the highest value observed being 1.75, and the lowest 0.6. The intermediate values ranged between these limits, but were quite irregular and seemed

to depend more upon the existence of wind and its velocity than other changes in the weather. The occurrence of showers did not exert a noticeable effect on the amount of radioactivity excited. It was frequently observed that exposures made in the morning gave greater induced radioactivity than those in the afternoon, but the exposures made after sunset gave values generally equal to and sometimes greater than those of the morning.

The observations were made in Toronto between September 5 and 12, and between the 19th and 23d, those made at Niagara occupying the interval. The values in the two series at Toronto exhibited similar variations and were confined in both cases within the limits mentioned. The weather throughout, except for a few showers, was uniformly fine.

In conducting the experiments at Niagara Falls the electrometer and its attachments were set up in the basement of the landing station at the foot of the inclined railway. It was a large room, having stone walls and a cement floor, and, being fairly dry, was well suited for making the tests. The electrical machine which was intended for charging the wire was also located in this room, and the wire to be exposed was led from it out of a window and suspended in three stretches of about 30 m. each, the insulation being secured as already described.

The situation was admirably adapted for making the exposures, as the wire could be led within a meter or two of the vast mass of falling water. The first length of wire was generally enveloped in a very fine spray, that which surrounded the second was heavier, while the downpour upon the third resembled the heaviest rain.

A point of peculiar interest in connection with the first exposure was that the wire, upon suspension in the spray, immediately became negatively charged to a potential of about 7,500 volts. This voltage was maintained with but little variation both day and night during the period covered by the experiments and the sign of the electrification was invariably negative.

The results of a number of tests with the different sections of the wire showed that the removal of the third generally caused a drop in the potential of the remaining sections from 7,500 volts to about 5,500 volts, while the first section, when exposed alone, gave a potential varying from 3,000 to 4,000 volts. From this it was evident that the spray was the cause of the electrification and that the potential of the wire was largely determined by its density. On account of the permanency of this negative electrification of the exposed wire, the electrical machine necessary in the Toronto experiments was not employed.

In testing for induced radioactivity, the wire was exposed for periods of about two hours and was removed in sections, each section being replaced by a new length. The same method of testing was used as in

Toronto and the results expressed in terms of the standard radioactive substance.

Exposures were made on four different days with the following results, *A*, *B*, and *C*, denoting the sections of the wire, of which *A* was that nearest the observing station and *C* that nearest the falls.

Date.	Section of Wire Tested.	Induced Radioactivity. Current of Standard Substance	Saturation i.o.
September 12.	<i>A</i>	0.1	
	<i>B</i>	0.31	
	<i>C</i>	None observed.	
" 13.	<i>A</i>	0.16	
	<i>B</i>	None observed.	
	<i>C</i>	Not exposed.	
" 16.	<i>A</i>	0.13	
	<i>B</i>	0.14	
	<i>C</i>	0.11	
" 17.	<i>A</i>	0.15	
	<i>B</i>	0.12	
	<i>C</i>	None observed.	

From these numbers it will be seen that but little variation was observed in the radioactivity excited in the sections *A* and *B*. In *C*, however, which was nearest the falls, measureable radioactivity was present in only one of three exposures.

Ordinarily this section was drenched by masses of falling water in addition to being surrounded by fine spray such as enveloped the rest of the wire. But on September 16, the air currents during exposure were such as to drive aside the sheets of water and permit only the fine spray to come in contact with the section.

As already stated the spray enveloping the section *B* was generally heavier than that in contact with *A*, but the condition was not permanent, as the spray was blown about by gusts of wind whose direction was continually changing in the gorge.

The chief interest attaching to the results was that the radioactivity excited was much less than that in Toronto. It will be noticed that the greatest amount of radioactivity observed at Niagara Falls was 0.3, while the least observed in Toronto during the period covered by the experiments was 0.6. The numbers also show that on the average the radioactivity excited at Toronto was at least from six to seven times greater than that induced at the falls.

The observations were not made in the two places at the same time, and a direct comparison is therefore impossible, but, as there was no break

in the weather during the progress of the experiments, and as the values found in Toronto before going to the falls were almost the same as those obtained on returning, one is warranted in concluding that the air at the foot of the falls permanently possesses less power to excite radioactivity than the air in localities at some distance.

While the experiments were in progress the spray was frequently examined for radioactivity. This was done by collecting a quantity, evaporating it and testing the containing vessel. In no case was any trace of radioactivity observed.

UNIVERSITY OF TORONTO,
October 30, 1902.

EXPERIMENTS CONCERNING VERY BRIEF ELECTRICAL CONTACTS.¹

BY HERSCHEL C. PARKER.

A SERIES of electrical contacts giving a fairly accurate range of adjustment from 0.1 second to 0.00001 second would furnish a valuable means of investigation. A gravity contact key devised by Dr. Charles Forbes gives promise of fulfilling the above conditions. The writer has made many determinations of the times of contact given by the various devices employed on this key, and has also investigated the times of contact of several forms of pendulum. The method employed was as follows: a condenser of known capacity (F , farads) was charged during the time of contact (t) and the deflection on discharging noted. This deflection (if a good mica condenser is used which has no absorption) is proportional to the electromotive force (E) and the capacity (F). The condenser is again charged through a resistance (R) and the deflection (Q) observed.

Then: $Q = EF \times (1 - e^{-\frac{t}{RF}})$ and, $t = -RF \times \log e \times \left(1 - \frac{Q}{EF}\right)$.

The method while very easy to apply allows a wide range in the resistances and deflections with but little effect on the values of the time. An example will best show this. 1st, $F = 0.00000048$, $R = 1000$, $EF = 120$, $Q = 43\frac{1}{2}$, $t = 0.00026$ second. 2d, $F = 0.00000048$, $R = 10,000$, $EF = 120$, $Q = 5\frac{1}{2}$, $t = 0.00027$ seconds.

The "gravity key" consists essentially of a rectangular weight falling on metal guides, the key being furnished with a scale divided in fractions of a second according to the law of falling bodies, and the weight actuating the various forms of switches employed. If two switches are used,

¹ Abstract of a paper presented at the meeting of the Physical Society held on December 30, 1902.

one to make the contact and the other to break the contact, by placing them at different distances apart on the scale times of contact varying from 0.4 second to 0.001 second may be obtained. For shorter times, a single switch that makes and breaks the contact is made use of and the time made faster or slower by placing in different positions on the scale so that the falling weight strikes it with varying velocities.

In one form, the weight moves the short arm of a lever, the long arm passing over a contact strip. Another form is one in which the fulcrum of the lever changes, first giving contact and then breaking the circuit immediately afterwards. By placing this form of key in different positions on the scale, times of contact varying from 0.0015 second to 0.000035 second may be obtained. In still another type, which seems the most rapid and positive contact of the three, the falling weight striking a lever arm releases a spring which makes the contact and a further motion of the lever breaks the contact, thus giving a differential effect between the velocity of the weight and the rapidity of the spring. With this key, while a contact as slow as 0.02 second can be made, the rapidity of 0.000017 second may be obtained and it seems possible with the most careful adjustment to reach 0.00001 second. The limiting condition in all these keys is the mechanical adjustment, for as the time becomes very short the contact becomes uncertain and may frequently fail together.

Experiments made with pendulums consisting of a steel ball suspended by a wire and striking against a steel anvil, gave very positive and satisfactory contacts. These contacts, while very brief, give only a slight variation in time. Using a pendulum with a suspension wire about 4 meters long and the steel ball 2 inches in diameter, an arc of $\frac{1}{2}$ degree gave 0.00039 second and an arc of 15 degrees, 0.00016 second, while a pendulum with a short suspension wire using a $\frac{1}{2}$ -inch steel ball, through an arc of 90 degrees gave 0.000079 second.

It is interesting to note that in working with condensers, the best mica condenser gives no appreciable variation in capacity for practically the briefest times of charge, while a paraffine condenser may show a reduction in capacity of some 60 per cent. from a time of charge of 1 second to that of 0.001 second.

THE CRITICAL CURRENT DENSITY AND THE FALL OF POTENTIAL AT THE CATHODE IN VACUUM TUBES.¹

BY CLARENCE A. SKINNER.

THIS investigation was undertaken in order to arrive at an explana-

It has been repeatedly observed that the discharge from the cathode tends to maintain a certain current density by confining itself to a part of the cathode surface only, if the current drops below a certain value. With increasing current the discharge gradually spreads over the cathode until the exposed surface is wholly covered. Within these limits the cathode fall, as shown by Hittorf, remains constant. H. A. Wilson has shown that the current density also remains constant—equal to about $0.4 \text{ } \mu$ milli-amp./square cm. in air, where μ designates the pressure of air in mm. When the cathode is wholly covered by the discharge its fall of potential increases with the current, that is, with the current density. These observations were made with a wire serving as cathode.

With a disk cathode the author has previously shown that the cathode fall is a linear function of the current density, the gas pressure remaining constant. Stark, using a wire, finds it to be a parabola. He embodies in his results the condition that the current density cannot, as described above, fall below a certain value, which we may designate as the *critical* density. With a disk, the author finds this critical density to be a remarkably uncertain quantity, though the relation between cathode fall and current density does not vary with these changes. This characteristic is especially marked at gas pressures below 1.5 mm. In nitrogen the critical density varies (with the same gas) from about $0.2 \text{ } \mu$ milli-amp./square cm. as a maximum value, to *less* than $.01 \text{ } \mu$ as a minimum—this lower limit being that of the milli-ammeter. At higher pressures the critical density is larger and fluctuates less—being about $0.3 \text{ } \mu$ at 4.5 mm.

Simultaneous observations with two cathodes of the same material, one a disk and the other a wire, showed the cathode fall to be practically the same when the cathode current density was the same, but the critical current density for the wire to be about one and a half times the maximum value for the disk. By manipulation the critical density for the wire also was reduced to less than one half its normal value (at a gas pressure of 0.85 mm.) in which case the cathode fall was reduced accordingly.

These results disprove the supposition that the critical current density represents the minimum cathode fall obtainable, that the fall would increase if by any means we should be able to force the current density below this value. It may be caused by the resistance of the gas under the influence of the cathode increasing with decreasing current density, as shown by the author. According to the latter view the critical density would be that for which the potential difference between the cathode and a point in the stream at the boundary of its influence remains a minimum as the current varies.

ARE FALLING BODIES DEFLECTED TOWARD THE SOUTH?¹

BY E. H. HALL.

IN *Science* for November 29, 1901, Professor Cajori published an article under the heading "The Unexplained Southerly Direction of Falling Bodies," in which he gave a summary of the experimental results obtained by the various observers of this deviation. The main parts of this summary can be put into tabular form as follows: S. D. standing for southerly deviation, and E. D. for easterly deviation:

Observer.	Time.	Place.	Fall.	S. D.	E. D.	S. D. + E. D.
Hooke,	1680	London	8.2 m. (?)	+	+	?
Guglielmini,	1791	Bologna	78.3 "	1.19 cm.	1.89 cm.	0.63
Benzenberg,	1802	Hamburg	76.3 "	0.34 "	0.90 "	0.38
Benzenberg,	1804	Schlebusch	84.4 "	0. "	+	0.
Reich,	1831	Freiburg	158.5 "	0.44 "	2.84 "	0.16
Rundell,	1848	Cornwall	400 "	(25 to 50) cm.	?	?

To this table the author now adds from his own experiments,

1902	Cambridge	23 m.	0.0045 cm.	0.149 cm.	0.03,
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the probable error of the S. D. being nearly as large as the value found for the deviation itself.

Gauss and Laplace have by theoretical discussion arrived at the conclusion that the S. D. for experimental heights is inappreciable. It is therefore worth while to examine carefully the original accounts, so far as they are available, of the various experimental researches regarding this deviation, to see whether any explanation of the discrepancy between theory and observation can be found. The most thorough examination which I have been able to give of these researches does not yield any satisfactory explanation. The testimony from the work of any one experimenter in favor of a southerly deviation is not entitled to much confidence, but the general agreement of the testimony that there is such a deviation makes further experiments desirable.

My own results were obtained from 948 trials made under a considerable variety of conditions, but the distance available was too small for the most conclusive results.

THE CHANGE OF ATMOSPHERIC NUCLEATION DURING COLD WEATHER.¹

BY CARL BARUS.

THE author has continued the work with his coronal method of counting atmospheric nucleation, and shows the remarkably high values obtained during the very cold weather of December and January. The ten-

¹ Abstract of paper read before the Physical Society at the meeting held on December 31, 1902.

dency of the nucleation to increase coterminously with a rapid fall of temperature and to fall off with rise of temperature is particularly brought out in a series of parallel curves. The author observes that fall of temperature must also be favorable to the formation of water nuclei and three hypotheses are advanced: A current from the upper air regions rich in nuclei may be brought to the observer by the cold wave; or the formation of water nuclei may bring down an air stratum overlying cities; or the water nuclei may be radioactive. Special experiments are in progress to test the latter question.

IONIZATION AND NUCLEATION OF THE PHOSPHORUS EMANATION.¹

BY CARL BARUS.

THE author endeavors to ascertain whether in an emanation neutral as a whole like that from phosphorus, evidence may be obtained that the negative ions are more efficient as condensation nuclei than the positive ions: for in such a case the emanation after successive partial precipitation in the condensation chamber, should become continually more positive. The results however show that the ionization vanishes practically in a few minutes, while the nucleation is *by contrast* indefinitely persistent and constant. With water nuclei it may be demonstrated that before and after the time in which the ionization nearly quite disappears, the coronas scarcely change in character. Hence in these experiments no relation between ionization and nucleation can be detected.

EXPERIMENTS WITH A HOLTZ MACHINE AND THE RATIO OF THE VELOCITIES OF THE IONS.¹

BY CARL BARUS.

BY attaching a point electroscope, adjusted both to dissipate charge and to measure the potential at which the dissipation takes place, alternately to the positive and the negative electrode of a Holtz machine, the author finds from the potentials observed on open circuit, V'_p and V'_n and from the residual potentials of the closed circuit, V_p and V_n

$$(v_n/v_p)^2 = V'_p(V_n - V'_n)/V'_n(V_p - V'_p)$$

where v_n/v_p is the ratio of the negative and positive ionic velocity.

THE RÔLE OF THERMOELECTROMOTIVE FORCES IN A VOLTAIC CELL.¹

BY HENRY S. CARHART.

THE writer presented briefly the thermo-dynamic theory of a voltaic cell, so far as relates to its properties dependent on temperature. It was shown that all these could be completely explained by means of electrolytic thermoelectromotive forces between a metal and the liquid in contact with it. Thermoelectromotive forces exist without temperature difference at the junctions, for a current will either absorb or generate heat at a junction according to its direction in relation to that of the thermoelectromotive force there.

Data were given showing that the temperature coefficients of a Daniell cell, a Carhart-Clark cell, and a calomel cell are all accounted for numerically by the thermoelectromotive forces at the metal-liquid junctions.

It was also shown that the heat represented by the second term of the Gibbs-Helmholtz equation is the difference between the heat generated at the negative electrode, where the current flows against the thermoelectromotive force, and that absorbed at the positive, where both current and electromotive force are in the same direction. The effects are thus localized in the cell.

It was also demonstrated by curves and numerical data that the electromotive force of a concentration cell is explained for dilute solutions by the thermoelectromotive force at the two electrodes, because this electromotive force increases with the density of the solution. For this last reason also thermoelectromotive forces explain the change in the electromotive force of a Daniell cell when the density of either solution is changed. All these conclusions have been confirmed by numerous measurements.

ON THE MECHANICAL EFFICIENCY OF MUSICAL INSTRUMENTS AS SOUND PRODUCERS.²

BY A. G. WEBSTER.

EXPERIMENTS were made on the efficiency for the tone of frequency 256 of the following instruments: Cornet, French horn,

water gauge connected with a tube in the corner of the mouth. The volume delivered to the instrument was obtained by measuring the air inspired from a spirometer and that expired to the same after blowing for a measured period. For the voice, the pressure was estimated by singing steadily, closing the mouth on the gauge while maintaining the same muscular effort on the lungs. From pressure, volume and time the input of energy was obtained. For the violin, the bow was supported so as to be played mechanically by a measured pull with a measured velocity. The output in sound was measured by the author's phonometer, by comparison with the output of the standard "phone" placed in the position occupied by the instrument. Preliminary results follow. The small efficiency of the oboe is due to the fact that most of the work goes to the production of harmonics, which are not here taken account of.

	Pressure.	Volume / sec.	Energy Input.	Sound Output.	Efficiency.
Cornet.	13 cm.	52.6 $\frac{\text{cm.}^3}{\text{sec.}}$	.067 watts.	770 $\frac{\text{ergs}}{\text{sec.}}$	.00115
French horn.	18.2	70.0	.125	4,734	.0038
Bombardino.	17	58.1	.097	12,310	.0127
Saxophone.	22	102.5	.222	1,970	.00086
Clarinet.	16	46.7	.073	3,076	.0042
Oboe.	25	24.0	.059	31	.00005
Voice.	6	199.0	.117	11,070	.0095
Violin.			.048	254	.00052

DIFFUSION AND SUPERSATURATION.¹

BY HARRY W. MORSE AND GEORGE W. PIERCE.

THIS is an account of a research on diffusion and supersaturation in gelatine solutions, by a method based on Liesegang's "A-lines." When the end of a capillary tube containing a gelatine solution of potassium chromate is dipped into a water solution of silver nitrate, the silver nitrate diffuses up into the tube and throws down a precipitate of silver chromate. The silver chromate, instead of growing continuously as diffusion proceeds, forms in distinct layers widely separated in comparison with the thickness of the layers.

By means of the mathematical theory of diffusion and the equation expressing the equilibrium of the silver ion, the chrom ion and the undis-

it is possible to supersaturate a solution with silver chromate. The only the bottom of the tube and the times at which they form. By employing various initial concentrations of the diffusing substances it is found that the concentrations of the ions at the limit of supersaturation satisfy a formula similar to the corresponding formula for saturation in the presence of the solid phase, namely,

$$\bar{A}g^2 \times \bar{Cr}\bar{O}_4 = H,$$

in which $\bar{A}g$ is the concentration of the silver ion, $\bar{Cr}\bar{O}_4$ the concentration of the chrom ion, and H a constant called the "Metastable Solubility Product" in distinction to the "Solubility Product" for saturation in the presence of the solid phase.

The value of H obtained for silver chromate is 1.4×10^{-6} . The ordinary solubility product is 5.1×10^{-18} . By taking the cube root of the ratio of these numbers it is seen that at the limit of supersaturation the solution contained 145 times as much silver chromate as is required to form a precipitate in the presence of the solid phase. It was found that the metastable limit is a perfectly definite quantity. So far as the authors know, the numerical value of the metastable limit has not previously been published for any substance.

Incidentally the diffusion constant for silver nitrate in gelatine was determined.

The method is applicable to the determination of the diffusion constant and the limit of supersaturation of a great variety of slightly soluble substances.

JEFFERSON PHYSICAL LABORATORY,
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A SIMPLE GEOMETRICAL PRINCIPLE AND ITS POSSIBLE SIGNIFICANCE IN CONNECTION WITH A GENERAL PHYSICAL THEORY.¹

By JOHN MILLIS.

THE various possible arrangements of a collection of spherical bodies of equal size are considered and the employment of "ping-pong" balls fastened together by adhesive material for experiment and illustration is suggested.

The arrangement commonly regarded as the most compact possible, when the balls are in contact throughout and the mutually tangent planes form the oblique dodecahedron about each ball, is capable of indefinite extension. In such a collection any one ball may be taken as the cen-

¹ Abstract of a paper presented at the meeting of the Physical Society held on February 28, 1903.

tral one of a group of thirteen and the twelve balls tangent to it will have the same disposition in all of such groups, being of course tangent also to each other. The arrangement is a rigid one throughout but not entirely symmetrical or homogeneous as the void spaces have two general forms, one form of space being surrounded by four balls and the other by six. The above arrangement is called arrangement *A*.

Twelve balls may also be placed about and tangent to a central one so that no two of the side balls are in contact with each other, the mutually tangent planes forming about the central ball the regular dodecahedron. This is called arrangement *B*. It cannot be extended on the same system beyond thirteen balls as a simultaneous condition, but in a collection of an indefinite number each ball may become temporarily and in turn the central one of a group having arrangement *B* by proper relative movements throughout the mass.

If to each of the two groups of thirteen balls having arrangements *A* and *B* one additional layer of balls is added by placing a ball over each interval on the outer surface of the collection, group *A* will have twenty-seven balls and group *B* thirty-three. The circumscribing tangent sphere for the twenty-seven will have a greater diameter than the circumscribing tangent sphere for the thirty-three, and all the balls in this latter group except the central one may be moved radially so the balls are nowhere in contact with each other and they can still be included in a tangent sphere smaller than that enclosing the twenty-seven, notwithstanding these latter have the arrangement commonly considered the most compact possible. By removing the central ball of the thirty-three group the remaining thirty-two may be included in a sphere still smaller relatively than the one enclosing the twenty-seven group and yet have no two of the thirty-two group in contact.

A third arrangement, called arrangement *C*, is that in which the mutually tangent planes form cubes.

The assumption is now made that the balls come together in a collection under their mutual attractions according to gravity laws. They will not assume or remain in arrangement *C* because while this gives symmetry throughout it is not the most compact possible and the equilibrium of the collection would be unstable.

Arrangement *A*, while the most compact possible for an indefinite number of balls in contact throughout, is not entirely symmetrical.

Arrangement *B* gives a perfectly symmetrical disposition of twelve balls with respect to a central ball but it is geometrically impossible throughout a collection of a greater number than thirteen

that will give minimum total volume, symmetrical disposition, and therefore fixed stable equilibrium throughout.

In arrangement *B* if regular dodecahedrons be formed about the balls these solids will approximate to a "fit" but there will be of course vacant spaces, since the diedral angle of the regular dodecahedron is somewhat less than one third of three hundred and sixty degrees.

Similarly regular tetrahedrons formed by drawing right lines joining the centers of four balls placed as close as possible will not quite stack up without voids, though this condition is approximated to.

As an assistance to an understanding of conditions as they actually exist it is suggested that it might be interesting to speculate upon what the consequences would have been had it so happened that the diedral angles of the regular dodecahedron and the regular tetrahedron were exact fractional parts of three hundred and sixty degrees.

Reference is made to some apparent relations between the totals that result from adding layers of balls in arrangement *B* and the series of differences in the table of atomic weights when the numbers representing these weights are arranged according to the periodic law.

The simple geometrical principle referred to in the title is concisely stated as follows:

In an aggregation of an indefinite number of equal spherical bodies an arrangement giving minimum total volume and perfect symmetry throughout is impossible.

The possible significance of this in connection with a general physical theory is simply that it appears to offer an explanation — independent of any assumption of molecular repulsion or of a special medium or any special attribute of matter other than gravity — of the impossibility of the existence of a homogeneous plenum of matter. From this follows a possible elucidation of the vexed problem of elasticity in its various forms and consequently more or less advance towards an understanding in nearly the whole range of physical phenomena.

No special assumption is made as to the shape or size of an ultimate particle of matter or whether there is in fact such a thing as an ultimate particle of matter. It is sufficient that matter having the attribute of gravity only cannot exist as a homogeneous plenum. If such a plenum were possible it would be possible to describe within it a system of equal spheres of any size, small of course with respect to the dimension of the body considered, not limited as to location, and arranged so that the matter in the volume not contained in the spheres could be removed leaving the spheres in a condition of fixed and stable equilibrium; and conversely so that the spheres could be removed leaving the remaining matter in the same condition. This cannot be done.

It is suggested that the above may indicate some progress towards a

generalization in which all physical phenomena are reduced to the irresolvable conception, *time, space and matter-gravity*.

SEATTLE, WASH.,

February 21, 1903.

A METHOD OF DETERMINING INTERNAL RESISTANCE, APPLICABLE TO RAPIDLY POLARIZING CELLS.¹

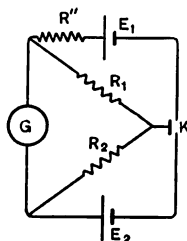
BY J. R. BENTON.

THE methods of determining internal resistance may be divided into two classes: (1) Those in which the current through the cell is varied, and (2) those in which it is kept constant. Methods of the first class cannot be used with cells which polarize rapidly, for in such cells the internal resistance is a function of the current and of the time it has been flowing.

Some of the methods of the second class have the disadvantage that they require an instrument for measuring current or difference of potential, whose constant must be known. The method of Beetz, which can be used with rapidly polarizing cells if they are in circuit only for short intervals of time, avoids this inconvenience, but has several other disadvantages.

The method about to be described is similar in principle to that of Beetz, but appears to possess some advantages as compared with it, as follows: (1) There is no limitation to the E.M.F. of the cell to be investigated; Beetz's method cannot be used unless this E.M.F. is greater than that of the smallest available auxiliary cell. (2) The method can be used for cells of very low internal resistance; Beetz's method cannot be used to advantage under such circumstances. (3) While the accuracy of the results must depend in either method on a great many different considerations, it seems that in general this method gives greater accuracy with a galvanometer of the same sensitiveness.

The apparatus required is the same as in Beetz's method, except that an additional resistance box is needed. The connections are shown in the accompanying diagram. At K is a key which can be thrown so as to complete at the same time the circuits $KE_1R''R_1$ and KE_2R_2 ; E_1 is the cell whose resistance is to be determined; E_2 is an auxiliary cell; G a galvanometer, preferably of low resistance and sensitive; R_1 , R_2 , and R'' are resistance boxes.



¹ Abstract of a paper presented at the meeting of the Physical Society held on February 28, 1903.

The determination is made as follows: Taking all the resistance out of R'' , so that its resistance is zero, set R_1 at a convenient small resistance; then adjust R_2 so that the galvanometer gives no deflection when the key K is thrown. Next set R_1 at a large resistance; then, keeping R_2 unchanged, adjust R'' till no deflection of the galvanometer occurs when the key is thrown. Denoting the first value of R_1 by R_1 , the second by R_1' , and the resistance in R'' by R'' , we have the internal resistance of the cell equal to

$$\frac{R_1 R''}{R_1' - R_1}$$

NEW BOOKS.

A Laboratory Manual of Physics for Use in High Schools. By HENRY CREW and ROBERT R. TATNALL. Pp. 234. New York, The Macmillan Company, 1902.

A Manual of Elementary Practical Physics (second edition). By JULIUS HORTVET. Pp. 276. Minneapolis, H. W. Wilson, 1902.

Laboratory Exercises in Physics for Secondary Schools. By GEORGE R. TWISS. Pp. 193. New York, The Macmillan Company, 1902.

As we pass on to a stage of fuller provision for the needs of teachers and pupils in secondary schools, three types of books must be furnished in connection with the instruction in physics. These will correspond to three elements now generally recognized as necessary to a sufficiently broad presentation of the subject—at least where large classes are taught. First, a collection of illustrative experiments, to be carried out by the teacher before the class, introducing fundamental relations among phenomena (more usually on their qualitative side) with greater variety than the scanty laboratory time allows. Secondly, the laboratory manual proper, designed as a guide for the individual experimental work of the pupils (mainly quantitative). And, thirdly, the text-book as a basis for discussion in the class-room. It may be objected that a capable teacher should devise his own demonstrative experiments; but it remains practically true that there is still a field for a book which shall gather up and record the dispersed traditions of many class-rooms for the common benefit of all. Such an enterprise would not be so remunerative as a successful text, but it would be a piece of public service to compile the data.

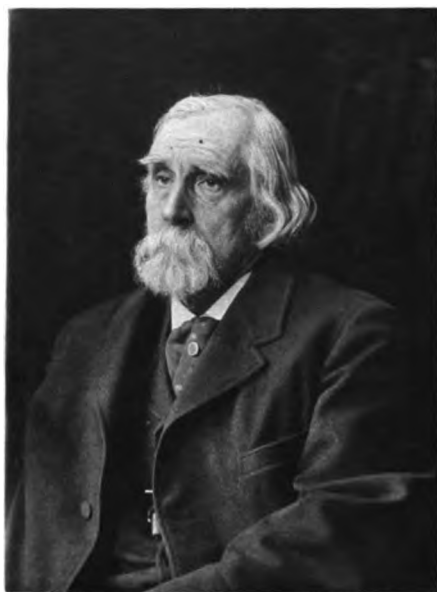
The three books here given by title are intended to fall into the second class; and all of them possess distinctly the merit of putting into circulation suggestions of practical teachers, tested and tried out under working conditions. Any one of the three offers a treatment that will be judged satisfactory of those laboratory exercises which, under slightly varying forms, are becoming standard material. The choice among these manuals will then be determined by other features, expressing more particularly the points of view of their authors. Thus the "Laboratory Exercises" (Twiss) announces a consistently limited purpose—it is written for the *pupil* and the *laboratory*, leaving the teacher's experiments

and the text for the class-room to be supplied from other sources. In the main, too, it offers one set program of forty-three exercises for a (full) year's work.

The "Elementary Practical Physics" (Hortvet) is more composite in its selection of material, and includes so much explanation introductory to the experiments that it may be said to occupy ground half-way between text and laboratory manual. Here, again, we have a "course" including about forty exercises. To be sure, if the text-book used as an accompaniment is too abridged, it is desirable to supplement it; yet, assuming a good text, a short and disjointed statement of theory in a laboratory manual is likely to confuse the pupil rather than to satisfy him. On matters more closely related to the laboratory, such as sources of error, and figures "significant" of accuracy in the result, this book gives many good suggestions.

The "Laboratory Manual" (Crew and Tatnall) shows commendable unity of purpose in avoiding any considerable overlapping on the field of the text-book. It is also, to some extent, fresher and less stereotyped in its forms of experiment. And, with the possibility opened that the "Teacher may select, say, thirty or sixty out of a list of more than ninety exercises," the suggestion can be taken that the remaining experiments may be recast (and perhaps expanded) into a set of class-room demonstrations, making the manual serve a legitimate double end — aiding both teacher and pupil — until the book of directions for the former comes to be published.

It is to be regretted (to express a personal opinion) that "measurements for the measurements' sake" still hold any place in the laboratory training of beginners. Every hour of their time should be made to count for the cultivation of interest in physical thought and methods, and of insight into common physical phenomena. Neither of these ends is noticeably served by an exercise "to measure a length," or "to weigh by oscillations," or "to find the ratio of the inch to the centimeter" — taken alone; is it not agreed (pedagogically) that the manipulation of such measurements is better learned where they enter into a combination of *physical* interest? I should like, also, to raise continually the question, for consideration by teachers of elementary physics, whether the formal and abstract ideas of mechanics are best put at the threshold of the subject? Must the completed logic of the C.G.S. sys-



OGDEN NICHOLAS ROOD

THE PHYSICAL REVIEW.

AN EXPLANATION OF THE FALSE SPECTRA FROM DIFFRACTION GRATINGS.

BY THEODORE LYMAN.

IN a previous paper¹ the author has shown that the principal spectra produced by concave diffraction gratings are complicated by the presence of false spectra of lower order than the first and that these false spectra are common and often of excellent definition. It is the purpose of this paper to show that these false spectra are not of the nature of "ghosts" and that, while the theories developed to explain the latter cannot be made to fit them, a theory, proposed by Prof. Carl Runge after examining one of the plates which formed the basis of the earlier paper, furnishes an explanation of the phenomena.

The "ghost," so-called, is a faint reproduction of a real line and in general occurs within a few Ångström units of its parent. The false spectra not only occur in regions many hundred units from that occupied by the principal line, but also differs fundamentally from "ghosts" in other respects.

Mr. C. S. Peirce² has made a careful mathematical study of the subject and his paper also contains experimental data on the position of "ghosts" showing a good agreement between the theory and the observed facts. His treatment, however, deals only with "ghosts" occurring very near the parent line. Rowland³ has

¹ PHYSICAL REVIEW, I., 1901.

² American Journal of Math., II., 1879, p. 330.

³ Rowland's Physical Papers, p. 525.

given a theory, the formulæ of which may be extended to the case of false lines occurring at a considerable distance from the parent. The author, however, has never been able to fit the positions or intensities of any of his observed false spectra into these formulæ in the form given by Rowland. In the case of most gratings this does not seem surprising since the false spectra are very numerous, of small intensity and with wave-lengths which bear no simple relation to one another. The error in ruling which produces them must be of a very complex nature and we cannot expect the phenomena to be brought readily under mathematical analysis. In some cases, however, an exact treatment should be possible, for with certain gratings some few of the false spectra are very much stronger than the others. Here it is fair to suppose that we have a comparatively simple error combined with a complicated one. Under the circumstances it seemed profitable to attempt to modify the ordinary treatment as given by Rowland.

A grating of N lines may, if $N = nm$, be considered as a grating of m groups, each consisting of n lines. The spectrum of the first order may be considered as the spectrum of the n th order of a grating of m groups. Each group is in itself a grating of n lines. The intensity I_n any point of a spectrum of a grating of n lines is known to be proportional to

$$A^2 \left[\frac{\sin na}{\sin a} \right] \text{ where } a = e (\sin i - \sin r) \frac{\pi}{\lambda}$$

A^2 is the intensity of one line, which may be different in different directions, and e is the grating space or distance between two consecutive lines.

If we consider the grating of m groups the intensity at any point of its spectrum will be proportional to $I_n \left[\frac{\sin ma'}{\sin a'} \right]^2$ where $a' = na$. The 1st, 2d, 3d, $\dots$ $n-1$ order will vanish for any value of A because I_n vanishes for $a = \frac{\pi}{n}, \frac{2\pi}{n} \dots \frac{(n-1)\pi}{n}$.

Now let us assume that in the ruling of n consecutive lines there is an irregularity, such that the grating of n lines which we take to be repeated m times is of itself an imperfect grating. Then I_n will

not vanish at all places where $a = \frac{\pi}{n}, \frac{2\pi}{n} \dots \frac{(n-1)\pi}{n}$, and some of the first $n-1$ orders of the grating of m groups will be visible. So far this is the ordinary theory of false lines.

Further, let us assume that the grating of n lines has a periodic error. To make the matter definite let us take this error as occurring every third line. That is to say every third line is slightly out of place while the other lines remain in their correct position. Then it is probable that I_n will have some appreciable value at $a = \pi/3$ and at $a = 2\pi/3$. If we take the error to be somewhat irregular the intensity I_n will spread to both sides of those positions where $a = \pi/3$ and $a = 2\pi/3$. The intensity of the light given by the whole spectrum for the wave-length λ will be appreciable for those values $a = \pi/n, 2\pi/n \dots [(n-1)\pi]/n$ which come near to the values $a = \pi/3, a = 2\pi/3$. Here n may be taken at pleasure. For the sake of a definite case let $n = 70$. Then I_n will have an appreciable value at $a = 23\pi/70, 24\pi/70, 46\pi/70$ and $47\pi/70$ for $\frac{2}{3}$ and $\frac{2}{3}$ are the nearest values to $\frac{1}{3}$; $\frac{4}{7}$ and $\frac{4}{7}$ are the nearest values to $\frac{2}{3}$. Moreover, since $\frac{1}{3}$ of 70 is 23.33, the 23d spectrum will be nearer this position than the 24th and therefore the stronger of the two. Similarly, as $\frac{2}{3}$ of 70 is 46.66, the 47th spectrum will be stronger than the 46th. Therefore, taking the number 70, the two spectra which are the stronger will be the lower of the first pair and the higher of the second.

If, then, we let $n = 70$ and consider the error to occur every third line in the grating, for each line in the spectrum there will be four repetitions between its normal position and the direct image of the slit. These repetitions correspond to the 23d, 24th, 46th and 47th order of the grating of m groups, the 70th order corresponding to the normal position.

This simple treatment of false spectra was suggested to the author by Professor Runge. The values used are those which seem most nearly to fit the case of the false lines obtained by the author from the grating called No. 1, and illustrated in a former article.

Before proceeding further to the numerical verification of this theory, it may serve to illuminate the matter if we place in contrast

with it the results which may be expected from the theory of Rowland.¹ The theory of Rowland is divisible into two parts, one dealing with the production of ordinary ghosts, the other part dealing with the production of lines at a considerable distance from the parent. It is this second part which could alone be expected to fit the case in hand. It does not do so, however, for it demands lines whose apparent wave-lengths bear simple ratios to the parent line. Professor Runge's theory shows the possibility of the total absence of lines at these positions indicated by Rowland, and it shows the probability of the formation of lines on either side of these positions at different distances and of an indicated relative intensity.

Thus in the present case the theory of Rowland might be made to call for lines at positions corresponding to $\frac{2}{3}$ and $\frac{1}{3}$ the wave-length of the parent line. The lines actually observed do not fulfil this condition but occur in flanking positions. For example, in the spectrum of magnesium, which is illustrated in Plate I, the line of wave-length 2790.8² which forms the most refrangible member of a characteristic group is seen to be reproduced four times. Two of these false spectra are in the region between wave-lengths 1,800 and 1,900 and two in the region between 900 and 1,000. It is to be noted that of the two groups near 1,800 the less refrangible is the stronger, while of the groups near 900 the more refrangible is the stronger. This is in accord with the theory, for if the two strong reproductions are considered to be the 47th and 23th spectra of the grating of m groups, the two weak reproductions are the 46th and 24th spectra, while the real line 2790.8 corresponds to the 70d spectra.

When, however, the wave-lengths as measured in the previous paper were compared with those calculated from the fractions $47/70 \times 2790.8$; $46/70 \times 2790.8$; $24/70 \times 2790.8$; $23/70 \times 2790.8$ the observed facts did not seem to agree with the theory as accurately as might be expected. It seemed worth while to remeasure

in the matter. In fact the method employed in remeasurement was suggested by him.

The arrangement of the apparatus is shown in Fig. 1. Two slits, *A* and *B*, were placed upon the circle whose diameter was the radius of the concave grating. The grating itself was kept fixed in position and the normal to its surface fell midway between the two slits. By this method the image of *A* was formed at *B*. An arm carrying the plate holder *C* was pivoted at the center of the circle. The result of this plan was that the first spectrum obtained when *A* was used as source was shifted with respect to that obtained with *B* as source by a definite amount. The heights of the two slits were so arranged that the *A* spectrum fell directly over the *B* spectrum upon the photographic plate.

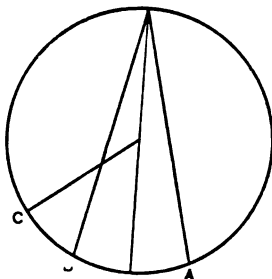


Fig. 1.

The method of procedure was to illuminate *A* with the light from a magnesium spark and to place the plate holder *C* in such a position that a photograph of the false lines under investigation was obtained. Next *B* was illuminated by light from a spark between iron terminals and thus the standard iron spectrum was photographed under the false magnesium lines. To determine in Ångström units the amount by which the one spectrum was shifted with respect to the other, both *A* and *B* were illuminated with light from the iron spark and the relative position of two known lines was measured. As an example of the method, the false line, which by the theory should be the forty-seventh order of the grating of *m* groups, was compared with the line in the iron spectrum 3541.2¹ and found to have the relative position 3541.0. Next the line 2737.0¹ in spectrum *A* was compared with the line 4404.9¹ in spectrum *B*. The difference between them was .5 Ångström units. Thus the shift of spectrum *A* with respect to *B* was $(4404.9 - .5) - 2737 = 1667.4$, and the apparent wave-length of the false line was $3541 - 1667.4 = 1873.6$. In this way the apparent wave-lengths of the four false lines were determined. In order to check the accuracy of the method, slit *B* was moved toward

¹ Exner and Haschek, compare also Kayser and Runge Arc.

slit *A* and the grating was readjusted. In this way the shift of one spectrum with respect to the other was altered and the unknown spectra were compared with a new region of the iron spectrum. The following table gives some idea of the accuracy of the method and of the agreement between the observed and the calculated values.

Calculated.	First Observation.	Second Observation.
917.	917.0	916.8
956.8	956.7	956.4
1834.	1834.1	1833.9
1873.8	1873.7	1873.7

The method seems to be accurate to within .3 of an Ångström unit. The average difference between the observed and the calculated values is 0.15 units. Thus the difference between theory and practice is within the errors in observation.

It might seem that values different from $n = 70$ and a period of 3 would even better satisfy the conditions when substituted in the equations. This is not the case, as may be shown by trial. If there are to be but four reproductions and if the relative dispersions and intensities of these reproductions are to be explained, the values 70 and 3 give the result best in agreement with experiment.

Up to this point it has been the object of the discussion to contrast, in the light of these experiments, the theory of Professor Runge with that part of the theory as given by Rowland which would seem most obviously applicable to the case. This part of Rowland's theory was called "One line in m displaced;"¹ the other part of the theory which deals with "ghosts" came under the title "Periodic error."² A moment's consideration will show that the lines under discussion are not of this latter class.

In the case of "Simple periodic error" the position of any groove in the grating ruling is given by the equation $y = a_0n + a_1 \sin(en)$. Thus the n th groove from a fixed line of reference is out of its true position by a term which varies as a sine function with period e ; the maximum value of the displacement is of course a_1 . Thus no groove in the grating surface is exactly in its proper place unless

¹ Physical Papers, p. 535.

² Physical Papers, p. 536.

$\sin(en) = 0$. The system of ghosts to which this form of error gives rise is characterized by the following properties:¹ Ghosts of any order must occur in pairs. Of a pair one lies to the right of the parent line, the other to the left. Ghosts of the second order must lie exactly twice as far from the real line as ghosts of the first order; ghosts of the third order three times as far.

The distance of a ghost of a given order from its parent line is a constant independent of the order of the spectrum in which the parent line is measured. These three properties are not in any way possessed by the lines under discussion. This, together with the fact that no numerical application of the theory of ghosts to the case in hand seems possible, excludes it from further consideration.

In short then, Professor Runge's explanation of the false spectra seems to fit the facts accurately. It is perfectly possible to extend the theory to even the more complex cases where there are more than four reproductions of every real line. The period of the error and the value of n may be taken at pleasure so that the treatment can be made to fit a great variety of cases. In practice, however, the errors of ruling in those gratings which give a great number of faint false spectra are too complex to make calculation profitable.

It may be of interest to remark that false spectra are not confined to concave diffraction gratings, but are to be found in the spectra produced by plane reflection gratings as well. The author has examined two instruments ruled upon speculum metal by Rowland's engine. In the experiment the light was collimated and brought to a focus by quartz lenses. In order to reduce the phenomena to the simplest possible form, a line in the visible blue spectrum of magnesium was separated out by a prism spectroscope and thrown upon the slit of the collimator. The camera was focused on the image of this line in the first spectrum. The first grating was ruled in 1883 and had 14,438 lines to the inch. The image of the line used appeared in its proper place in the first spectrum — = 4481.3 Ångström units — and was not accompanied by any ghosts of the common kind. That is to say, there was no doubling of the line, nor were there any faint reproductions very near it. In the region near wave-length 3,000, however, four sharp lines

¹ Rowland's Physical Papers, p. 519.

occurred, and again near wave-length 1,550, four more reproductions were present. Thus this grating produces eight false spectra of a lower order than the first, corresponding to the real line 4481.3. Besides these eight, numerous faint reproductions may be detected, but they are of extremely feeble intensity.

The second grating examined was ruled in 1887 and had also 14,438 lines to the inch. The spectrum obtained with it, however, was very different from that given by the first instrument. The first spectrum of the line 4481.3 was, as before, sharp and without ghosts, but the eight distinct false spectra were replaced by at least seventy reproductions of very feeble, but nearly equal intensity. These extended between positions corresponding to wave-lengths 2,000 and 900.

The results obtained with these two plane gratings are exactly similar in character to those obtained with the concave gratings called No. I. and No. II. and recorded in a previous paper. These gratings seem to belong to two types, the one in which the false spectra are all of nearly equal intensity and feeble, the other in which some few of the false spectra are many times more intense than the others. In the one case the grating gives a background of faint lines, in the other sharp, strong false spectra are present.

The author wishes to call attention to the plate which accompanies this article. It is from a concave grating of six-foot radius and shows the false lines whose positions have been discussed in this paper. The plate is taken directly from a negative by photographic process. The two groups at positions corresponding to wave-lengths 956 and 1834 are so faint in this reproduction as hardly to be visible. Their positions are indicated, however. The character and dispersions of the two stronger groups is well shown. All the lines in this plate are false.

The pairs of sharp lines in the immediate neighborhood of the characteristic groups to which the treatment has been confined, are reproductions of the real lines at 2936.8 and 2928.9. They form the 47th, 46th, 24th and 23d spectra of the system in which the real lines are the 70th spectra. The single line near the middle of the plate is the 23d false spectrum of the real line at 4481.3. The lines not far from it are the 23d false spectrum corresponding to

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the real lines 3838.5, 3832.5 and 3829.5. The 24th spectra of these lines are too feeble to reproduce. Their 46th and 47th spectra occur among real lines in the region between 3050 and 2500. The apparent wave-lengths of these false lines are marked upon the plate; the decimal place is omitted. The following table gives the exact measured apparent wave-lengths, the wave-lengths of the real lines of which the false spectra are the reproductions and the order of the false spectra.

Apparent Wave-lengths.	Wave-length of Parent Line.	Order of False Spectra.
916.9	2790.8	23d.
956.5	2790.8	24th.
961.8	2928.8	23d.
1258.4	3829.5	23d.
1472.8	4481.3	23d.
1834.0	2790.8	46th.
1873.7	2790.8	47th.
1966.5	2928.9	47th.

It is well to remember that this grating is not unique, but that most reflection gratings produce false spectra. It is in the extreme ultra-violet that these false lines are most easily seen, and it is in this region that they may be most readily taken for real lines. The lines may serve to show by their strength and sharpness the danger which they offer in spectroscopic work.

In conclusion, it may be well to repeat the chief result of this paper. False spectra differ fundamentally in character from the commonly observed "ghost." The former are due to a so-called periodic error in the grating ruling, an error which operates to displace every groove in the grating surface by an amount depending on a sine function of the position of the groove. It seems probable that the false spectra are due to an error of another type. Here the error operates to displace one, out of a given number of grooves, slightly, leaving the remainder in their proper positions. In order to make theory and the observed facts agree, this error must be considered somewhat irregular over the surface of the grating. While the theories proposed by C. S. Peirce and Rowland account in every way for the phenomenon of ghosts, they do not either qualitatively or quantitatively account for these false spectra,

whereas the theory proposed by Runge and given above explains the phenomena qualitatively and quantitatively. That is to say, it explains the production of lines far from the parent line, lying entirely on one side of it; it explains their relative intensity, and it explains very nearly indeed their exact position. The departure between the positions of these false spectra demanded by the theory and observed in the plates is never more than 0.4 Ångström unit. This is slightly greater than the errors of observation, but it is to be remembered that the measurements depend upon the wave-lengths of the parent group and of the comparison iron lines, both of which are borrowed data.

JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY,
January, 1903.

ON THE RELATION OF THE DIELECTRIC CONSTANT OF WATER TO TEMPERATURE AND FREQUENCY.

BY A. DE FOREST PALMER, JR.

SOME observations made during the progress of an investigation¹ "On the Dielectric Constant of Dilute Electrolytic Solutions" led me to suspect that the variation of the dielectric constant of water with changes in temperature was greater than one would expect from the best published values of the temperature coefficient involved. The frequency of the electrical oscillations of my apparatus was considerably different from that employed in the determinations with which I was familiar and it occurred to me that this fact might explain the discrepancy. Accordingly the present inquiry was undertaken to determine whether the indicated difference was genuine or spurious. The results recorded below entirely corroborate the anticipated large value of the temperature coefficient for the frequency employed in my previous work and also show a value in good agreement with published results for frequencies of about sixty cycles per second.

The apparatus and general method of observation employed in this work have been very fully described in a previous communication;² the only modifications being the substitution of more accurate thermometers, the removal of the test electrodes used in determining changes in conductivity, and the provision of means for varying the temperature of one of the electrometers. The arrangement was essentially as follows: An electrical oscillator, of the Hertzian type, consisting of two Leyden jars with their outside coatings connected through a spark gap and their inside coatings connected to two long parallel copper wires was energized by a storage battery

¹ Palmer, *PHYS. REV.*, 14, p. 38, 1902.

² Palmer, *l. c.*, p. 44. The end of line 10, p. 46, of this description should read 1.5 mm. in place of 15 mm

and a large Ruhmkorff coil. Two quadrant electrometers *A* and *B*, of the cylindrical electrode and needle type, were connected in multiple between points on the two parallel wires equidistant from the jars. The needles were suspended from torsion heads by very fine platinum-rodium wires and carried concave mirrors so adjusted that they cast superimposed images, of an illuminated wire, on the central division of an observing scale, about two meters distant, when the needles were symmetrically situated with respect to the fixed electrodes.

Each electrometer was supplied with two mercury-in-glass thermometers, so placed that their long thin bulbs were just outside the spaces between the insulated and uninsulated quadrants and in line with the horizontal axis of the needle. These thermometers were graduated in fifths of a degree from zero to fifty degrees Centigrade and were very carefully calibrated by comparison with a normal thermometer, graduated in tenths of a degree, made by Max Kohl and supplied with a certificate from the Physikalisch-Technische Reichsanstalt.

Electrometer *A* was surrounded by a copper jacket through which water from the city mains was allowed to flow continually, and its temperature never varied during a series of observations by more than a few tenths of a degree Centigrade. A tin plate tank, of about ten liters capacity, surrounding electrometer *B* and containing cracked ice or water, served as a bath for regulating the temperature of the instrument. Any desired temperature between 0° C. and room temperature could easily be obtained by adding ice or warm water to the bath and stirring thoroughly till the thermometer readings became stationary. Then, on account of the great heat capacity of the bath and the small radiation constant of its bright tin surface, the change in temperature of the electrometer did not exceed two tenths of a degree per hour.

Both electrometers were filled with distilled water, conductivity 7×10^{-6} reciprocal ohms per centimeter cube, and complete series of observations between about 0°.3 C. and 16° C. were obtained with three different frequencies of the applied electrical field. The two higher frequencies were obtained with oscillators of the type above described, using jars of different capacity in the two cases,

and were respectively about 1×10^6 and 3×10^6 complete vibrations per second. For the lowest frequency the jars were removed and the parallel wires connected through a variable transformer to a source of alternating current of sixty cycles per second.

Before commencing a series of observations the temperature of electrometer *A* was allowed to become stationary at the temperature of the water supply of the laboratory and the bath surrounding electrometer *B* was brought to the lowest temperature for which observations were to be made and maintained constant sufficiently long for the temperature of the water in the electrometer to become uniform. The oscillator was then started and the images from the mirrors on the two needles brought to superposition on the central scale division by suitably turning the torsion heads. When this had been attained the current was turned off and after the images had come to rest their positions on the scale were noted and the readings of the thermometers recorded. The temperature of the bath was then raised a few degrees and the above operation repeated. This process was continued until the temperature of the bath had reached the room temperature and consistent results were always obtained. Above this point it was found impossible to maintain a sufficiently uniform and constant temperature, with the appliances at hand, to produce satisfactory results and, as time was not available for modifying the apparatus, the observations were discontinued. As indicating the uniformity of temperature attained in the electrometers it may be noted that the indications of the two thermometers situated some ten centimeters apart never differed by so much as one tenth degree Centigrade.

The method of obtaining the angular deflections δ_A and δ_B , of the two electrometers, from the scale readings of the images and the constants of the apparatus has been very fully discussed in my previous paper⁵ and need not be recapitulated here. If *R* represent the ratio of the angular deflections, *C* the ratio of the constants of the electrometers. *K* the dielectric constant of the water in *B* and

When the temperature of A is maintained constant, K' is constant and any change in R , coincident with a change in the temperature t of B , must be due to a corresponding change in K , provided C is independent of t . In general this is not the case, but a calculation of the variation of C for the extreme temperature range of the observations, from the known dimensions and expansion coefficients of the materials from which the electrometers were made, showed that the error introduced by assuming C constant, would be much less than the probable errors of observation. Hence we may write

$$R_t = \frac{C}{K_t} K_t.$$

Differentiating with respect to t and dividing by R_t

$$\frac{1}{R_t} \frac{\partial R_t}{\partial t} = \frac{1}{K_t} \frac{\partial K_t}{\partial t} = \beta_t.$$

Also

$$\frac{R_t}{R_{17}} = \frac{K_t}{K_{17}},$$

and if R is found to fulfil the linear relation

$$R_t = R_{17} \{1 + \beta_{17}(t - 17)\}$$

K will satisfy the equation

$$K_t = K_{17} \{1 + \beta_{17}(t - 17)\},$$

in which the numerical value of β_{17} is the same as in the preceding equation.

The observations were first reduced graphically on the assumption that K' was constant and the first approximation to β_{17} thus obtained was used to correct the observed values of R for the small changes in K' due to slight variation in the temperature of the water in electrometer A . The final values of β_{17} and the probable errors were calculated from the corrected values of R and t by the method of least squares.

The individual observations are sufficiently represented by the points marked on the chart, Fig. 1, where the ordinates correspond to corrected values of the ratio R and the abscissas to the tempera-

ture of the water in electrometer *B* in degrees Centigrade. Line 1 represents the observations with frequency sixty cycles per second, line 2, those with frequency 1×10^6 , and line 3 those with frequency 3×10^6 . The relative position of the three lines on the chart is without significance since the constant C/K' was different in the three series of observations owing to difference in the constant temperature of the water in electrometer *A*.

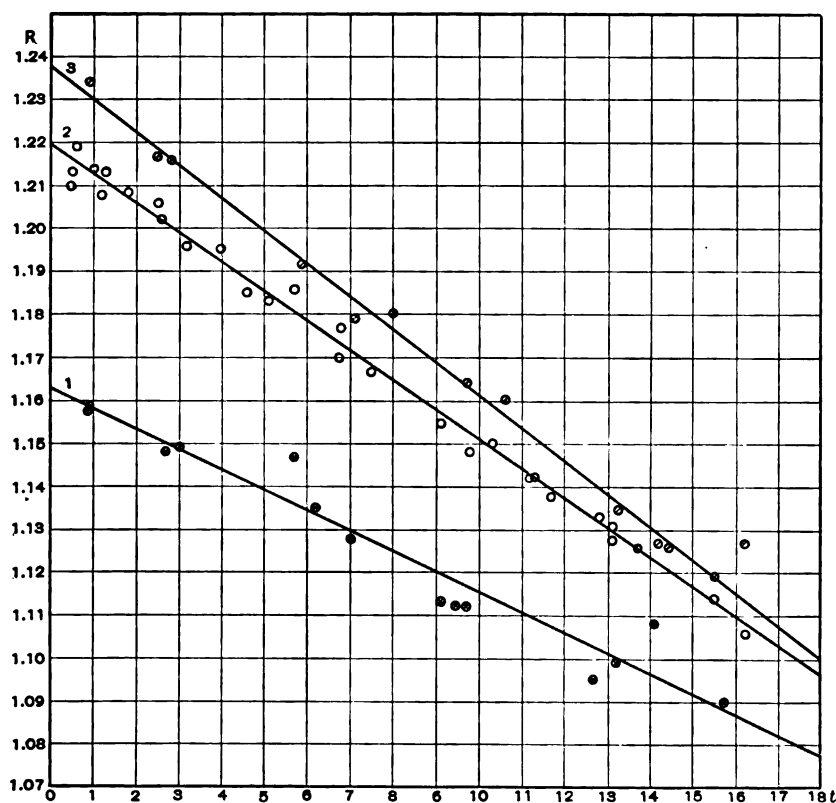


Fig. 1.

It should be noted that the scale of the plot is such that one division on the vertical axis corresponds to a change of less than one per cent. in the value of R and consequently that the observed

tenths per cent. The plot shows clearly that R and t are connected by the linear relation, within the errors of observation, between 0° C. and 17° C., and there is no indication of a maximum value of R at any point between these temperatures.

TABLE I.

Line.	R_{17}	β_{17}	Frequency.
1	$1.082 \pm .0023$	$-.0044 \pm .00023$	60
2	$1.103 \pm .0007$	$-.00624 \pm .000066$	1×10^6
3	$1.107 \pm .0012$	$-.0070 \pm .00012$	3×10^6

The calculated constants of the three lines with the corresponding probable errors are given in Table I. Taken by themselves they indicate a consistent increase in the numerical value of β_{17} with the frequency of the electrical field that cannot be due to the errors of observation. Electrometer methods, similar to the above, are, however, subject to three sources of constant errors that might, if sufficiently large, account for the observed result. These are polarization of the electrodes, variation of the constant C and inequality of the potential differences applied to the two instruments.

In a previous communication¹ I showed that the first of these errors was negligibly small, for the apparatus and method above described, when the frequency of the electrical field was 1×10^6 per second. Moreover in order to account for the observed increase in β_{17} it would be necessary to assume that the rate of change of the polarization with temperature increased with increasing frequency and in this case the observations might be expected to be more widely scattered for high than for low frequencies. But on inspection of Fig. 1 it is apparent that exactly the reverse is the case and that in all probability the observations at sixty cycles per second are more affected by polarization than the others. Yet the value of β_{17} derived from this series is in good agreement with the results obtained by other observers.

The constant C has been shown to be independent of the temperature, within the errors of observation, and it is obvious that a uniform continuous change in its value with lapse of time would

¹ Palmer, l. c., p. 55.

affect all the series alike and hence would not influence the relative values of β_{17} . Irregular variations would appear as errors of observation and are therefore included in the probable errors. A continually accelerated change is hardly to be expected and, since the observations at sixty cycles per second were made between the other two series, it is obvious that no such change did occur, for

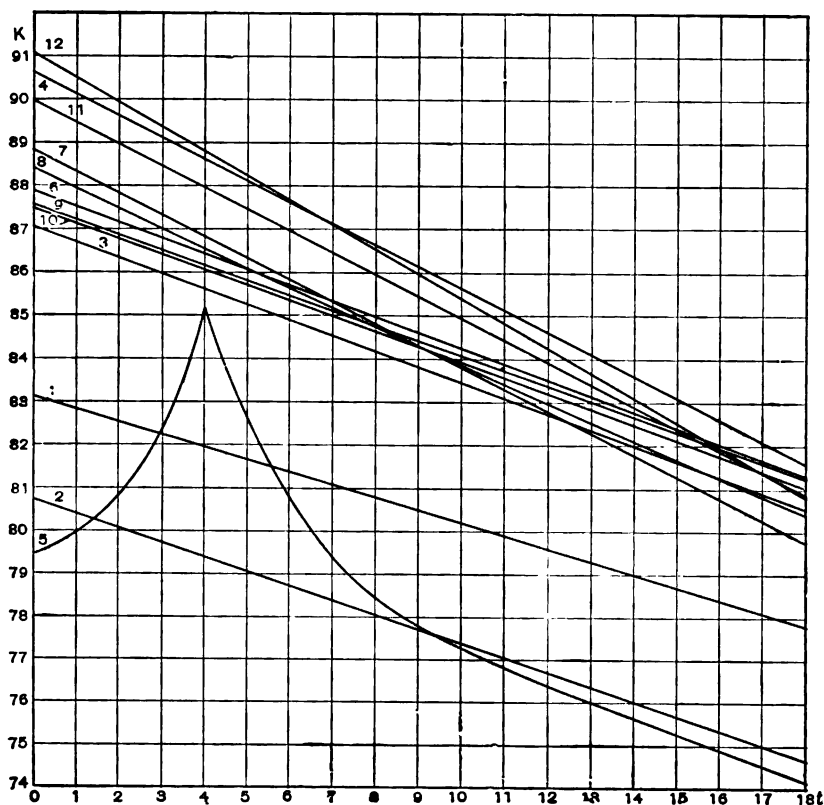


Fig. 2.

otherwise the value of β_{17} corresponding to frequency sixty would lie between the values at 1×10^6 and 3×10^6 instead of below both of them.

For the purpose of reducing the third source of error to a minimum the two electrometers were constructed as nearly alike as possible and the constant temperature of the water in *A* was chosen

about midway between the extreme temperatures of the water in *B* so that the capacity of the two instruments was never greatly different. The connecting wires to corresponding parts were as short as possible, of the same length and diameter, and symmetrically arranged, thus producing very small and nearly the same self induction in the two electrometer branches of the circuit. Under these conditions the effective potential differences at the terminals of the two instruments could not have been sufficiently different to produce sensible error provided their ratio was independent of the temperature. However, since the capacity and resistance of *B* decreased as the temperature increased, it is possible that all of the above results are affected by a residual error of this nature, but a calculation, from approximate values of the resistance, capacity and self induction of the two circuits and the variation of these quantities for the extreme temperature range, shows that this error is negligible in comparison with the errors of observation.

To facilitate comparison the results of previously published investigations of the variation of the dielectric constant of water with the temperature, so far as they have come to my notice, are collected in Table II. and graphically represented on the chart, Fig. 2, where values of *K* are plotted as ordinates and values of *t* as abscissas. The numbers in the first column of the table correspond to those on the lines of the chart. The second column gives the authors of the determinations, the third the values of the dielectric

TABLE II.

No.	Observer.	K_{17}	$-\beta_{17} \times 10^6$	Frequency.	Temperature.	Date.
1	Rosa.	78.10	375	50	3°-31°	1891
2	Cohn.	75.00	445	1×10^6	9°-35°.3	1892
3	Heerwagen.	80.88	448	42-85	4°-26°	1893
4	Franke.	82.10	614	40	2°.6-25°.3	1893
5	Thwing.	74.70	626	1.5×10^7	6°-20°	1894
6	Drude.	81.67	451	4×10^6	0°-25°.8	1896
7	Ratz.	80.25	632		4°-30°	1896
8	Abegg.	80.84	526			1897
9	Coolidge.	81.60	432	2×10^6	3°.5-24°.7	1899
10	Palmer.	81.40	440	60	0°-16°	1903
11	Palmer.	81.40	624	1×10^6	0°-16°	1903
12	Palmer.	81.40	700	3×10^6	0°-16°	1903

constant at 17° C. (K_{17}), the fourth the values of $\beta_{17} = 1/K \partial K/\partial t$ at 17° C., the fifth the frequency of the electrical field employed in making the determinations, the sixth the range of temperature for which the linear relation holds or the total range of the observations, and the seventh the year in which the work was published. The values of β_{17} attributed to Thwing and Abegg were calculated by the author and represent the straight lines nearest to their results, but since neither of these observers found a linear relation between K and t the values thus obtained must be regarded as rough approximations.

Rosa's¹ observations were made with an electrometer of the attracted disk type. The source of electromotive force was a gravity battery and the direction of the current was reversed about fifty times per second by a mechanical commutator. Potential differences were measured by a Weston voltmeter connected to the battery side of the commutator.

Cohn² generated a system of stationary electrical waves along two parallel wires and determined the ratio of the wave-lengths on parts of the wires surrounded by air and water respectively. The refractive index n thus found was reduced to dielectric constant K by Maxwell's relation $K = n^2$.

Heerwagen³ used two quadrant electrometers placed one above the other with their needles on the same suspension and determined the ratio of the potential differences that would reduce the deflection to zero when applied to the two parts of the instrument in opposite directions.

Franke⁴ observed the deflections of two quadrant electrometers connected in multiple between the earth and one terminal of a Kohlrausch adjustable condenser. The other terminal of the condenser was connected to earth through the secondary of an induction coil the primary circuit of which was interrupted about forty times per second.

Thwing's⁵ method was based on the principle of electrical reso-

¹ E. B. Rosa, *Phil. Mag.* (5), 31, p. 188, 1891.

² E. Cohn, *Wied. Ann.*, 45, p. 376, 1892.

³ F. Heerwagen, *Wied. Ann.*, 49, p. 272, 1893.

⁴ A. Franke, *Wied. Ann.*, 50, p. 163, 1893.

⁵ C. B. Thwing, *PHYS. REV.*, 2, p. 35, 1895; *Zeit. Phys. Chem.*, 14, p. 286, 1894.

nance. Two neighboring circuits of constant self-induction but variable capacity were adjusted to unison, by a parallel-plate adjustable condenser in one of them, while a cylindrical condenser of fixed dimensions was included in the other. The capacity of the fixed condenser was thus determined first with air as dielectric and then with water at various temperatures. The frequency of the electrical oscillations of the system is not stated in the published description and I take pleasure in expressing my thanks to Professor Thwing for his courtesy in calculating the same for me with the result given in the above table.

Drude¹ adopted a method similar to that used by Cohn and developed it to a high degree of accuracy and convenience.

Ratz² employed Nernst's³ bridge method and his results are relative to Nernst's value $K = 79.6$ at $18^{\circ}.1$ C. The frequency of the alternating current used is not stated.

Abegg's⁴ results were calculated with the formula

$$K = 372 E^{-\frac{T}{190}}$$

in which T is the absolute temperature, E the base of the Napierian logarithm, and the constant 372 represents the "imaginary" dielectric constant of water at absolute zero. This formula was derived, with the aid of the rule for mixtures, from two series of observations, by Nernst's method, extending over a temperature range -86° to 0° C. One of the series was carried out with ethyl-alcohol and the other with a mixture of ethyl-alcohol and water of known proportions.

Coolidge⁵ inserted a water condenser in an oscillating system connected to two parallel wires and determined the changes in wavelength produced by changes in the temperature of the water. The absolute value of his results is based on Drude's value at 19° C.

In order to include the results of my own observations in Fig. 2 I have adopted 81.4 as the dielectric constant of water at 17° C. and drawn the lines 10, 11 and 12 through the corresponding point

¹ P. Drude, *Wied. Ann.*, 59, p. 17, 1896.

² F. Ratz, *Zeit. Phys. Chem.*, 19, p. 109, 1896.

³ W. Nernst, *Zeit. Phys. Chem.*, 14, p. 622, 1894.

⁴ R. Abegg, *Wied. Ann.*, 60, p. 54, 1897; *Wied. Ann.*, 62, p. 257, 1897.

⁵ W. D. Coolidge, *Wied. Ann.*, 69, p. 125, 1899.

on the diagram with the inclinations required by the values of B_{17} given in Table I. The value 81.4 for K_{17} is the mean of the independent determinations given in Table III. and is probably accurate to within less than one per cent.

TABLE III.

Observer.	K_t	t	K_{17}
Heerwagen. ¹	80.88	17°	80.88
Franke. ²	82.10	17	82.10
Drude. ³	81.76	17	81.76
Nernst. ⁴	79.6	18.1	80.00
Tereschin. ⁵	83.8	13.5	82.50
Turner. ⁶	81.1	18	81.50

The first column of the table gives the author of the determination, the second the value of the dielectric constant K found at the temperature stated under t , and the fourth the value of K_{17} calculated from K , when necessary, with the value — .0044 for β_{17} .

In drawing the three lines through the same point on the chart it is not intended to intimate that the dielectric constant of water is the same for all frequencies at 17° C., for this is probably not the case. However, sufficient data is not available to determine the relative values of K_{17} for the frequencies in question and the relation between β_{17} and the frequency is more clearly shown by adopting a common point for the three graphs.

The close agreement of the values of β_{17} corresponding to frequencies of less than one hundred cycles per second, obtained by Rosa, Heerwagen and the author, with the values corresponding to frequencies of the order 10^8 per second, found by Cohn, Drude and Coolidge, is strong evidence that β_{17} is independent of the frequency. On the other hand all determinations with frequencies between these limits have resulted in values of β_{17} that increase numerically with the frequency. Hence taken as a whole the results given in Table

¹ Heerwagen, l. c.

² Franke, l. c.

³ Drude, l. c.

⁴ Nernst, l. c.

⁵ S. Tereschin, Wied. Ann., 36, p. 792, 1889.

⁶ B. B. Turner, Zeit. Phys. Chem., 35, p. 385, 1901.

II. are inconsistent unless we assume that β_{17} first increases to a maximum and then decreases to its initial value as the frequency increases from about 10^2 to about 10^8 per second. The data in hand are obviously insufficient to establish this proposition and the only definite conclusions that can be drawn from it are that β_{17} is not independent of the frequency and that further investigation is necessary to establish the exact relation between these quantities.

BROWN UNIVERSITY,

January 19, 1903.

THE INFRA-RED EMISSION SPECTRUM OF THE MERCURY ARC.

BY W. W. COBLENTZ AND W. C. GEER.

THE present investigation of the radiation from the mercury arc was undertaken to account for the high efficiency¹ of the Arons lamp.

The spectrum was produced by means of a mirror spectrometer, and a rock salt prism, while a Nichols radiometer was used to measure the distribution of energy. The whole is shown in Fig. 1 in which *N* is the source, *S* the shutter, *P* the rock salt prism and *R* the radiometer. A full description of the adjustment and calibration of the apparatus is given elsewhere² and need not be mentioned here.

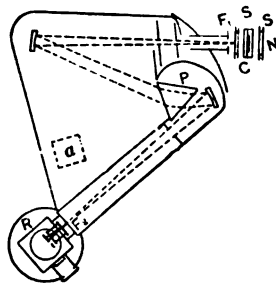


Fig. 1.

THE MERCURY ARC.

The arc was formed in a vacuum tube which was a modification of that first described by Arons.³ The vertex of the arc was used as a source of radiation. For this purpose a side tube, having a window of fluorite or rock salt, was placed at right angles to the plane of the arc.

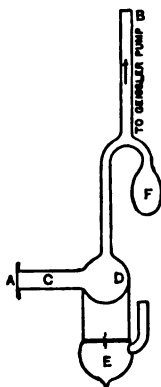


Fig. 2.

The side elevation is shown in Fig. 2. The diameter of *C* was 1 cm., that of *D* 2.5 cm., with the other parts in proportion. Throughout the work the lamp was connected to a Geissler pump and when in use the stopcock at *B* was closed. The whole was enclosed in a tin box containing water with the side tube, *C*, projecting through a water-tight sleeve. A fluorite window was at *A* for wave-

¹ Geer, *PHYSICAL REVIEW*, Vol. XVI., No. 2, 1903.

² Coblentz, *PHYSICAL REVIEW*, Vol. XVI., Nos. 1 and 2, 1903.

³ Arons, *Wied. Ann.*, 47, 767, 1892.

lengths up to $6\ \mu$, while a window of rock salt was employed from 6 to $9\ \mu$.

The mercury was purified by running it through a separatory funnel containing mercury, on the surface of which was a layer of mercurous sulphate and concentrated sulphuric acid. The mercury treated in this way was some that had been standing in contact with the cleaning agent for several weeks. By this means it was freed from moisture and metallic impurities. It left no weighable residue upon evaporation.

During the exhaustion the mercury was continually flowed from one leg of the lamp to the other to free it from all traces of air. Throughout the investigation the lamp was pumped out at frequent intervals in order to be certain that the vacuum was maintained. On starting the arc the vacuum was always less than 0.03 mm.

A direct current of six ampères was used throughout the work. The water in the tank varied from 50° to 80° . It was allowed to become warm since it produced a larger arc. Larger currents were not used for fear of breaking the lamp.

CURVES AND RESULTS.

It was found that the energy radiated consists of a series of emission bands at $1\ \mu$ and $5\ \mu$ with a slight indication of a band at $3\ \mu$. Nowhere in the spectrum was the intensity of the radiation from the arc very great. This indicates a low temperature like that of gases in vacuum tubes,¹ although Arons² has computed a value of several thousand degrees.

The distribution of the maxima is best shown by the curves in Fig. 3. For the region at $1\ \mu$ the mean of several readings has been plotted, while beyond $4\ \mu$ individual readings are given.

In Table I. a typical set of readings is given for the region from $\lambda = 0.92$ to $\lambda = 2.0\ \mu$. It shows that the variation in the individual

¹ Salisbury first observed the low temperature of such gases. *Phil. Mag.* (4), 45, p. 241, 1873.

E. Wiedemann, *Calorimeter*, *Wied. Ann.*, 6, p. 298, 1878; 10, p. 202, 1880.

readings is not great, when one considers the smallness of the deflections.

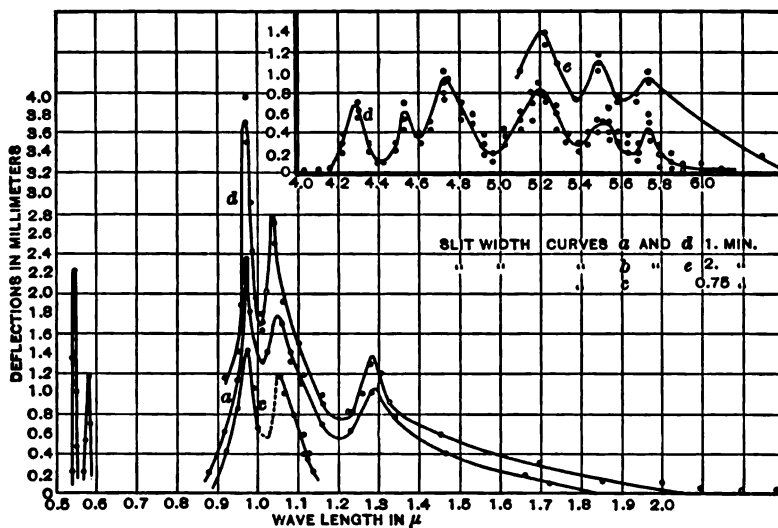


Fig. 3.

TABLE I.

Deflections in Millimeters. Wave-lengths in μ .

λ	.92	.95	.98	1.01	1.02	1.06	1.08	1.10	1.16	1.23	1.28	1.34	1.46	1.72	2.03
Deflec- tions.	0.5	0.9	1.7	1.5	1.5	1.7	1.3	1.2	0.8	0.7	1.0	0.8	.03	00	00
	0.4	1.4	2.0	1.2	1.4	1.7	1.4	1.3	0.6	0.5	1.1	0.9	.05	00	00
	0.6	1.0	1.8	1.1	1.3	1.8		1.2			1.0			.02	00
	0.8		1.8	1.4		1.6		1.1			0.8				
Mean,	0.5	1.1	1.8	1.3	1.4	1.7	1.4	1.2	0.7	0.6	1.0	0.9	.04	00	00

λ	.92	.95	.96	.97	.98	.995	1.01	1.02	1.04	1.06	1.16	1.26	1.28	1.32	1.34
Deflec- tions in mm.	1.2	1.4	1.6	3.8	3.0	1.9	1.6	2.0	2.2	1.6	0.8	0.8	1.1	0.9	0.7
	1.1		1.9	3.5	2.8	2.0	1.8		2.4	1.7	1.0	0.5	1.0	0.9	0.8
				4.2					2.7	1.9			1.3		
									2.5				1.3		
Mean,	1.1	1.4	1.8	3.7	2.9	2.0	1.7	2.0	2.5	1.7	0.9	0.6	1.2	0.9	0.7

The difficulties in observing the deflections were extreme on ac-

conditions, a cylindrical acetylene flame gave a deflection of 50 cm. at the maximum part of its spectrum, while the CO_2 emission band of a bunsen acetylene burner at $4.4\ \mu$ threw the deflection clear off the scale.

Although the deflections were small they were trustworthy. It should be remembered that the radiometer at the sensitiveness used was unusually stable. The mirror suspension was not the lightest, so that earth tremors rarely caused annoyance. No readings were made when such vibrations were noticeable. The period of the radiometer was about thirty seconds. In order to reduce the errors due to temperature changes it was necessary to shorten the period at a sacrifice of the sensitiveness of the instrument. The radiometer was packed in wool, in a tin box, and the zero shift, due to changes of temperature, for single readings was generally only 0.1 to 0.2 mm., while often it was not to be noticed at all. The whole apparatus stood in an inner room and the deflections were observed through a hole in the wall. When there was no disturbance in the room the zero shift was always extremely small. On exceptional days—not for this work, however—the zero shift could not be detected during ten minutes, on actual count, while in four hours it amounted to only 1.6 cm.

In the first exploration (slit 1 mm. in width), up to $6\ \mu$, the apparatus was not so sensitive and no radiation from the hot walls of the tube could be detected. But when the fluorite window was replaced by one of rocksalt, and an increased sensitiveness of the radiometer was combined with a slit of 2 mm. width, the hot glass radiation was perceptible beyond $5\ \mu$. Consequently, for each setting of the spectrometer the radiation from the hot glass was measured and deducted. For the latter measurement the shutter at the window was raised at the instant that the arc was extinguished. It was found that beyond $6\ \mu$ up to $9\ \mu$, which was so far as the exploration extended, the average deflections for the hot glass were as large as those due to the arc.

The spectrum was explored at every minute of the spectrometer circle up to $9\ \mu$, while in certain regions readings were made at

their relative heights. This variation of height, due to the changes in the sensitiveness of the instrument, is shown in Fig. 3. The best values for the relative heights is shown in the curve *b*. In Fig. 4 the complete curve is given. The deposition of mercury on the window was one source of annoyance and at times vitiated the readings. On three different days observations were made on the region from 2μ to 4μ , but no radiation could be detected except at about 3μ where there was a slight deflection at times. This is of interest in what follows.

The reflecting effect of the glass walls is not known. Since glass becomes opaque at 3μ it will be noticed that all the energy of

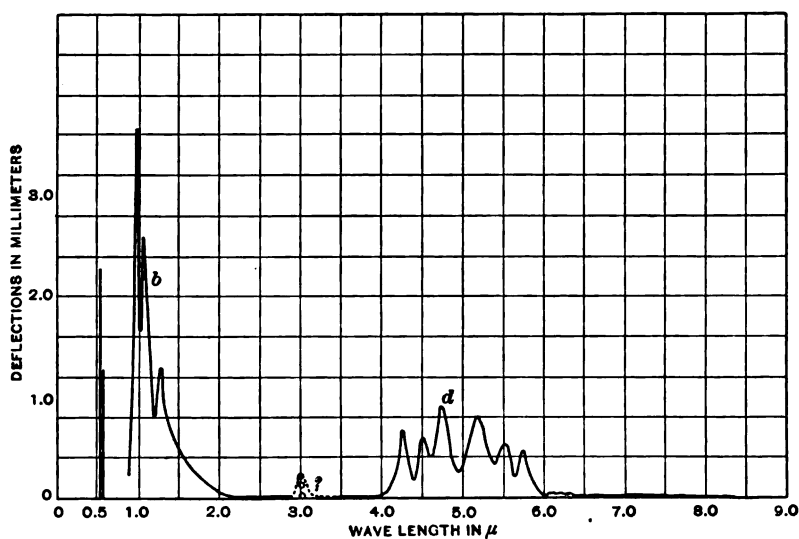


Fig. 4.

greater wave-length is absorbed by the glass in a lamp of the form described by Arons. Thus the apparent radiant efficiency of the arc would be greater than the true efficiency. This accounts in part for the unusually high values found in the previous investigation.¹

ADDENDUM ON SPECTRAL SERIES.

After the work had been completed, the senior writer repeated some of the observations to find the bearing of the emission maxima on laws for spectral series. Also to learn whether another line exists between 1.06 and 1.12μ where the curve is very asymmetrical. It was found that the mean of a great many readings was very nearly the same for $\lambda = 1.08$ and 1.10μ , and if another line exists it is very weak.

From their work on the elements of the second group of Mendeleeff's classification, viz., Mg, Ca, Sr, Ba; and Cd, Zn, Hg, Kayser and Runge¹ have found that there are groups of lines, triplets, distributed throughout the visible spectrum, whose position can be expressed by an empirical formula of the form

$$1/\lambda = A + Bn^{-2} + Cn^{-4}$$

where $1/\lambda$ is a function of the frequency and n represents the successive integral numbers. They have also found that the difference between the values of $1/\lambda$ for the members of any triplet is constant for any given element. In the case of mercury this difference is

$$v_1 = \begin{cases} (4584.6) \\ 4630 \end{cases}$$

$$v_2 = 1760.0.$$

By means of their formula, for $n = 3$, they predicted a triplet at $\lambda = .9497 \mu$, $.6595 \mu$, $.5906 \mu$. But they add that these lines may lie still farther toward the infra-red. This follows from the fact that the wave number, $1/\lambda$ in negative powers of n^2 forms a rapidly converging series, and three terms were sufficient for their work. Now the triplet, $n = 3$, of the first subordinate series, falls in the infra-red and the computed values of λ are obtained through wide extrapolation. They add that the formulæ require more terms with smaller powers of n , when n is less than 3. Hence they ex-

will be observed that the one at $.97 \mu$ agrees remarkably well with the predicted value $.9497 \mu$, especially when one remembers that Kayser and Runge expected the latter value to be larger. The fact that the radiometer failed to detect the other two lines is no reason for considering that this is not the third. Still more interesting is the fact that the difference between $1/\lambda$ for the maxima at 1.045μ and 1.285μ ($v_2 = 1787$) are unusually close to those in the visible spectrum, that is, they appear to belong to another triplet, of which the third component lies at 3.05μ . The latter is obtained by computation, using the value $v_1 = 4630$. Since at 3.0μ the radiometer showed slight deflections at times which were recorded as questionable, and since in all other parts of the spectrum from 2 to 4μ no such effect could be detected, one is led to believe that the deflections observed as 3μ were real. The great variation in the intensity of these lines may be due, in part, to the fact that the slit subtends different angles in the two regions, and that the suspected line at 3μ is isolated while the others occur in a group, each one of which is influenced by those adjoining it.

In Table II. are given the observed maxima and the products obtained by multiplying each one by the arbitrary factor 4.4. According to Kayser and Runge the first, second and third components of a sharp or diffuse series of triplets form, in each element, a line series. Whether the maxima at 4 to 6μ are thus related to those at 1μ has not been determined, but the coincidence, when the arbitrary factor 4.4 is used, is very striking.

TABLE II.

A°. U. from Kayser and Runge.		A°. U. Observed.	$10^3 + \lambda$.	Difference.
$\lambda = 3007.02$	} $v_1 = 4584.6$	$\lambda = (30500 ?)$ 12850 10450	7783 } 9570 }	$v_2 = 1787$
2642.70				
2524.80				
3680.74	} $v_1 = 4631.8$			
3144.61				
Maxima.	Maxima $\times 4.4$.		Observed.	
.97	4.27		4.28	
1.04	4.56		4.53	
1.29	5.68		5.73	

The unavoidable errors in drawing the curves account, in part, for the variations between the predicted and the actual positions of the maxima. Since most of the curves were drawn prior to the consideration of their bearing on the question of spectral series the position of the maxima is not biased by the latter. An inspection of Fig. 2 shows that, through the few points observed, curves can be drawn which have different trends, especially when they are asymmetrical. For this region a prism of greater dispersing power, *e. g.*, carbon disulphide, would have increased the accuracy of the work.

CONCLUSION.

From the foregoing investigation it is seen that the infra-red spectrum of the mercury arc comprises several well-defined bands with perfectly definite maxima. These maxima occur at the following wave-lengths: 0.97, 1.045, 1.285, 4.28, 4.53, 4.73, 5.20, 5.50 and 5.73 μ .

The presence of the bands of wave-lengths greater than 4 μ indicates that the true radiant efficiency of the arc is lower than that previously found.

The maxima of the curves at .97, 1.045 and 1.285 μ follow quite closely the law of spectral series as developed by Kayser and Runge

CORNELL UNIVERSITY,
November, 1902.

THE RELATION OF IONIZATION TO NUCLEATION IN
THE CASE OF PHOSPHORUS.

BY C. BARUS.

1. *Introductory.* — The opinion was expressed in my earlier papers, that wherever there is intense ionization, there one may also expect to find active nucleation ; for it is hardly probable that a group of ions neutral as a whole can ultimately escape combination with the medium in which they are suspended. If these combinations occur in the presence of water vapor and particularly in a saturated atmosphere, the nuclei due to solution are the result. When the nuclei are produced from dilute solutions by shaking, there is evaporation of the fog particle to the nuclear diameter ; when the solute is produced by any kind of radiation or emanation, each trace of solute grows in bulk by absorbing water to the nuclear stage. In case of an intense emanation like that from phosphorus, this process may actually continue until a visible cloud is produced and the nuclei attain the size of fog particles. Slow spontaneous condensation is induced even without exhaustion, while the fog on sudden cooling in the presence of water vapor is correspondingly intense.

2. If the original emanation, highly ionized, though neutral as a whole, is put through the process of condensation, then if the negative ions are more efficient as condensation nuclei than the positive ions, the nuclei after condensation or even after remaining in a saturated atmosphere, should become *continually more positive*—assuming that a greater number of negative ions are removed by condensation.

The investigation would therefore consist in testing the ionization immediately coming from phosphorus as to its power in dissipating positive and negative charges, and to compare these results with the degree of ionization after the emanation has produced condensation. In other words, it is to be ascertained whether the nuclei after a succession of condensations become continually more positive.

The results to be discussed in the following paragraphs have made this apparently straightforward investigation of little value: for before the emanation has been made available for condensation, scarcely 3 per cent. of the original ionization is left. The residue is then so small that a decision of a possible excess of positive or negative ionization is difficult, because the whole is now of the same order as the normal leakage of the electrometer. Meanwhile the nucleation or condensational activity, so far as discernible, has suffered no appreciable decline.

In fact, the decision as to whether the positive or negative ionization is in excess is now of very secondary interest, for the nuclei introduced into the condensation chamber have already lost all but a trifle of their original charges. All the successive and even the initial condensations thus virtually proceed without electrification.

The initial intense ionization nearly vanishes even in a moderately dry atmosphere. Indeed, it is hard to understand how a neutral, intensely ionized emanation can be produced from a body like phosphorus. It appears to me that the emanation is a molecular body which is stable in the presence of an excess of phosphorus, *i. e.*, at the surface, but which becomes unstable and breaks to pieces in presence of an excess of air, on leaving the phosphorus. The observed ionization is the accompaniment of this dissociation. If the ions were produced by phosphorus directly one would expect that they would be either positive or negative but not neutral.

3. *Water Nuclei.*—After finishing the work with phosphorus, correlative experiments with water nuclei were undertaken. It was found necessary, however, to produce them in greater number than is possible by mere shaking, to obtain marked effects. Hence jets were resorted to and studied in some detail, as has been shown elsewhere.¹ The results obtained, though closely resembling the phosphorus data in the main, differed from them inasmuch as the currents above a certain potential difference were constant and independent of the electromotive force of the condenser, while the ionization or charge is usually not neutral as a whole. Nucleation again remained equally effective after the ionization had all but vanished in the speedy way observed for phosphorus.

¹ Cf. *Am. Journal of Science* (4), XV., pp. 105–120, 1903.

4. *Comparison of the Steam Jet and the Condensation Chamber.*—A digression may here be made relative to the indications of the colors of the steam jet and of the coronal phenomena, in relation to the number of nuclei concerned. The usual and strong response of the steam jet is for axial blues and greens as far as the purples of the second order. These are already too weak to be of effective service for measurement. But at this stage of smallness of fog particle, the coronal display has but begun. The strong blues of the axial colors correspond to the diffuse gray fogs of the condensation chamber out of which the coronas are gradually evoked when the number of particles¹ has sufficiently diminished.

The two instruments are thus in a measure supplementary; the condensation chamber gives intense evidence of the presence of nuclei long after the steam jet would imply their absence. It is for this reason that ordinary smokes like sal ammoniac do not affect the steam jet where a number of nuclei exceeding a certain lower limit is necessary. The latter again is particularly active for those intense and fresh nucleations which produce the browns and yellows of the first order, implying a degree of abundance or fineness of nucleation which it is impossible to produce in the condensation chamber at all.

5. *Decay and Absorption.*—To account for the rapid diminution of the number of nuclei in the phosphorus emanation in the lapse of time, two hypotheses are prominent. With finely divided and in so far highly potentialized matter (particularly when ionized positively and negatively) combinations of nuclei may occur to the detriment of the number of independent nuclei. Such a decrease would take place as the square of the number. On the other hand it is equally probable that the initial and very small nuclei are in rapid motion much like molecules, and that the loss takes place by absorption or arrest at the walls of the vessel. In my memoir on the subject I included both hypotheses in the computation; but finding that the phenomena for phosphorus could be adequately explained by the latter. I ignored all spontaneous decay. Though

Indeed the evidence already instanced shows that it does not. In case of water nuclei which are much the more sparsely distributed, the original number of nuclei can be proved by coronas to have varied but little, neither increasing nor decreasing in the short time in which the ionization falls off to a few per cent. Hence the nuclei must be regarded as parting with their charges more rapidly than they are themselves absorbed in the lapse of time, and one will have to distinguish between the velocity of the uncharged and of the charged nucleus in the electric field, the latter being incremented by the electric forces.

6. *Apparatus.*—The apparatus used in the present experiments was capable of a great number of variations. The essential purpose is to enable the observer either to introduce phosphorus emanation at once into the electrical condenser, or else to introduce it after it has been saturated with water, suddenly cooled or otherwise treated.

The parts of the train of apparatus are the large copper Mariotte flask, with a supply of water sufficient for aspiration; the condensation chamber or receiver, used both for producing coronas and for the aspiration and storage of air laden with phosphorus nuclei; the vacuum reservoir, the phosphorus ionizer, the tubular electrical condenser and the electrometer. An accessory desiccator of the tower form may be inserted on the way when dry air is needed.

The vacuum reservoir is connected through a stopcock with a suction pump, with the atmosphere and (by wide tubing) with the condensation chamber. It carries a vacuum gauge.

The receiver is connected by a stopcock with a cotton filter, with the phosphorus ionizer (where the tall desiccator may be inserted), with the vacuum reservoir and also carries a vacuum gauge. The receiver is further joined by stopcocks with the Mariotte flask for aspiration, and is graduated in liters on its side. It holds about ten liters.

The ionizer is a large U tube containing calcic chloride for desiccation, kept in place by loose cotton plugs. One shank is nearly empty, and carries thin pellets of phosphorus between strips of wire gauze.

The condenser is tubular, 2.10 and .64 cm. in diameter and 50

cm. long, with the outer mantle permanently put to earth. The core is a brass rod, supported on hard rubber insulators about 15 cm. long and at a distance of 10 cm. from the end of the tube. This rod is highly charged by a storage battery, and its leakage found from the electrometer, one pair of quadrants of which are in connection with the rod and the other pair put to earth. A commutator enables the observer to use either the positive or the negative pole of the storage battery for charging the system.

The electrometer was specially built for the present purposes and the needle was kept charged by a water battery of 48 volts, one pole of which was earthed. The suspension is a silk bifilar moistened by a dilute solution of any hygroscopic salt, and the battery charge is conveyed through the fibers. The quadrants are supported on hard-rubber insulators 10 cm. long. Difficulties were encountered in using this apparatus as will appear elsewhere.

7. *Manipulation.*— On raising the Mariotte flask, and opening appropriate stopcocks, the emanation passes directly from the ionizer into the condenser and its ionization may be measured. A tower desiccator is here to be inserted ahead of the ionizer to dry the air.

On lowering the Mariotte flask, removing the desiccator, and reversing the ionizer, the emanation passes into the receiver. Here its nucleation may be tested by condensation, and it may therefore be introduced into the condenser, at once or after a number of condensations. The nucleation may also be stored in a dry vessel and subsequently transferred from it into the condenser.

Finally the phosphorus ionizer and desiccator may be quite removed and replaced with a pipe connection adjusted for spraying. Hydrant water passing under high pressure reaches a jet placed suitably within the receiver. The water nuclei thus producible may be tested either by coronas or electrically by passing them into the condenser.

IONIZATION OF PHOSPHORUS NUCLEI COLLECTED OVER WATER.

capacity of the apparatus in parallel, E the potential of the condenser, i the radial electric current through it, R its ohmic resistance for the given medium, s the deflection at the electrometer,

$$(dE/dt)/E = (ds/dt)/s = 1/CR, \text{ or } d(\log s)/dt = .434/CR = a,$$

where common logarithms are used and a is the constant sought. Thus a varies directly with the conduction of the ionized medium traversing the condenser.

TABLE I.

Conduction of room air aspirated over phosphorus, directly or indirectly into condenser.

	Remarks.	$a \times 10^3$
I.	Insulation of electrometer (morning).	7
II.	“ “ “ (afternoon).	10
V.	Slow current (liter/min.) of phosphorus nuclei in saturated moist air entering condenser.	8
VI.	Do., but faster current (10 liter/min.).	14-16
VII.	Phosphorus nuclei enter condenser by diffusion.	8
VIII.	Do., slowly blown in with moist air.	7
IX.	Phosphorus nuclei enter condenser <i>directly</i> from ionizer. Fast current of moist air.	320
XI.	Do., slower current.	200
XII.	Do., slow air current but fresher phosphorus.	320
XIII.	Ionized air current stopped; observations immediately after.	54
XIV.	Do., after a few minutes.	14
XV.	Insulation finally.	8

Cases V., VI., etc., tested for coronas by successive exhaustion gave the usual full series, beginning with diffuse dense fogs.

Table I. gives a summary of these results, among which may be mentioned the following: The insulation is not above $a = .01$ and usually lower. It is not exceeded when phosphorus nuclei have access merely by diffusion (VII.) nor on being blown in from a wide vessel (VIII.), the charge vanishing on the way. The leakage is not exceeded when a slow current of highly nucleated air stored over water, is passed through the condenser (V.), and but slightly for the case of a fast current of such nucleated air (VI.) taken out of the receiver or condensation chamber.

By contrast the excessive ionization ($a = .2 - .3$), if the original nucleation is at once introduced into the condenser, is striking enough. Hence less than 3 per cent. of the original ionization has survived after short storage in the receiver, in spite of the extreme

and undiminished density of nucleation which the coronas contemporaneously show. In fact the ionization dies out almost at once in the condenser (XIII.-XIV.), even in the absence of water vapor.

9. *Further Data.*—The experiments were now repeated and the résumé of results shown in Table II. These are substantially like the above. With fresh phosphorus the residual ionization of the nuclei bearing air after short storage over water is but a few per cent. of the original ionization. It was supposed that on drying the nucleated air over phosphorus pentoxide, before passing it into the condenser the original ionization might be in part regained, but the table shows not a trace of this.

TABLE II.

Conduction of room air aspirated over phosphorus directly or indirectly into condenser.

Remarks.	$a \times 10^8$ Observed.	$a \times 10^8$ Corrected.
Insulation (room air).	8-10	0
Filtered damp air.	11	0
Nucleated damp air (fresh phosphorus). ¹	24	14
Do., partially dried. ¹	21	11
Do., older phosphorus. ¹	17	7
Do. ¹	18	8
Phosphorus nuclei at once into condenser. ²	260	250
Do. ²	290	280

The charge in the condenser is negative as before. It should be more rapidly dissipated if negative ions are precipitated more rapidly in the receiver, than a positive charge. To obtain dense nucleation room air was again aspirated over phosphorus into the receiver, from which it was then discharged as expeditiously as possible, the time taken being about five minutes.

10. *Effect of Different Charges in the Condenser.*—In the next experiments the sign of the charge in the condenser was varied. To find comparable results it was thus necessary to maintain a definite current through the condenser and about 2.5 liters per minute was adopted compatibly with the dimensions of the apparatus. Nuclei were again aspirated into the receiver over phosphorus. The leakage was apparently different for charges of opposed sign; but this

¹ From ionizer, to condensation chamber, to condenser.

² From ionizer, to condenser. A faster air current would increase this conduction.

was due to the insulation of the condenser, which is greater for negative than for positive charges. Deducting this the values of the ionization α differ by quantities which lie within the errors of observation.

As the receiver in the course of the efflux of nucleated air shows fogs of continually increasing density, the spontaneous precipitation must have been equally effective for positive and for negative nuclei.

TABLE III.

Conduction of room air, aspirated over phosphorus into condensation chamber and thence to condenser. Air current 2.5 lit./min. charge at 10 volts.

Charge in Condenser.	Conduction Observed.	Conduction Corrected. ¹
Negative.	$\alpha = .025$	$\alpha = .009$
Positive.	22	6
Positive.	22	6
Negative.	18	8
Positive.	24	8
Negative.	17	7

In Table IV. similar results are given, but with the insulation tested after each passage of nucleated air through the condenser.

TABLE IV.

Conduction of room air aspirated over phosphorus, through condensation chamber into condenser. Air current 2.5 lit./min. charge at 20 volts.

Charge in Condenser.	Conduction Corrected. ²	Mean α .
Positive.	$\alpha = .009$	.010
Negative.	12	
Positive.	09	
Negative.	09	
Positive.	11	

There is slight excess of leakage for negative charges ; but as the insulation was $10^3 \times \alpha = 9-12$ for positive and 1-2 for negative charges, these differences are within the uncertainties of observation.

One may again note that if the dense spontaneous fogs in the receiver were associated with selective condensation, then negative charges should be more rapidly dispelled.

¹ Insulation $10^3 \times \alpha = 10-16$.

² Insulation tested after each passage of nucleated air.

11. *Dried Emanation.* — In the following experiments, Table V., the phosphorus was previously dried over calcic chloride, and then introduced into the receiver from the desiccator and the shortest possible connecting tube. Dry ions thus *suddenly* come in contact with water vapor, and it was supposed that an *unequal* reduction of positive and negative ionization might ensue. The ions were stored less than five minutes in the receiver, the shortest time practicable. Insulation of the electrometer and parts were determined before and after each measurement with nucleated air.

TABLE V.

Conduction of desiccated room air, aspirated over dried phosphorus, through condensation chamber, into condenser. Air current 2.5 lit./min.

Charge in Condenser.	Conduction Observed. ¹	Conduction Corrected.
Positive.	$a = .0152$	$a = .0023$
Negative.	.0070	22
Positive.	.0135	41

Contrary to all expectations the residual ionization so obtained, $a = .002-.004$ for positive and .002 for negative charges, is smaller than heretofore, but again practically neutral. Thus very dry phosphorus nuclei seem to lose their ionization quicker than if placed in ordinary air. The activity of the ionizer may account for the difference.

12. *Wet Emanation.* — For contrast, the nuclei were conveyed

TABLE VI.

Conduction of moistened room air, aspirated over phosphorus to the condensation chamber,¹ thence into condenser. Potential 20 volts. Air current 2.5 lit./min.

Charge in Condenser.	Conduction Observed.	Conduction Corrected.
Positive.	$a = .0207$	$a = .0109$
Negative.	.0117	.0046

into the receiver (Table VI.) in a wet current of air passing over phosphorus. A U-tube was used, one leg of which contained wet sponges and the other the phosphorus grid, the damp air from the former sweeping over the latter, into the receiver. The ionization

¹ Tested for coronas the nucleated air showed the usual strength.

found is distinctly greater¹ than the dry air data of the last table though it does not exceed the usual values for room air. The difficulty of keeping the ionizing activity constant is again involved.

According to the table, positive charges are more rapidly discharged than negative charges, that would indicate an excess of negative ions comparable to the case of water nuclei in the next chapter.

13. *Residual Ionization After One Hour.* — The storage of phosphorus nuclei over water in the above experiments did not exceed ten to fifteen minutes. It was thought that by giving the fog particles more time to subside, the sign of the residual ionization might become apparent, supposing that more negative nuclei are precipitated. The results in Table VII. are peculiar; whereas the positive charges in the presence of the nucleation vanish more slowly than for room air, the negative charge vanishes much faster. This would make an excess of $a = .005$ of residual positive nucleation over the negative nucleation, and it would follow that the negative nuclei are precipitated faster. But with a leakage in the electrometer of .011 and .004 respectively, without any ionized medium, the result is not guaranteed, particularly as the positive leakage is large.

TABLE VII.

Residual ionisation of phosphorus nuclei, stored 1 hour over water. Charge in condenser at 20 volts. Air current 2.5 lit./min.

Charge in Condenser.	Conduction Observed.	Conduction Corrected.	Remarks.
Positive.	$a = .0121$	$a = .0000$	Room air.
Positive.		$- .0014$	Nucleated air.
Negative.	$.0068$	$+ .0032$	Nucleated air.
Negative.		$.0000$	Room air.

14. *Nucleation Partially Precipitated.* — The nucleated air stored over water in the receiver was suddenly cooled and allowed to subside five to ten minutes. In this way greater chance was given for the differentiation of positive and negative nuclei. The corrected values of a , Table VIII., show that positive charge is removed faster than negative charge, by $a = .0010$ and $.004$, respectively, implying excess of negative nuclei. With the insulation varying from .003–

¹ Cf. Am. Journ. of Science (4), XII., p. 327, 1901.

.010 and .003-.013, before and after, some of the measurements, this result is again doubtful. It should be noticed that it is the reverse of the preceding.

TABLE VIII.

Residual ionisation of phosphorus nuclei, after partial precipitation. Charge in condenser at 10 volts. Air current 2.5 lit./min.

Charge in Condenser.	Conduction Observed.	Conduction Corrected.	Remarks.
Negative.	$a = .013$.015 and .003	$a = .004$	Nucleated air.
Negative.			Room air. ¹
Positive.	.022 .013 and .003	.014	Nucleated air.
Positive.			Room air.
Negative.	.019 .009 and .008	.010	Nucleated air.
Negative.			Room air.

15. *Ionization of Dry Phosphorus Nuclei.* — In the present experiments a dry vessel of ten liters capacity was introduced between the desiccator and the phosphorus tube. Phosphorus nuclei were now aspirated thence into this vessel. They were discharged after five minutes into the condenser after removing the phosphorus tube. Table IX. shows that almost all the ionization is lost by this dry storage, as the excess of leakage due to the discharge of this dry air through the condenser is $a = .000$ and .007, respectively. Thus dry air shows no preservative effect.

TABLE IX.

Residual ionisation after dry storage (2m-5m) of phosphorus nuclei. Charge in condenser at 10 volts. Air current 2.5 lit./min.

Charge in Condenser.	Conduction Observed.	Conduction Corrected.	Remarks.
Negative.	$a = .006$.006	$a = .000$	Nucleated air.
Negative.			Room air.
Positive.	.011 .004	.012	Nucleated air.
Positive.			Room air.

Intense ante-coronal fogs were obtained in the condensation chamber, due to incidental back aspiration of nuclei through the desiccators.

In Table X., by a modification of the apparatus, a current of dry air passes from the desiccator over phosphorus, and then into one

¹ Insulation given before and after experiment with nucleated air.

end of a dry vessel of ten liters capacity. At the other end of the vessel the air is continually discharged into the condenser. But the usual negative result appears.

TABLE X.

Residual ionization after continuous passage of phosphorus nuclei through a large dry vessel. Air current 2.5 lit./min.

Charge in Condenser.	Conduction Observed.	Conduction Corrected.	Remarks.
Negative, 20 volts.	$a = .017$	$a = .017$	Nucleated air.
" " "	.000		Room air.
" 30 "	.012	.012	Nucleated air.
" " "	.000		Room air.

INFERENCES.

16. *Inferences.* — The chief result of the investigation is the immensely rapid initial reduction of the ionization of the phosphorus emanation in contrast with the relative persistence and constancy of the nucleation. In other words only a few per cent. of the original ions are usually associated with the nuclei by which the dense fogs and coronal sequences are produced, even at the outset. It was with a view to possibly restoring some of this lost ionization that the great variety of experiments detailed in the chapter were undertaken.

Moreover, the whole of the original ionization vanishes symmetrically, for the nuclei as a whole are neutral throughout. At least with so insignificant a residue of the original ionization, the decision as to whether more positive or more negative ions have vanished is a delicate one and perhaps of trifling interest in this connection. For the phenomena are now all of the order of the leakage of the electrometer and appurtenances. If when 100 ionized nuclei of the phosphorus emanation are suddenly introduced into an atmosphere saturated with water vapor, the ionization of 96 has vanished without a record, while the remaining four are in equal number positive and negative, it is unlikely that negative ions can have greater affinity for water vapor or be more remarkable in their efficiency as condensation nuclei than positive ions.

Finally it does not appear that the ionization lost so soon after the removal of the emanation from the phosphorus surface can ever

be restored, notwithstanding the fact that in the condensation chamber, nuclei may be made to pass from the fog particle to the nuclear stage of size and density, an *indefinite* number of times.

That the original nucleation immediately at the phosphorus surface is so enormously reduced in number as to correspond to the loss if ionization; *i. e.*, that but a few per cent. of the initial number of nuclei can in any event be used for condensation, seems scarcely credible, though direct experiments would be difficult to devise with phosphorus. Placed in the condensation chamber over water phosphorus emits a dense filament of smoke and is relatively inefficient. All this would complicate the question uselessly. With water nuclei, however, the decision may be reached; for the nuclei are most effectively produced in the condensation chamber itself, while the ionization may be studied without loss of time; and here it makes no appreciable difference whether a corona is produced at once after the jet has been shut off, or after the short time needed for the ionization practically to vanish. The aperture and type of the corona and hence the number of nuclei involved are the same.

BROWN UNIVERSITY, PROVIDENCE, R. I.

EFFECT OF HEAT ON EXCITED RADIOACTIVITY.

BY FANNY COOK GATES.

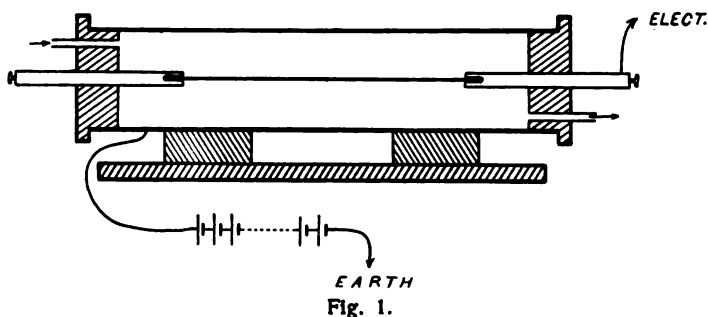
RECENT experiments on the nature of excited radioactivity have shown that by various mechanical and chemical means the radioactivity may be removed from the surface on which it is found, but no proof has been given that it can be destroyed by such treatment. On the contrary, when an excited wire loses its activity by friction, the substance with which it is rubbed becomes radioactive to a degree corresponding to the loss from the wire. When attacked by acids, the radioactivity lost from the wire is found in the solvent.

Professor Rutherford has found that a platinum wire excited by thorium emanations can be made red hot without appreciably changing its activity, but that the activity is almost entirely lost after heating the wire to incandescence. This sudden disappearance of the radioactivity at white heat seemed to furnish evidence of its destruction, and with the aim of either establishing or refuting such a view the following experiments were undertaken.

If the excited radioactivity is not destroyed by intense heat, it was argued that by heating the wire inside a closed vessel so as to prevent all escape, the active particles, although removed from the wire, would still remain and might be discovered in the surrounding air or on the walls of the vessel. For this purpose a hollow brass cylinder about ten inches in length and three inches in diameter was fitted with ebonite ends; in each end were two openings, one in the center through which to pass the wire, and another through which to send a blast of air when desired. The wires used were six inches in length and about one twentieth of an inch in diameter. The cylinder was used in the usual manner, as a testing vessel for measuring the amount of radioactivity within it. Its outside was joined to one terminal of a storage battery of 100 volts, and the

central wire to one pair of quadrants of an electrometer of which the other pair was connected to earth. Another testing vessel was used for comparing the amounts of activity produced by the wire before and after heating.

The wire was excited by making it the cathode of a P.D. of 100 volts and placing it for several hours inside a closed metal box containing 50 grams of thoria. When placed in position in the cylinder, an ionization current of the order of 10^{-11} amperes was produced. A current of 20 amperes was then passed through the wire, heating it to incandescence, after which the ionization current was again measured. Instead of showing a great loss, as might have been expected, the current was nearly as large as before; upon replacing the platinum wire by an inactive copper wire but little further



change was observed. The platinum wire was then put in the second testing vessel and found to possess only about 3 per cent. of its original activity. Clearly then the active particles had been removed from the wire and were still inside the cylinder. The air was then blown out but no decrease in current was thereby obtained, showing that the active particles had become attached to the inside surface of the cylinder. The rate of decay of the radioactivity within the cylinder was observed and found to be such that it was reduced to half value in eleven hours, the characteristic of excited radioactivity from thorium.

From these results we conclude that *intense heat does not destroy excited radioactivity*, and that under conditions as here described the active particles are transferred at a definite temperature without other change from the platinum wire to the inside surface of the cylinder,

The tests were repeated with wires excited by radium emanations with similar results, although on account of the rapid decay of the excited radium radioactivity during the first fifteen minutes, the tests had to be confined to the period between 15 and 35 minutes after the removal of the wire, since the rate of decay is known to be a minimum during this interval.

Some of the results of several sets of these observations are given in tabular form on a following page. (Table I., 1-4.) It cannot be assumed that, if all of the active particles were transferred from the wire to the cylinder, the ionization current would necessarily be unchanged, since in the latter case the radiations are not all radial as in the former. It would be difficult to determine experimentally how great a change this produces, but it is doubtless large enough to account for the slight numerical difference between the current before and after the wire is heated, when we also consider that some of the active particles may have escaped through the ends of the cylinder with the sudden outrush of heated air.

In order to get from the wire to the cylinder the active particles must have been in the air at some time, and since none could be blown out later, all must have been transferred during the heating. This was shown experimentally to be the case by the fact that the particles could be blown out of the cylinder while the wire was being heated, although at no other time. A strong blast of dust-free air was directed along the wire all the time it was being heated. Although this so cooled the wire that fewer particles were driven from it than in the preceding tests, the activity within the cylinder was reduced to less than one third its original value, showing that over two thirds of the particles had been driven out of the cylinder. It was found possible to blow the particles from one cylinder into another.

An attempt was made to observe the effect produced by blowing and heating in the presence of a strong electric field with the object of ascertaining the nature of the charges carried by the particles, but no definite conclusions were attained.

Two views may be taken as to the cause of the removal of the active particles from excited platinum at white heat :

1. Their removal may be due to a volatilization of the active material.

2. The platinum may itself disintegrate and carry the active particles with it.

It is known that platinum when white hot undergoes a surface disintegration in air, and the remarkable disappearance of the radioactivity at about this temperature is in harmony with the view that the active particles were carried off with the outside disintegrated film of the wire. If this were the true explanation,

TABLE I.

Heating without blowing.

Exp. No.	Wire Used.	Excited Radioactivity Due to	Activity from Wire Before Heating.	From Cylinder and Wire After Heating.	From Cylinder Alone After Heating.	From Wire alone After Heating.	Difference.
1	Platinum.	Thorium.	100	90	87	2	10
2	"	"	100	92	90	2	8
3	"	"	100	94	—	—	6
4	"	Radium.	100	92	88	2	8
5	German silver.	"	100	98	33	66	2
6	Plat. iridium.	Thorium.	100	97	81	15	3
7	Plat. in CO ₂ .	"	100	98	95	3	2
8	Plat. in H.	"	100	95	89	6	5

TABLE II.

Blowing while heating.

Exp. No.	Wire Used.	Excited Radioactivity Due to	Activity from Wire Before Heating.	From Cylinder and Wire After Heating.	From Cylinder Alone, After Heating.	From Wire alone After Heating.	Amount Escaped.
1	Platinum.	Thorium.	100	36	20	18	64
2	"	"	100	36	10	13	64
3	"	"	100	38	20	18	62
4	"	"	100	22	16	6	78
5	"	Radium.	100	25	—	—	75
6	"	"	100	23	15	8	77
7	Plat. iridium.	"	100	45	31	14	55

wires of other material, or even platinum wires when surrounded by other gases, should not show the same effects.

German-silver and platinum-iridium (10 per cent. iridium) wires were used instead of platinum, but although the wires lost less of their activity in these cases, the behavior of the liberated particles was the same as that of those expelled from platinum. When the

wire was heated in carbon dioxide instead of in air, it lost 97 per cent of its radioactivity, and nearly as much when heated in hydrogen. (See Table I., 5-8 ; Table II., 7.)

These results seem to completely refute the theory that a disintegration of platinum is necessary to cause the transfer of the excited radioactivity from the wires to the walls of the surrounding vessel and to point to the conclusion that, at the temperature of white heat for platinum, a volatilization of the active material itself takes place.

During the course of the above observations an interesting effect was noticed which has not been mentioned. When measurements of the ionization current were made immediately after heating, the current was always much less than the value which it afterward attained, sometimes being only a third that amount. It would then steadily increase for twenty or twenty-five minutes when the maximum was usually reached. It can easily be supposed that part of this effect is due to the cooling of the air since the ionization current varies inversely as the absolute temperature, but neither could so large a change be accounted for on this ground alone, nor would the current increase for so long a period. The phenomenon is completely explained by assuming that the air in the cylinder after heating is filled with particles of disintegrated platinum and other matter, which behaving as dust particles increase the rate of recombination and decrease the current for a given voltage. That this assumption is correct was shown by blowing a current of dustless air through the cylinder immediately after heating. The values which otherwise required twenty minutes to be reached, were attained immediately.

SUMMARY OF RESULTS.

1. Excited radioactivity cannot be destroyed by heat.
2. The active particles are removed from platinum and other wires at a temperature just below white heat, and are transferred unchanged to the surfaces of the cooler solids near.
3. By removing the surrounding gas as fast as the wire is heated, a large portion of the active particles may be carried off with it.
4. The removal of the excited radioactivity from the wire is probably due to a volatilization of the active material.

I desire to express my gratitude to Professor Rutherford through whose kindness the privileges of this laboratory have been extended to me, and under whose direction these experiments have been performed.

MACDONALD PHYSICS BUILDING,

MCGILL UNIVERSITY, MONTREAL, January, 1903.

RADIOACTIVITY OF FRESHLY FALLEN SNOW.¹

S. J. ALLAN.

C. T. R. WILSON has shown that freshly fallen rain is radioactive and that its activity decays rapidly with time. It seemed probable that freshly fallen snow would also show this effect. During the first snow storm of the season about a liter of snow was quickly collected from a thin layer on the surface, and evaporated down to dryness in a shallow tin dish. This dish which before filling with snow had been tested, and found to contain no trace of radioactivity was now able to ionize the air in its immediate vicinity quite strongly.

The apparatus used to test the presence of the activity is shown

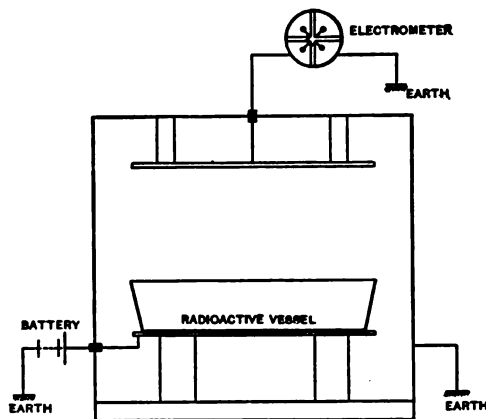


Fig. 1.

in Fig. 1 and consisted as follows: Two parallel zinc plates were suspended one above the other and both insulated. The upper plate was connected to one pair of quadrants of a sensitive electrometer, the other pair being earthed. The electrometer was of the usual type with upper and lower quadrant plates. The needle was of thin aluminium suspended by bifilar silk fibers. It was kept continually

¹ Communicated to American Physical Society January, 1903.

charged to 300 volts by a battery. When charged to this potential the electrometer needle gave about 600 scale divisions per volt, the scale being about 2 meters from the mirror.

The lower zinc plate was connected to one pole of a battery of small accumulators from which any voltage up to 300 could be obtained. The other pole of the battery was earthed. These two plates were enclosed in a zinc box connected to earth, which served as a guard ring. By operating a key from a distance, by means of a string, the quadrants could be separated. The radioactive vessel was placed on the lower plate. The rate of movement of the needle was thus a measure of the activity present. There was always a small leak, due to the natural ionization within the apparatus. As this was only .2 of a scale div. per sec., any increase in the ionization current, due to a radioactive substance being present could easily be detected. A specimen of uranium was kept as a means of standardizing the apparatus.

A great many tests were made extending over about six weeks. The snow was always collected from a thin layer on the surface, so as to get that which had recently fallen. It generally took from twenty to thirty minutes to collect, evaporate down and test. The activity had thus fallen in value during this time.

Two of the best methods of distinguishing between the various types of radiations are the rate of decay and the penetrating power in various substances. These two methods were therefore tried. Numerous tests on the rate of decay showed that it followed closely a geometrical progression with time, falling to half value in about thirty minutes. This rate of decay was never found to vary appreciably. Some of these results are plotted in the accompanying curves (Fig. 2). Curves *A* and *B* are from observations taken on the same day, *A* from one liter of snow and *B* from one half liter. The fall of snow on this day was heavy and thick, and gave the greatest amount obtained during any snow storm tested. Curve *C* shows one taken on another day when the fall of snow was much lighter. It is the amount obtained from one liter. In some of the tests the snow had been falling for several hours and in others only a short time. All of these curves show the activity to fall to half value in about thirty-two minutes. In curve *D* are plotted results,

taken from a test on the rate of decay of excited activity on a negatively charged copper wire in the open air. These are plotted on the same scale for sake of comparison. It will be seen that this falls to half value in about forty-eight minutes. There is thus a distinct difference in the rate of decay of the two activities.

Tests were made on the penetrating power of the activity from snow. For this purpose the activity was rubbed off onto a piece of leather, moistened with ammonia. The leather was then covered with sheets of thin aluminum foil, and the rate of leak taken when covered and uncovered. From these results the penetrating power

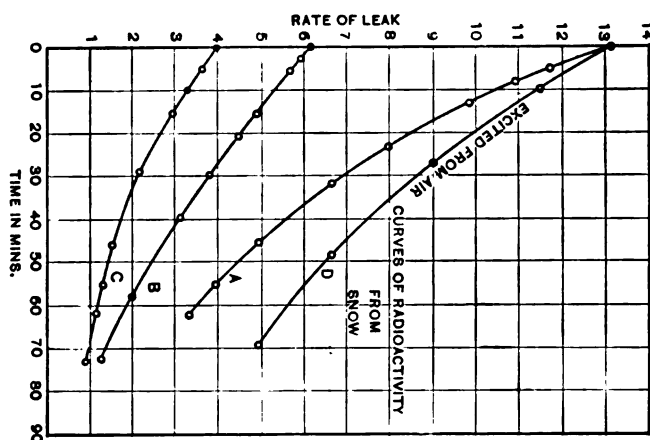


Fig. 2.

could be calculated. It was found to be about the same as for the excited activity from air, half of it being absorbed in .00076 cm. of aluminum.

In many other respects this activity behaved like the excited activity from air. If the activity was rubbed off onto a piece of cotton, which was then burnt to ashes, the residue was still radioactive. It could be heated to a bright-red heat without destroying any appreciable amount of its activity. It was found that during a steady fall of snow the amount of activity that could be obtained from a given amount of snow remained about constant, and no such thing as an exhaustion of the atmosphere could be observed. Twenty-four hours after a heavy fall of snow a very small amount could be still obtained. Experiments were tried to see whether

the decay was the same in the snow as when the snow was evaporated down to dryness. Owing to difficulties encountered nothing definite could be obtained.

The amount of activity obtained from a liter of snow was about equal in effect to one fifth of a gram of uranium. It might be of interest to show the amount that could be obtained from a square mile of territory, covered with a layer of freshly fallen snow 1 cm. thick. The maximum ionization current obtained from a liter was about twenty scale divisions per second. With this apparatus each scale division corresponds to about 3×10^{-13} amperes. From a square mile we ought therefore to obtain roughly about 1.5×10^{-4} amperes per second. This is a considerable amount and could be easily measured with a galvanometer, so that during a heavy fall of snow over a territory the size of Canada, a considerable amount of energy would be represented in the activity.

From the results of this investigation two general conclusions may be drawn: either that this activity is different from the excited activity from air, or that the excited is of a more complex character than was at first supposed. We know nothing as to the mode of production of the radioactivity in air. There may be several processes going on, each manifesting itself under different conditions, of which the activity in snow may be one. The rate of decay of the excited on a charged wire may be the resultant of several rates of decay, and that from snow one of them. In all other respects the two activities give the same results, the rate of decay being the only difference. Since this paper was first communicated, investigations by the author on the rate of decay of the excited activity from air on copper wires have brought out some interesting results. The activity was rubbed off from the wire, with the aid of ammonia, on to various materials, such as leather and felt. The activity on the leather fell to half value in thirty-eight minutes, whilst on a piece of absorbent felt it took sixty minutes to fall to half value. If a piece of cotton was rubbed over the wire, and then reduced to ashes, the activity on it fell to half value in forty-five minutes, the same as on the wire itself.

some light on the difference between the rates of decay of the radioactivity obtained from the air by these methods.

It seems probable that there is in the atmosphere some process going on producing positive radioactive carriers, since they are only collected on a negative wire. The particles of snow would act as collectors for the radioactive carriers, removing them from the air. The amount of activity which reached the ground with the snow would depend on the closeness together of the snowflakes, and on the rate at which they fell.

MACDONALD PHYSICS BUILDING,

MCGILL UNIVERSITY, MONTREAL, January, 1903.

NOTE.

Ogden Nicholas Rood.—Ogden N. Rood, son of the Reverend Anson Rood, was born at Danbury, Conn., on the 3d of February, 1831. He was graduated from Princeton College in 1852 and spent the two following years in part as a graduate student at Yale and in part as an assistant at the University of Virginia and as assistant to Professor Silliman in New York. In 1854 he went to Germany for further study.

During his four years' residence abroad (1854-58) Rood, in addition to his scientific study, devoted much time to the practice of oil painting. He had a natural taste for art and music and took advantage of the great opportunities afforded in Berlin and Munich for the development of his talents. In 1858 he married Miss Prunner, of Munich, and in the same year returned to America and began his career as a teacher in the Troy (N. Y.) University.

Before going abroad Rood had already published two papers on microscopical subjects. During the five years which he spent in Troy he completed a series of experimental investigations chiefly on light, which were printed in the *American Journal of Science*. In 1864 the chair of physics in Columbia College became vacant and he was appointed to the professorship in that subject, which position he held up to the time of his death, on November 12, 1902.

Rood's work during his residence in Troy, which included, in addition to the optical experiments already mentioned, an elaborate research on the performance of projectiles, and likewise a series of papers on the application of photography in physical investigation, received prompt and cordial recognition on the part of men of science at home and abroad, and in 1865 he was elected to the newly-established National Academy of Sciences. His unusual skill as an experimenter was specially shown by his success in obtaining measurements of extreme accuracy and delicacy by comparatively simple appliances. In his measurements of the changes of the dimensions of bodies by means of the horizontal pendulum, in 1874, he was able to attain a degree of sensitiveness fully equal to the best performance of the interferometer. By the use of the revolving disc he determined with great accuracy the duration of the electric spark and of the lightning flash, demonstrated the multiple character of lightning and estimated the minimum time of illumination capable of producing distinct vision. When, in the seventies, the work of Toepler

and of Crookes upon the electric phenomena observed in high vacua was attracting the attention of the scientific world, Rood turned his attention to this fascinating subject. In 1876 he published a paper on the force involved in Crookes' radiometer. In 1880 he described an improvement in the Sprengel pump, and in the following year he brought out his paper on a method of obtaining and measuring high vacua with a modified form of the mercury pump. Few problems are better adapted to test the skill, ingenuity and patience of an experimenter than that of attempting to produce the highest attainable vacuum, and no one, probably, among the numerous workers in this difficult field has recorded the measurement of a lower pressure ($\frac{1}{1880000}$ of an atmosphere) than that attained by Rood in this research. While later determinations have thrown into serious doubt the accuracy of such measurements by means of the McLeod gauge, it is safe to say Rood pushed his investigations to the very limits of human possibility.

The period of Rood's greatest scientific activity lies in the twenty years between 1860-1880. His attention, during these years was chiefly, although by no means exclusively, directed to physiological optics and especially to the subject of color. His long-continued studies in this field were embodied in the well-known volume entitled "Modern Chromatics" which he published in 1879 and which was reissued later under the less appropriate title of "A Text-book of Color." This little book will ever be a delight to students of color. The treatment is admirable in its directness and simplicity, in which respect it reminds one of the popular works of Tyndall. It is like the work of Tyndall also in being no mere compilation of the results of others and in consisting of original experiments described in language intelligible to every educated reader. The book has a double charm. On the one hand, it is good physics, while on the other, it is enriched throughout by the author's artistic experience and enlivened by his enthusiasm for the beauties of color as observed in nature. The subjects of color mixture, of contrast, of change in luminosity, and of color gradation are illustrated by descriptions of the effects observed in the landscape. These are matters of which Rood was entitled to speak from the standpoint of both artist and physicist. Painting in oil and water colors was with him a passion. It was a pastime to which he devoted much of his leisure, and by means of which his powers as an observer of the phenomena of color as displayed in nature were developed in an extraordinary degree. The development of the æsthetic and artistic side throughout the work is systematic and progressive. The book begins with the pure physics of color and goes over gradually to the consideration of the application of the principles in art. The concluding chapter on the use of color in painting and decoration, although brief, is one of the most interesting and suggestive contributions to this subject which has ever been written.

After 1880 published contributions to science from the hand of Rood became comparatively infrequent, but the falling off was of quantity only and not of power. In 1893 he brought out a new principle in photometry of great theoretical interest and practical importance, the soundness and utility of which was subsequently further corroborated at the hands of Whitman and others. In 1900 he published a second important contribution to this subject in which an ingenious form of the flicker photometer was described. In 1896 he published a paper on the reflection of X-rays, and in 1900 and 1901 he made his final contributions to science upon the measurement of very high resistances.

Among the notable characteristics of Rood's personality were his disdain for the commercial and industrial aspects of physics, and his scarcely less intense dislike of ostentation and publicity. Always more or less inclined to be a recluse, the love of retirement seemed to grow upon him with the passing years. Instead of yielding, like too many others, to the demands of the time for something practical and utilitarian, he resisted absolutely the influence of the philistine atmosphere of the great city in which he lived and clung resolutely to the ideals of pure science.

E. L. N.

NEW BOOKS.

Physics. By FREDERICK SLATE. Pp. i + 404. New York, The Macmillan Company, 1902.

High School Physics. By CARHART & CHUTE. Pp. i + 424. Boston, Allyn & Bacon, 1902.

Elements of Physics. By FERNANDO SANFORD. Pp. i + 414. New York, Henry Holt & Company, 1902.

It will perhaps be generally admitted that the success with which students may pursue the subject of physics is more largely a function of the teacher than of any other objective condition: and yet it must be conceded that the text-book employed is a condition not entirely devoid of influence. Such a book is, therefore, always interesting from at least two points of view; first, as giving us an almost infallible judgment as to the author's conception of the great body of truth which is to-day grouped under the head of physics; and secondly, as a message from the author to teachers of physics. The first point of view is a purely psychological one, the second, a purely pedagogical one.

It is in this second regard, viz: the message from the author to the instructor, that lies the chief merit (and perhaps also the *raison d'être*) of many of our text-books. The fact that the book serves the student as a compendium of fact and as a syllabus of the subject is merely incidental as compared with the influence of the book upon the teacher, who actually handles the class. The student's intellectual growth and his ideas of the subject depend so much more upon the teacher's point of view and his mode of presentation than upon any other factor that, in most classes, a text-book might easily be changed without producing a ripple upon the mental surface of the class.

Looked at from this two-fold viewpoint, three interesting volumes have recently made their appearance in the high school world.

We shall consider first Professor Slate's "Physics." This book is characterized, above all else, by the author's philosophical view of the subject; even the simplest phenomena are made interesting in the light of the philosophy which underlies them. The really deep insight into the physical processes of nature and the keen epistemological appreciation of the author combine to make the volume well worth reading to every instructor of physics. Illustrations in the form of diagrams are rather

meagre; but the illustrations of principles by means of citation of familiar phenomena are so abundant as to form one of the marked features of the book. As to English, the phrasing is at times very effective; take, for example, the description of "weight" (p. 219) as "a partnership effect in which earth and body are both concerned." As a rule, however, the style, while clear, is involved; it is by no means so simple as it might be, and is, therefore, less adapted than it might be to the grade of students for which the book is intended.

Another feature which serves to give this volume a marked individuality is the entire omission from its pages of such simple mechanical ideas as centrifugal force, simple harmonic motion, wave motion, Newton's laws of motion (except implicitly); not to mention such minor (?) subjects as thermo-electric currents, E.M.F. due to cutting lines of force, the idea of electrostatic quantity, electrostatic capacity. The manner in which the use of these terms is avoided, and the proper idea still conveyed, is always clever and often ingenious. One cannot help feeling, however, that even this skillful author now and then realizes the need of some of these well tried tools of the profession. Witness the following explanation (p. 221) of the diminution in the acceleration of gravity as one approaches the equator. "The earth is turning on its polar axis once in twenty-four hours, the consequence being that gravitation at any place is active *in other ways than in adding to the speed of falling bodies*, and that the value of g is smallest at the equator, where the other demands upon gravitation are greatest." The phrase we have placed in italics is the one by which the use of centrifugal force is neatly avoided.

The clever explanation of musical quality (p. 234) without the use of the word "wave" is certainly clear to one who already understands the explanation; but whether the average beginner would comprehend the idea without several re-readings is perhaps doubtful. The frequent use of the principle of minimum potential energy is an admirable feature. In a certain sense the discussion of mechanics is distributed throughout the entire volume. This accords perhaps with the fact that most successful teachers of physics find it necessary to make constant reference to the chapters on dynamics.

On the whole, this excellent treatment impresses your reviewer as somewhat in advance of the demands of the times; but not at all in advance of the needs of the times. The clear and scholarly unitary view of the subject which is here presented is certain to prove helpful both to teacher and taught until the present viewpoint of the physicist has been decidedly modified.

The excellent *High School Physics* of Professors Carhart and Chute, although appearing under a slightly changed title, is really only a new edition of their *Elements of Physics*, which has for many years enjoyed

a well-deserved popularity. The reason which the authors give for making any changes is the following: "The advances in physics are so rapid and the point of view from which many topics are now considered is so different from that of half a dozen years ago that it seemed best to make an entirely new book rather than a revision of the former one."

We select the following as fairly representing the trend of these changes:

1. Matter is no longer defined as "that which occupies space." Indeed any ability to define it at all is distinctly disavowed, and the student is referred to the study of the various properties of matter as yielding the best notion of what matter is.

2. Those convenient little bodies, called atoms and molecules, are given a much less prominent place in the earlier pages.

3. The discussion of surface tension has been transferred from the first chapter to the chapter on the "Mechanics of Fluids," where it undoubtedly belongs.

4. The term "speed" is introduced to denote the scalar factor of velocity.

5. The chapter on Sound is removed from the latter half of the work and is placed in a much more natural position, immediately following Mechanics.

6. The three statements concerning spectra hitherto honored by the title of "*Laws*" and printed in italics, have been robbed of their former dignity, degraded to the ranks of common type, and confined to a single portion of a single paragraph. Certainly no one will doubt the wisdom of this change.

7. The introduction of the definition of the electrostatic unit of quantity makes possible a logical definition of electrostatic capacity which was impossible in the earlier edition.

The above changes are perhaps sufficient to characterize the improvements introduced into the new edition, though it must be confessed that they are hardly such as flow from any new "point of view" which has found its way into physics within the "last half dozen years."

The language of the text is simple, clear, and well adapted to the average high school student. The illustrations are excellent and almost without exception place the emphasis where it would appear to belong.

The introduction of the Nernst lamp, the oscillatory discharge, wire-

Professor Sanford's "Elements of Physics" represents, in an excellent way, both the concrete and the discrete presentation of the subject. It is concrete in the sense that most of the volume is occupied with certain definite and particular phenomena including 145 laboratory exercises for the student; while it is discrete in the sense that it minimizes (perhaps very properly) the general discussions and abstractions of the lecture room, and because there is found throughout the volume comparatively little "connective tissue," if one may so describe those general principles which make physics one subject instead of half a dozen. The prominence which is given to mechanics is pleasing to those who have faith in the unifying power of dynamics. On the other hand, there are some evidences of haste here and there which may easily be remedied in a second edition. For instance, one finds acceleration everywhere expressed in the same unit as velocity, namely, "centimeters per second." And again, it is difficult to see how the following statement of Newton's First Law (p. 36) harmonizes with the facts:

"Every body perseveres in its state of rest or uniform motion in a straight line unless it receives energy from or gives off energy to some other body." Such a statement assumes that when a body is neither receiving nor giving out energy, it is not acted upon by any force. That this is not the case may be seen in a uniformly rotating wheel where the energy and speed of each particle remain constant while the linear velocity of each particle is constantly undergoing change. The same would be true of a car moving without friction on a circular horizontal track.

The following definition (p. 138) will hardly meet the approval of those who believe that energy and matter are not conditions but objective realities. "In the study of physics, we define heat as the *physical condition which may give rise to the sensation of warmth in our bodies.*"

Again on page 268 we find electric force and electromotive force apparently interchanged. E.M.F. is defined as "a name given to whatever tends to move an electric charge," whereas E.M.F. is ordinarily (J. J. Thompson. Art. 173) defined as the line integral of this quantity.

The author's clearness and simplicity of style and the wealth of experimental illustrations which he has given will be appreciated both by students and teachers. While thoroughly up to date, the treatment is still marked by conservatism both in ideas and in nomenclature. But perhaps the volume is to be recommended above all else for its keen sympathy with the student and with the student's point of view, and by a realizing sense of the fact that acquisition of power by the student is the main object.

High school teachers everywhere are to be congratulated on having three such excellent volumes added to the already long list at their disposal.

HENRY CREW.

Die Electricität in Gasen. By JOHANN STARK. Pp. xxviii + 509. Barth, Leipzig, 1902.

In this volume the varied phenomena produced by the passage of the electric discharge through gases are brought together and are treated from the point of view of the ionic theory. Dr. Stark's book opens with a brief chapter on the apparatus and methods of experimentation. This is followed by a historical description of the ionic theory and by a very full discussion of the recognized methods by which ionization takes place. Part III. deals in a complete and detailed manner with the characteristic features of the electric current in gases and part IV. with the mechanism of the current considered from the point of view of the ionic theory. Under the general title of *ionic rays* the author describes the production and properties of kathode rays and canal rays.

Part VI. deals with the force by means of which the motions of ions are effected and the forms of energy which they may be regarded as possessing. The author considers the ion to have kinetic energy, magnetic energy, electric energy in so far as the ion possesses a field of electric force and finally potential or contact energy; which is defined as the energy which manifests itself when the ion without acceleration changes its molecular condition. This occurs for example whenever an ion passes out of the sphere of action of a metal surface and enters a region where it is acted upon by gas particles alone.

The volume closes with a series of chapters upon the thermal, optical and chemical effects of ionization. Under the first of these heads are considered the variations of temperature within vacuum tubes, the heating of the electrode, the production of heat in the electric arc and in the spark discharge and the phenomenon of the disintegration of the kathode. Under the head of optical effects comes first of all the electric glow of gases when subjected to the discharge. This comprises, from the nature of the case, a complicated set of phenomena. We have, in the first place, radiation due to the temperature of the gas and also radiation which is ascribed to the impact of an ion against a molecule. Both positive and negative ions are to be considered as producers of radiation. In ionized gases, according to the author's view, there are in the most general case five different varieties of particles present. The neutral molecule, the negative ion, the positive ion, the positive molion and the negative molion. The molion is defined as an ion, which has taken to itself several neutral particles by virtue of mutual attraction. The molion may be either positive or negative according to the character of the ion in question. Radiation due to the negative ion is assumed to be identical with *Roentgen radiation*. The author regards the light produced by the two classes of molions to be so small as not to be observable in the spectrum of the glowing gas. He assigns gas spectra to two principal causes: First, To

the breaking up of a neutral molecule through impact with a positive ion (which is accompanied by an intense emission), and secondly to the impact of negative ions with molecules. The author inclines to the view that of the two spectra commonly observed in electrically glowing gases the line spectrum is due to the action of the positive ion while the band spectrum is produced by impact of the neutral molecules with one of the other of the two sorts of ions. Concerning the source of the continuous spectrum observed at high pressures no definite explanation is offered.

Considering the newness of this domain and the fact that widely different views are still held concerning the fundamental theories, the most that can be expected of the writer of any treatise on the phenomena accompanying the passage of the electric discharge through gases is that he shall give a fairly complete and intelligible account of our present knowledge of this important but very complicated department of physics. This Dr. Stark, in the volume under consideration, has done. His book is descriptive rather than controversial and its value is greatly enhanced by the very full set of references to original sources. These references of the literature are brought up to 1892 and it is especially gratifying to the American reader to know that the important contributions of Rutherford, Zeleny, Child, Barus, Graham and other physicists on this side of the water have been duly recognized. The absence of an index, a rare fault in German works, is especially unfortunate in the case of a volume containing so great a mass of detail. It is an omission that will be deplored by every reader.

E. L. N.

Practical Electro-Chemistry. By BERTRAM BLOUNT. Westminster ; Constable & Co. ; New York, The Macmillan Company, 1901. Pp. xi + 374.

This treatise is practical in the sense that it deals for the most part with electrolytic processes on the large or commercial scale, such as the electrolytic winning and refining of metals in aqueous solution, in igneous solution, and in the electric furnace. The last-named section closes with a short description of the production of carbides, borides and silicides.

The foregoing matter which occupies nearly three-fourths of the entire volume is followed by short chapters upon electro-deposition, the production of alkali and chlorine and the electrolytic manufacture of organic compounds.

Finally there is an interesting chapter on *power* in which the relative claims of water and steam power are discussed and the various attempts to obtain energy cheaply by means of primary batteries in which the carbon is consumed are considered. The author appears not to have been acquainted with the work done by Case in this direction but the addition of that to his description of other investigations would have in no wise

vitiating his conclusion that "the present position of the problem of converting the energy of carbon into electrical energy by means other than the boiler, engine and dynamo is one of attempt, not of achievement."

Some idea of Mr. Blount's method of dealing with his subject may be obtained from the following brief summary on the chapter of copper, the refining of which is the largest of all electrolytic industries. After giving the analysis of the average sample of the crude copper, the usual size of anode and cathode is stated and their arrangement in the vat is shown by means of diagrams. The current density, difference of potential, composition of the electrolyte, together with the devices for regulating its flow, are considered at some length. Tables are given showing the quality of the product. The working of the anode sludge for gold and silver is briefly described and the value of this by-product, which in 1896 amounted, in the United States alone, to fourteen million ounces of silver and sixty-eight thousand ounces of gold is given. The chapter continues with the discussion of the various attempts which have been made to place the electrolytic production of a copper directly from the ore upon a successful commercial basis.

The other metals, the electrolytic production of which from aqueous solution is considered are lead, gold, silver, nickel, tin, antimony and zinc. In the section dealing with the winning and refining of metals in igneous solution the production of aluminium, magnesium and sodium are described. The author distinguishes between the electric furnace as developed by Cowles and Moissan and the devices for fusing the solvents used in the reduction of the foregoing metals, although the heat in the latter case is likewise obtained by means of the electric current flowing through the cell. Moissan's work with the electric furnace is very briefly and cursorily treated. It is true that his investigations are fully described in his recent book upon the electric furnace, but many readers of Mr. Blount's treatise will regret that somewhat more space was not given to the description of these remarkable and suggestive researches, which seem likely to have increasing technical importance. The products of electric furnace work described in this section are chromium, molybdenum and tungsten which find their commercial use in the manufacture of special steels; and carbides of calcium and silicon. The remarkable properties of carbon boride are briefly described and the possibility that on account of its extraordinary hardness, the industrial preparation of this compound for use as an abrasive may prove remunerative is touched upon. The fact that graphite is produced in electric furnace work is alluded to but the electric production of graphite which has grown to be an important commercial matter in the hands of Acheson is not even mentioned. This is doubtless too recent a development to have found place in the present volume.

E. L. N.

THE
PHYSICAL REVIEW.

A STUDY OF THE EFFECTS OF TEMPERATURE
UPON A TUNING FORK.

BY EUGENE CYRUS WOODRUFF.

THE temperature coefficient of a tuning fork has been frequently investigated during the past fifty years. In most cases, however, the range covered was limited to little more than ordinary temperatures, and the results threw but little light upon either the actual character of the effect of temperature or upon the cause of this effect. In other cases where a wider range of temperature was attempted, the methods employed left something to be desired in convenience or in sensitiveness. With the method used in this research the ease and accuracy of the observations were nearly as great at high temperatures as at the lower ones, and the range of temperature employed left little to be desired, either as concerns the effect on the frequency of the fork or as regards the effect of temperature upon the elasticity of steel.

In the course of the investigation the question of the effect of temperature on the damping of the vibrations of the fork presented itself. Hitherto this phenomenon seems to have been almost wholly overlooked. In this research it came to light as a natural result of the method employed.

Mayer,¹ McLeod and Clarke,² Ellis,³ Shearer,⁴ Michelson,⁵

Mercadier,¹ Kayser,² and König,³ have all found the frequency of a diapason to be a linear function of the temperature. Pierpaoli,⁴ found otherwise. Most of these investigations, however, covered but a small range of temperature, and many distinctly specify that their coefficient applies only within that range. The function is practically linear, as Kayser says, from 0° C., to 30°, but not exactly so, and the departure from linearity becomes more marked with increasing temperature.

The following table shows the results of these investigators:

	Observer.	Frequency of Fork.	Range of Temperature.	Coefficient per 1° C.	Coefficient per 1° F.
1	Mayer.	256			0.000046
2	McLeod and Clarke.	256	15-26°	0.00011	0.000054
3	Ellis.				
4	Shearer.		20-80		
5	Michelson.	128 v. s.	12-24	0.00010	
6	Mercadier.	56 "	3-26	0.000096	
7	Kayser.	144 "	0-26	0.000088	
8	"	144 "	"	0.000105	
9	"	170 "	"	0.000098	
10	"	170 "	"	0.000100	
11	König.	512 "		0.000111	
12	"	512 "	26-56	0.000107	
13	"	128 "	3-26	0.000111	
14	Pierpaoli.	145 "	0-30	0.000097	
15	"	145 "		0.000101	

Mayer used the graphical method. McLeod and Clarke used a stroboscopic method in which the fork was compared directly with a ruled, rotating drum, which was timed by an electric clock. Shearer kept one fork at constant temperature while another was heated electrically and the two were compared by means of Lissajous' figures. The heated fork was alternately bowed and electrically driven. König used his *Stimmgabel-uhr*, running it for hours at a time, at various temperatures, in a room whose temperature was

fork. Kayser compared two forks by means of Lissajous' figures. The forks were heated by surrounding them with hot-water jackets. Michelson compared his fork stroboscopically with a pendulum, using substantially the method described by Rayleigh.¹ A modification of Michelson's method described by Reed,² combines greatly increased accuracy with ease of observation. This method has been used in this investigation.

Wertheim³ found that the coefficient of elasticity of iron and steel increased from -15° to $+100^{\circ}$ and then decreased up to 200° to a value less than that at ordinary temperatures. Kohlrausch and Loomis⁴ investigated the elasticity of steel wire, using torsional vibrations, from 0° to 90° , without finding any suggestion of the approaching maximum reported by Wertheim. Miss Noyes⁵ investigated Young's modulus by stretching, between the temperatures 15° and 180° , quite carefully, and made a few observations even up to 600° . She differs from Wertheim concerning the existence of a point of inflexion in the elasticity curve between 100° and 200° .

Concerning the effect of temperature upon the damping of a tuning fork but two citations can be made.

König⁶ says: "Die bis zur Temperatur des kochenden Wassers erhitzten Gabeln nur noch eine ausserordentlich kurze Schwingungsdauer haben." On account of this he found difficulty in his observations at the higher temperatures.

Mayer⁷ also reports a similar effect of change of temperature on the duration of the motion of a vibrating body. He says the sound from brass, bell-metal, aluminum, steel, zinc, silver and glass falls off in intensity and duration with rise in temperature between 0° and 100° C. Steel that vibrated for 80 seconds at 0° C. vibrated for but 5 seconds at 100° C. However neither König nor Mayer pushed the matter beyond 100° C.

¹ Phil. Trans. Roy. Soc., part 1, p. 316, 1883.

² PHYS. REV., p. 279, 1901.

The same difficulty presented itself in this investigation. *Moreover the duration of motion was found progressively to decrease with increasing temperature up to a certain point. On raising the temperature still higher, the duration of motion progressively increased until at last it was almost as great as at ordinary temperatures.*

The three points investigated are :

1. The temperature coefficient of a tuning fork between 0° and 200° .
2. The variation of the coefficient of elasticity of steel with rise of temperature.
3. The damping effect of temperature upon the vibrations of a tuning fork.

The thesis falls into three parts : Part one covers the method and apparatus employed ; part two the accumulated data ; and part three a discussion of the results, the curves, and the theoretical deductions.

PART I. METHOD AND APPARATUS.

1. *Method.* — As mentioned above, the method is stroboscopic. But one fork is used and that is compared directly with a free special pendulum. The nearest whole number of vibrations of the fork is determined graphically. The fraction of a vibration plus or minus is then obtained by observing a stroboscopic cycle between fork and pendulum as described by Rayleigh, Michelson and Reed. Some means of securing a cycle of any desired length, whatever the frequency of the fork, is necessary if the temperature, and, therefore, the frequency of the fork is to be varied over any wide range. The free pendulum used was the one described by Reed. The period of the pendulum was varied by placing small weights in a pan carried by the upper ball. Thus, whatever the frequency of the fork, any desired length of cycle was readily secured.

The period of the pendulum was determined by comparing it with the laboratory clock by means of flash coincidences as explained below ; then the fork and pendulum were compared by Reed's method. The equation for the frequency of the fork is $N = \frac{n + c}{P}$; where N is the number of complete vibrations per second of the fork, P is the period of the pendulum, n the nearest in-

tegral number of vibrations in time P , and c the reciprocal of the number of flashes per cycle between the fork and the pendulum. To determine the sign of c , a very small rider was placed in the pendulum pan, thus increasing P slightly but not enough to change the value of n . Now if n remain unchanged and the number of flashes per cycle be observed to increase the stroboscopic motion is a retrograde one and hence c is negative. On the other hand, if the number of flashes per cycle decrease under the same conditions, the motion is progressive and c is positive.

2. *Apparatus.* — The pendulum and its accessories were mounted upon a brick pier. The supporting framework, which was strong and sufficiently rigid for the purpose, was firmly clamped to the stone slab. The bearings of the pendulum consisted of two polished steel planes secured face downward to the under side of the supporting collar. Two steel bicycle balls, about one millimeter in diameter, were set in recesses in copper plates fastened to the framework on either side of the pendulum. On these balls the pendulum planes swung with almost perfect freedom. The total mass of the moving parts was about twenty-five kilos.

Owing to the large mass of the pendulum the two systems of rotating mirrors caused little decay in amplitude during the time required for a complete set of observations. By tuning the pendulum with weights instead of mercury it could be restored to any previously used condition, thus making data regarding the provisional period and other characteristics peculiar to any given condition immediately available. The amplitude was measured by placing immediately below the lower end of the pendulum a paper millimeter scale. A distant incandescent lamp cast a sharp shadow of the edge of this part of the pendulum upon the scale, and the motion of the shadow could be readily observed to tenths of a millimeter.

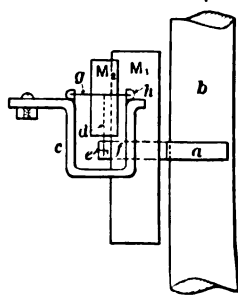


Fig. 1.

yet the arrangement of the mirrors was changed so as to give much greater sensitiveness, in fact any degree of sensitiveness desired, and the method of illumination was such as to make the process of actual observation largely automatic.

One mirror M_1 , 2.5×7.5 cm., was fastened to an arm a , projecting from the pendulum rod b twenty centimeters below the pendulum bearings. The mirror thus moved through the same angle as the pendulum itself. The mirror was made long so that *all the flashes*, and not merely those near a coincidence, would appear in the telescope. Then a skilled observer could, by observing the character and position of a few flashes, judge what would be the time and character of the next coincidence. The pendulum thus required very little watching, and the operator was free to make observations on the fork between coincidences.

Immediately in front of the mirror M_1 was placed a smaller mirror M_2 , about 1×3 cm. This mirror was supported on horizontal bearings g, h , of fine needle points in glass centers, drawn from thick-walled capillary tubing. These latter were mounted on an adjustable arm c , projecting from the framework that supported the pendulum. The planes of both mirrors when at rest were vertical and perpendicular to the plane of vibration of the pendulum.

Projecting downward from M_2 is a fine cambric needle d , which engages at right angles a second similar needle f , attached to the arm a by a short wire e . These needles cross about two centimeters below the axis of M_2 . Calling this distance r and the distance from the crossing of the needles to the pendulum bearings R , it is evident that M_2 will move through R/r times as large an angle as the pendulum. M_2 was so balanced as to cause the needles always to be in light contact.

One meter from the mirrors was placed the flash apparatus. The flash was furnished by an H-shaped Geissler tube illuminated by an induction coil whose primary circuit was controlled by the pendulum of the laboratory clock. Immediately above the tube was placed the observing telescope. When the pendulum was at rest the mirrors were so adjusted as to throw images of the capillary part of the tube upon the horizontal cross hair of the telescope. The two images were made to overlap throughout a small distance

for greater ease in determining a coincidence. If now the pendulum were swung at full amplitude the flashes in each mirror would divide into two sets, and these would step along the mirrors in opposite directions. Figs. 2 and 3 represent the typical appearance of the two sets of flashes. Each flash was normally double, the light and heavy parts being due respectively to the closing and opening of the primary circuit by the relay attached to the clock. M_2 , having the greater angular velocity, separated the parts of its flash images and also the successive flashes correspondingly more than M_1 . Assuming the flashes to move in the direction indicated by the arrows, if those marked 1 belonging to the downwardly moving set are observed at a given beat of the clock, then those marked 2, belonging to the upwardly moving set, will be seen at the next beat one second later, and so on as numbered.

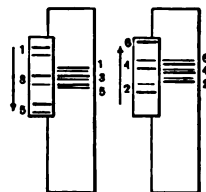


Fig. 2. Fig. 3.

If the period of the free pendulum be greater than that of the clock, then the distance 1 to 3 represents the amount by which the pendulum fails to reach the position it had two seconds before. Under these conditions the set of flashes in Fig. 2 occur while the pendulum is moving towards the telescope, and those in Fig. 3, while it is moving away. Not more than six or eight consecutive flashes could be observed in M_2 , owing to its shortness and comparatively great angular velocity; the flashes in M_1 , however, remained visible all the time. Flash 7 in M_2 , as figured above, would of course be out of the field of the telescope. At the moment of coincidence the flashes in corresponding sets again form a straight line.

Only those flashes belonging, say, to downwardly moving sets were observed, and only every other coincidence of these, in order to take the coincidences with the two pendulums going in the same direction, and thereby secure from the clock the two-second interval elapsing between two successive passages through the mercury globule in the same direction. The steps of the flashes in M_2 were large even for very small amplitudes of the pendulum. Therefore the coincidences were never ambiguous as they sometimes were with the old arrangement of mirrors.¹ By extending the time of

¹ G. R. Putnam, U. S. Coast and Geod. Surv., 1894, app. 1.

observation over even so short a period as half an hour, the possible error would appear only in the seventh place of decimals, and even that error would be due to irregularities in the rate of the clock. Usually, however, the pendulum would be started, two or more coincidences observed, and then observations completed on the fork. After an hour or so coincidences were again observed, and the periods of the pendulum and the fork then computed. Each time the quality of the coincidence was noticed and drawn. For accurate work only those coincidences equally good were used. A typical coincidence is shown in Fig. 2, flashes 3. The coincidence was always observed between the heavier parts of the flash images, although the lighter parts had also a valuable function. If, as the flashes moved along the mirrors, the lighter parts *preceded*, then the period of the free pendulum was known to be greater than that of the clock. Now, during an actual observation, if the pendulum's period was greater than one second, and if two successive flash images in M_1 had their lighter parts pointed towards each other, a coincidence was approaching, and the proximity of the two flashes would tell how soon the coincidence was to be expected. Likewise, at the moment of coincidence, all four sets of flashes appearing, the operator had to observe only those flashes whose lighter parts were below, and which consequently belonged to the downwardly moving sets. Thus the flashes labeled themselves, the time spent in actual observation was greatly shortened, and the mind of the operator was left comparatively free at the moment of coincidence, so that the quality of the same could be observed with much greater accuracy.

The energy taken from the pendulum by this set of mirrors was very small, as may readily be seen from the curves of Fig. 4, showing the decrease in amplitude with the time. The upper curve is for the perfectly free pendulum, the lower for the pendulum with both mirror systems in action.

The apparatus for comparing the fork with the pendulum consists of four parts: The flash system, the observing telescope, the starting mechanism, and the arrangements for heating.

1. *The Flash System.*—The periodic short flash of light by means of which the fork was compared stroboscopically with the pendu-

lum, was furnished by a mirror system similar to that described by Reed. In order, however, to reduce as far as possible the friction and moment of inertia of the moving system, a smaller mirror 20 mm. in diameter was used, and the pivot bearings were made from needle points set in shallow glass cups drawn from capillary tubing. The horizontal arm of the rotating mirror crossed the needle projecting from the pendulum, at a distance of five millimeters from the axis of the mirror when the pendulum was at rest.

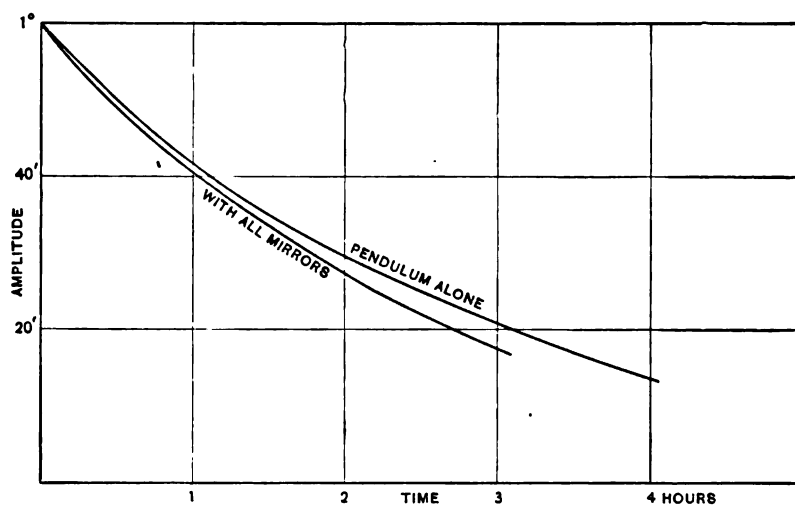


Fig. 4.

The curves of Fig. 4 showing the decrease in the amplitude of the pendulum, with and without the mirror systems, give some idea of the small amount of energy absorbed by the two mirror systems.

Instead of the slit illuminated by a small lamp, as described by Professor Reed, a short straight section of the filament of an incandescent lamp was used. By screening off the light from other portions of the lamp, the flash in the telescope was of sufficient intensity to allow the room to be comfortably lighted during observations.

2 *The Observing Telescope.*—The image of the filament formed by an achromatic lens was thrown by the rotating mirror upon the slit of an Abbé telescope. After passing through a totally reflecting

prism and an adjustable slit in the focal plane of the objective, the light leaves the telescope in parallel rays, falls normally upon the mirror carried upon one prong of the fork to be rated, and returns upon its path to form an image of the slit in the focal plane of the telescope. Fig. 5 shows the arrangement of the optical surfaces

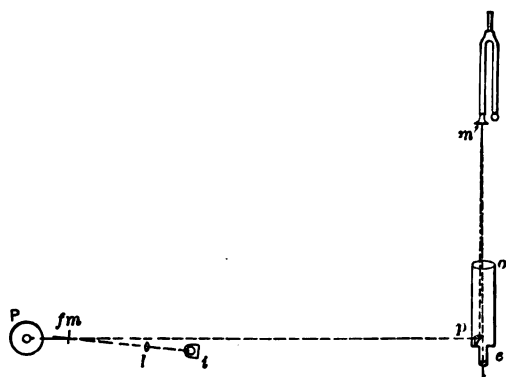


Fig. 5.

and the path of the light from the incandescent lamp i to the observer's eye at e ; l is the projecting lens; fm , the flash mirror; P , the pendulum; m , the fork mirror; p , the reflecting prism; and e , the eye-piece of the observing telescope.

The image of the slit with constant illumination appeared as a broad rectangular band of light when the fork was vibrating, and as a bright slit when the fork was at rest, thus furnishing a means of regulating and measuring the amplitude of the fork. In the focal plane of the eye-piece of the telescope were two cross hairs. One marked the center of the field, and the other, 2 mm. to one side, marked the outer limit of amplitude at which observations on the fork were begun.

3 *The Starting Mechanism.*—Owing to the necessity of enclosing the fork for heating it, some special means of putting it in vibration was necessary. Two methods were used in the preliminary work:

check upon the other. The wedge method had two defects: If the fork were already vibrating the wedge would, of course, bring it to rest, while by a skillful use of the second method the operator could add to an already existing amplitude with approximately any desired change of phase. This gave more control over the amplitude at the beginning of an observed cycle. Difficulty would also have been found in applying the wedge method to forks of different dimensions.

In the second method the electromagnet *m* (Fig. 6), arranged to slide back and forth between the prongs of the fork, was connected in series with an outside electric reed capable of being tuned through a considerable range. The reed was a square steel bar, $500 \times 6.4 \times 6.4$ mm., supported on a wooden base by two firm brass uprights with squared holes to fit. The base was supported free from the operating table by two large rubber stoppers, sometimes one at each end and sometimes the rear one moved forward twenty centimeters, according to the method used in tuning the reed. Neither part of the platinum contact was fastened to the reed. The moving part was a thin sheet of platinum, 6×25 mm., one end resting on the reed and the other clamped to a universally adjustable brass pillar. The foil would follow the reed perfectly without interfering with its frequency or with tuning. At the other end of the base was an upright supporting a disk threaded to receive a screw, the head of which was fastened to the end of the reed. By turning the disk the free reed was shortened or lengthened, while vibrating, until the fork would spread. Other methods for obtaining exact unison were frequently used, especially with the more rapidly vibrating forks. One was with the rear supporting stopper moved forward. Then one pressed on the rear of the base, or raised the rear end entirely off of its support and held the base at a suitable angle. It was found that an elevation of 45° would change the frequency by several whole vibrations per second. This change was very gradual at first, thus making the method as sensitive and accurate as one could wish.

The electromagnet *m* and its sliding ways *w* are shown in Fig. 6. The magnet was wound with bare wire, the layers being separated by thin asbestos sheet. One end of the wire was fastened to

the brass magnet spool, thereby making connection through the side wings s , sliding ways, and cast-iron heating shell S , with one end of the circuit of the electric reed. The other end of the magnet winding was fastened to the central draw rod d . This rod was insulated by asbestos from the magnet to which it was fastened, and from the heating shell where it passed out through the front cap c_1 , just below and at one side of the telescope. It served the double function of moving the magnet back and forth on the ways, and of forming the second terminal of the magnet. The circuit, then, included in series the magnet, a key, a resistance box, the electric reed and one or two storage cells.

The key was closed, the reed started, and the magnet drawn forward between the prongs and kept there until the fork was suitably

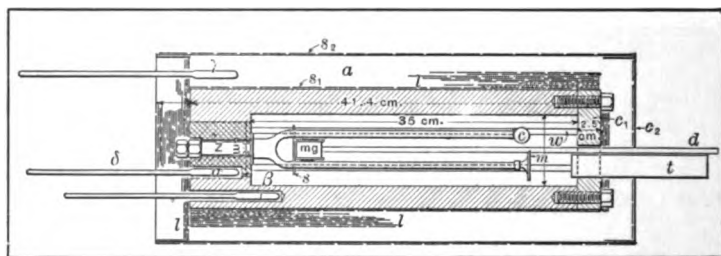


Fig. 6.

spread. Then the key was opened, the magnet pushed back near the base of the fork, and the cycle observed. The dragging effect of the magnet on the frequency of the fork when the circuit was open was closely observed. The maximum effect, when the magnet was at the outer limit, was found to be in the third decimal place. With the magnet half way back the effect was very small, while, when the magnet was pushed back to the limit between the prongs, no effect whatever could be observed.

4. *The Heating Arrangements.*—Fig. 6 shows a longitudinal section through the center of the mounting of the fork. The cast-iron cylinder S , including the cap c_1 , was 44 cm. long, 12.5 cm. outside and 7.5 cm. inside diameter. The side walls were therefore 2.5 cm. thick, the cap was of the same thickness, and the rear end wall had an average thickness of 6.5 cm. The zinc plug Z , in which the fork was mounted so as to insulate it magnetically, was

5.8 cm. in diameter. Such a mass of metal and such large metallic contact with the fork made uniform temperature of the fork easily and quickly obtainable. The total mass of metal to be heated was about twenty-five kilos.

Passing through the cap, immediately in front of the fork mirror, was a heavy-walled brass tube t , 2.5 cm. in diameter and 15 cm. long. The outer end carried a small glass plate set slightly inclined to the plane of the mirror in order to avoid troublesome reflections from its surfaces. Through this tube the observations on the cycle were made. The arrangement of the fork and magnet in the shell is sufficiently clear from the figure.

Surrounding the shell S_1 was a layer of thin asbestos, and over this was wound a layer, of number twenty-eight, tinned, iron wire, seven turns per centimeter. Then came a second layer of asbestos and a layer of wire wound in the reverse direction and connected in parallel with the first layer. The total resistance, cold, was about sixty ohms. Over all was wrapped nine more layers of asbestos. The cap was protected with several layers of asbestos, while the rear wall, where the thermometers were, and where the fork made metallic contact with the shell, was covered with asbestos disks to the depth of four centimeters. The outer shell S_2 was made of sheet iron lined with asbestos and supported 15 cm. above the table on six legs. An air space a , of about four centimeters, was left all around between the two shells, excepting at the rear. The inner shell rested on two curved blocks of wood. In the top of the outer shell were two asbestos-packed openings to admit the terminals of the heating circuit. The head was perforated to admit the observation-tube and the draw-rod. Two thermometer holes were drilled in the rear wall of the heating cylinder. One, α , was in the outer edge of the zinc plug, $1\frac{1}{4}$ cm. in diameter and 5 cm. deep. The other, β , was 2.5 cm. out from this, $1\frac{1}{4} \times 12$ cm., and extended into the side wall. The thermometer bulbs were packed into these holes with tinfoil. The reading of the inner thermometer was taken as the temperature of the fork. Six thermometers in all were used: Two in the shell, α and β , one in the air space, γ , one just outside, δ , near the stem of α , one for the room temperature near by, ϵ , and one near the pendulum, ξ . The thermometer in

the shell wall always read a little higher than that in the zinc plug.

The perfection of the heat insulation was shown by the facts that the temperature of the air space never rose much above 50° C., even when that of the fork was near 200°; and that thermometer, δ , never read more than 2° or 3° higher than thermometers ϵ and ξ . The hand held near the outer shell felt but little heat, even when working at high temperatures and after the high temperature had been maintained for several hours.

The source of heat was a 220-volt, direct current. The maximum current used was about three amperes. The method of heating was as follows: Suppose it were required to raise the temperature about 10°. The largest current possible was turned on. In from 5 to 20 minutes, after the reading of the thermometers had risen from 5° to 8°, depending on whether the original temperature was low or high, the current was shut off. In 5 to 10 minutes more the temperature would have risen the required 10° and become nearly uniform. Next a comparatively small current, to compensate for radiation losses, was turned on and adjusted frequently during the next half hour or until the readings of the thermometers remained constant, and the constancy of the length of the cycle showed that the fork had assumed the temperature of the zinc plug. As an example of the rate at which the temperature became constant the following table is taken from the record of one day's run:

Time.	Temperature		Amperes.	July 21, 1899.
	α	β		
9:45	38.7°	40°	1.84	Heating current.
9:53	41.6	45.3	off	
10:03	44.42	46.8	0.452	Compensating current.
10:08	44.6	46.6	0.440	
10:13	44.66	46.7	"	
10:21	44.68	46.6	"	
10:32	44.66	46.6	"	

Frequently at untried temperatures the compensating current would have to be changed several times before the temperature could be held constant. Working in this way, under favorable conditions observations could be made at six different temperatures in one day, beginning at 9:00 A. M. and finishing at midnight.

The following extract from the note-book is given as an example of a complete determination :

Time.	Images.	Interval.	Pendulum Amplitude.	Temperature.	The Cycle increases with P , $\therefore c$ is negative.
2:47:43	— —	2:40	15 mm.	45.5°	$n = 96$
2:50:23	— —				<i>Fork Cycles</i> 21 + ; 22 - ; 21 + ; 43 ; 65 - $c = -2/43 = -0.046511$
3:22:19	— —	34:36			$N = 95.953489$
2040	— —				$N = 1.0129096$
2014					$= 94.7305$
$\frac{2040}{2014} = 1.0129096 = P$					

Time.	Current Amperes.	Temperature.	
		α	β
1 : 56	3.080	23.6°	24°
2 : 12	off	42.2	43
2 : 15	0.620	43.4	47
2 : 22	0.600	44.3	47.4
2 : 27	0.592	44.7	47.5
2 : 37	0.440	45.3	48
2 : 42	"	45.5	"
2 : 52	"	"	47.9
3 : 02	"	"	47.6

The pendulum lost two swings in 2 minutes and 40 seconds. It would therefore lose 26 in 34 minutes and 36 seconds. Hence $\frac{2040}{2014}$ gave the period. The recorded fork cycles were observed just before the last observation on the pendulum. By reference to the table of times and temperatures and from observations in addition to those given above, it was seen that the temperature of the fork, as shown by thermometer α , became constant, 45.5°, at 2.42. The fact that the cycle increased with P was determined before the first pendulum observation by changing slightly the pendulum load, the table showing that the temperature of the fork was then nearly the same as that at which the other observations were made. From previous data n was known to be 96, so that the formula gave $(96 - 2/43)/1.0129096 = 94.7305$ as the frequency of sol_{1a} at 45.5°. Immediately after taking the fork cycle the maximum current

was again turned on and allowed to heat the cylinder to the next desired temperature while the final observations on the pendulum were made, and so on.

To determine the effect of temperature on the damping of the vibrations a third cross-hair was inserted in the eye-piece of the telescope between the other two. The distances from the cross-hairs to the central one were measured by means of a micrometer cathetometer. From the outermost hair to the central one was 2.038 mm.; from the second to the center 0.757 mm. This with the focal length of the objective ϕ , and the distance from the fork mirror to the lens, gave the actual amplitude of the fork. The extreme amplitude for ut_1 was about one millimeter, and that of the other forks was in proportion to their length.

The magnification of the eye-piece was about 13. The fork was spread at the given temperature and the time noted for the edge of the rectangular illumination of the field to fall in from the outermost cross-hair to the second one, and again from the outermost to a point half way between the other two. This latter distance represented the extreme decrease in amplitude permissible in the fork before the cycle flashes would overlap, and the corresponding time was the longest time a set of cycles could run in any one observation. When this period was short the cycle length was adjusted so as to nearly equal it. When it was long the cycle was made some submultiple of it.

PART II. RESULTS OF OBSERVATIONS.

Four forks were used. They formed a set made by König some twenty years ago, of uniform shape and cross-section and differing only in length. They were marked: one, ut_1 , 128 *v. s.*, two sol_1 , 192 *v. s.*, and one ut_2 , 256 *v. s.* The principal dimensions were as follows:

Breadth of prong	14.5 mm.
Thickness of prong	6.0 mm.
Between prongs	26.0 mm.

When the forks were rated one prong carried a nickel-plated steel mirror 30 mm. in diameter and extending out 16 mm. from the end of the prong. The other prong carried a brass ball as a counterpoise. The mass of ball and of mirror was 20 grams each.

The first fork used was the ut_1 . It was used for all the preliminary work from January 11 to June 20, 1899. This preliminary period included 45 complete determinations of the frequency at temperatures varying from 16.3° to 26.3° ; 22 in the open air, and the remainder with the fork mounted in the cast-iron cylinder but before the addition of any asbestos or wire. In most of the determinations made in the open air the fork was started with a bow. Besides these many partial determinations were made in which the fork was alternately bowed and electrically started. The close agreement of all the results obtained showed that the disturbing effect of the electric starter when used as indicated above, was much smaller than errors due to variations in the rate of the clock.

From the middle of May to the middle of June the heating arrangements were added and perfected. From June 20 to July 1, the 37 determinations with ut_1 recorded in Table I. were made. This table also shows the results of five determinations with one of the sol_1 forks, hereafter to be designated as sol_{1a} .

Table II. shows the effect of temperature upon the damping of the fork. In the first column under each date are given the temperatures; in the columns marked y_1 are the times required for the amplitude to fall from the outer cross-hair to the second one; and in the columns marked y_2 are the times required for the amplitude to fall from the outer cross-hair to a point half way between the other two. The particular fork used on each date is designated at the head of the appropriate column by ut_1 , sol_{1a} , sol_{1b} , and ut_2 respectively. ut_1 and sol_{1a} were the forks whose frequencies were determined.

TABLE I.

Number. Fork n_1 .	Date, 1899.	Period of Pendulum.	Cycle Length.	Frequency of Fork.	Temper- ature.
1	June 20	1.0074953	16.0	64.4544	37.2
2	" "	1.0084269	14.0	64.3859	47.0
3	" 21	1.0055866	10.0	64.5394	25.1
4	" "	1.0070281	17.6	64.4900	32.4
5	" "	1.1079523	16.25	64.4263	41.2
6	" "	1.0074813	20.5	64.4708	35.35
7	" 22	1.0056054	10.6	64.5438	24.38
8	" "	1.0079545	13.8	64.4151	43.19
9	" "	1.0084122	15.5	64.3939	46.0
10	" "	1.0074836	12.5	64.4377	39.84
11	" "	1.0074627	18.0	64.4633	36.35
12	" "	1.0072522	20.5	64.4835	33.5
13	" 23	1.0055896	9.0	64.5282	26.3
14	" "	1.0074844	15.0	64.4509	38.65
15	" "	1.0084112	11.0	64.3676	49.6
16	" "	1.0093777	13.0	64.3199	54.75
17	" "	1.0098328	14.0	64.2963	57.75
18	" "	1.0102929	13.7	64.2655	61.55
19	" 28	1.0103016	10.0	64.2382	66.25
20	" "	1.0112069	11.6	64.1943	71.55
21	" "	1.0122180	13.5	64.1422	77.35
22	" "	1.0126705	10.5	64.0926	83.3
23	" 29	1.0140575	22.0	64.0541	88.35
24	" "	1.0141271	11.0	64.0048	94.05
25	" "	1.0141079	7.0	63.9548	100.41
26	" "	1.0159680	11.6	63.8945	107.25
27	" 30	1.0170793	12.4	63.8292	114.95
28	" "	1.0055918	7.6	63.7749	121.4
29	" "	1.0055918	15.0	63.7104	128.7
30	" "	1.0070211	10.75	63.6461	135.9
31	" "	1.0079448	11.2	63.5841	142.7
32	" "	1.0088731	12.4	63.5170	150.0
33	July 1	1.0097767	12.8	63.4577	157.0
34	" "	1.0107527	24.5	63.3595	167.3
35	" "	1.0117302	23.0	63.3009	176.3
36	" "	1.0135922	16.0	63.2034	185.8
37	" "	1.0155102	15.0	63.0881	197.0
Fork					
$sol_1(a)$					
1	" 20	1.0102612	31.0	94.9929	23.6
2	" "	1.0129096	21.5	94.7305	45.5
3	" "	1.0159744	11.5	94.4049	75.75
4	" "	1.0097943	15.5	94.0146	109.35
5	" "	1.0152526	20.0	93.5334	148.1

TABLE II.

<i>ut</i> ₁ , July 3.			<i>sol</i> _{1a} , July 5.			<i>sol</i> _{1b} , July 10.			<i>ut</i> ₂ , July 11.		
Temp.	γ_1	γ_2	Temp.	γ_1	γ_2	Temp.	γ_1	γ_2	Temp.	γ_1	γ_2
27.6	31	56	53.0	25	45	22.2	38	66	29.7	28	50
44.5	15	26	68.5	21	35	32.6	34	58	37.2	25	44
52.6	11	20	86.7	18	30	40.3	29	48	34	27	47
63.5	8	14	98	21	36	49.6	23	38	56	18	31
73.5	7	13	76	15	25	63.0	18	31	72.2	13	23
58.4	8½	16	110	21	36	72.5	15	25	85	11	19
88.6	11	19	124.5	24	44	83.3	14½	24½	100	13	23
95.2	14	24	137.5	23	41	95	16	26	122	17½	30
107.3	22	40	147.1	22	38	117.5	17	28	143.5	18	31
116.0	27	50	157.8	21	36	135.6	16	26	166.5	16	28
125.4	32	58	166.2	21	36	149.6	14	22	177	15	26
136.7	33	60	176.1	20	35	159.6	13	20	<i>ut</i> ₁ , Jan. 30		
144.6	34	62	185.8	20	34	173	11	18	to Feb. 9, 1900.		
157.2	35	64	201	19	31	187.8	10	15	—9°	56	98
166.	36	68	212.9	18	30	202.4	10	15½	—2.3	55	96
177.3	33	62	220	18	30	219.5	9½	15	24.33	45	75
186.	32	60	226	17	27	227	9	15	49.1	18	30
196.2	31	58	232	15	25	<i>sol</i> _{1b} , July 12.			83.3	11	18
209.	30	56	242	15	25	23.5	42	70	117.7	10	20
<i>ut</i> ₁ , July 4.			247	15	25	26.6	35	55	152.3	22	44
27.4	32	54	256	15	25	35.6	26	42	188.6	30	52
40.4	19	33	<i>sol</i> _{1a} , July 7.			49	15½	25	232.2	26	46
48.1	14	22	24.4	42	70	62.3	11	18	256.8	28	50
51.0	12	21	35.4	36	60	80	9	14			
64.0	9	16	42.8	30	50	99.3	10	16			
72.8	8	13	51	23	38	119	12	19			
83.5	9	17	64.1	16	27	139.5	12	18			
98.7	15	25	71	13	23	146	10	16			
<i>sol</i> _{1a} , July 20.			83.5	13	22	162	10	16½			
23.6	46	80	89	15	25	184	11½	19			
45.5	30	48	100.5	19	32	200	12	20			
75.7	13	21	114.2	26	42						
109.4	28	48	120	31	51						
148.1	30	51	132.7	33	54						
			143.3	33	54						
			152	31	52						
			163.8	30	50						
			171.6	29	48						
			185.7	26	43						
			210.5	21	35						
			218.5	19	31						

In all cases the fork carried the usual mirror and counterpoise just as when the frequency was being observed, excepting on July 4, 1899. When the phenomena of the effect of temperature on damping was first noticed with ut_1 , and before the temperatures had been reached at which longer time of motion began, it was thought that differences in the loads on the two prongs might be the cause. Accordingly two light loads were made exactly alike except that one was polished to a mirror surface. The values under July 4, 1899, show results practically the same as before.

The values for sol_{1a} are under three dates. On July 5, 1899, the recovery of longer time of motion was not very marked, and other irregularities appear. This was the first time this fork had been heated. But on July 7 and 20, under the second and third heatings respectively, sol_{1a} behaved practically as ut_1 , *i. e.*, with

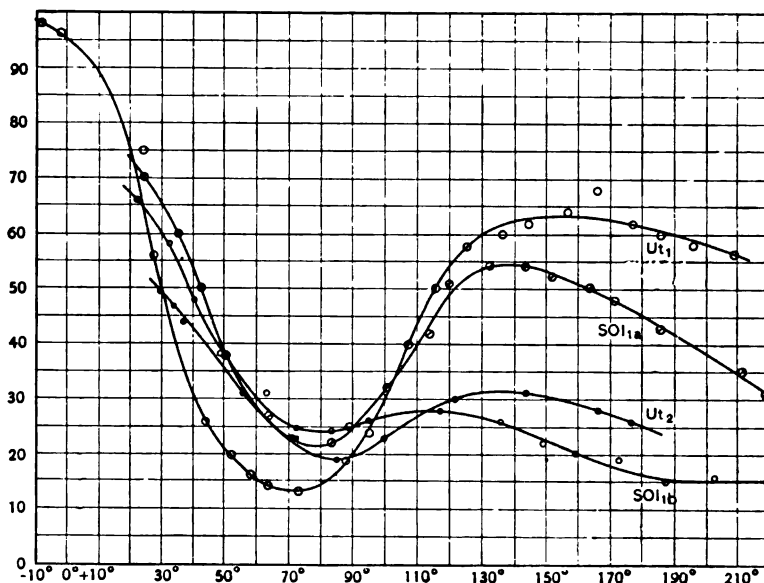


Fig. 7.

a large and continuous decrease in duration of motion followed by almost complete recovery at higher temperatures. July 10 sol_{1b} on being heated for the first time, behaved somewhat as sol_{1a} under similar conditions, but with even smaller recovery. On the second heating, July 12, the decrease was almost as much as that

of ut_1 . The recovery, however, was still very small and irregular, unlike sol_{1a} on its second heating. But a third heating in January, 1900, gave results with sol_{1b} very similar to those with ut_1 . Lastly, ut_2 , when heated for the first time on July 11, showed the same characteristics as ut_1 though in a less degree.

Reference to the curves of Fig. 7 will make these points more evident. The times of motion as given in the columns marked y_2 are plotted as ordinates and the temperatures as abscissæ. The data of July 3 gave the curve marked ut_1 . The second heating of sol_{1a} , July 7 gave the curve marked sol_{1a} , while the first heatings of sol_{1b} and ut_2 on July 10 and 11, respectively, gave the curves labelled sol_{1b} and ut_2 . These latter curves are typical of first heatings, the first heating of sol_{1a} giving very similar results; while the curves labelled ut_1 and sol_{1a} seem to be normal curves, resulting from repeated heatings, and representing, perhaps, a more stable condition of the forks.

To recapitulate: *All the forks agree in having a minimum duration of motion, and this occurs at about the same temperature, $80^\circ C$.* Repeated heatings seem to increase the effect, *i. e.*, the decrease in time of motion becomes more marked and the recovery becomes more complete. The phenomenon seems to be entirely independent of the effect of temperature on the frequency, the frequency curves being nearly linear throughout.

Thermometers. — The seven thermometers used were frequently compared. Three of them constituted a set of normal thermometers made by W. Haak, of Jena, September, 1897, and standardized by the *Reichsanstalt* in 1898. They were graduated in fifths of a degree, and had the following ranges and numbers:

No. 11021	+ 20° to + 105°
No. 11022	— 5° “ + 105°
No. 11023	— 90° “ + 210°

These three were used as standards for the others.

Rate of the Clock. — The clock with which the pendulum was compared was a laboratory clock of fair accuracy. When the

was found to be losing quite uniformly one second in ten hours. Therefore to obtain the absolute values of the frequency of ut_1 , $\frac{1}{86400}$ of its apparent frequency should be subtracted therefrom. When its observed frequency was 64.8000 its real frequency was 64.7982. However, the rate of the clock being practically uniform, the relative frequencies would remain as given in the tables. In any case the form of the curves, the real object of the research, would remain unaltered.

Limits of Accuracy. — The method, as finally perfected, eliminated in large measure such errors of observation as might have affected the desired accuracy, *i. e.*, the third or fourth decimal place. The most probable remaining sources of error were: variations in the rate of the clock, uncertainties in determining the length of the cycle and the temperature of the fork. The method of timing the pendulum admitted of no error greater than 0.00001 provided the rate of the clock remained constant. Now to produce an error of that magnitude the rate of the clock would have to vary undetected by one second in ten hours. But such an error in the period of the free pendulum would produce an error of less than one in the third decimal place in the frequency of the fork. There remains one other way, however, in which the period of the pendulum might be in error. Let us suppose the most unfavorable case that might arise in the observation of the last coincidence and also assume conditions under which it would give the greatest error. Suppose the flash in mirror M_2 that was taken as the time of coincidence was as far above the flash in mirror M_1 as the next flash, of the same system two seconds later, was below it, the approximate double-coincidence period to be six minutes, and the last coincidence to be the sixth. Then the true period of the pendulum would be 1.0055834 instead of 1.0055866, as it would have been if the coincidence had been exact. The resulting error, 0.0000032 second, would be much less than the above-mentioned error of one in the fifth place and so much the more negligible. However, by observing the quality of the coincidences, and always testing those for first and last that were alike, this source of error was removed within the limits assigned.

The possible error in reading the thermometers was, from the nature of the instruments used, $\pm 0.025^\circ$. But even here an error

as large as 0.1° would affect only the fourth place in the frequency of the fork. Whether or not the reading of the thermometer *a* was the exact temperature of the fork could not be determined with absolute certainty. But since the fork and thermometer both made such good metallic contact with the zinc plug, since the reading of the thermometer was kept constant until the length of the stroboscopic cycle between fork and pendulum became constant, and since a variation of 0.1° in the temperature of the fork would ordinarily change the length of the cycle by the readily detected amount of two per cent., it seems reasonable to assume the error in taking the reading of the thermometer as the temperature of the fork to be no greater than $\pm 0.1^{\circ}$ C. This would mean a possible average error of ± 0.0006 in the frequency of ut_1 .

The greatest chances for error, after irregular clock-rate, were in the observation of the fork cycle. This observation required by far the most skill; since accurate results depended to so great a degree on the ready detection and elimination of irregular workings of fork and flash system, and on the degree of fatigue of the eye. The time the flash took to pass the eye-piece slit was calculated to be approximately 0.0001 second. The time could of course be increased or decreased at pleasure, either by changing the amplitude of the pendulum or the length of the needle arm of the flash mirror, or by changing the distance from pendulum to fork. The limits of the room kept this latter distance constant at about four meters. The velocity mentioned, however, was found the best with the forks used so that the image would not be unduly spread out in the field by the motion of the fork. In fact this velocity was so great that very little spreading was observed when the fork was at full amplitude, and none at all when the amplitude had decreased so that the flashes came close together in the field; in which case three successive flashes might fall very near the cross-hair, making exact location of the coincident flash especially desirable. But very little error, then, could arise from this source.

Supposing, however, that a fractional number of flashes formed a cycle; then either the fractional part must be estimated from the position of two successive flashes on either side of the cross-hair, or the counting of the flashes must continue until the fractions have

added up to a whole number. The latter method was used whenever the fork would vibrate long enough without too great a variation in amplitude. But when the duration of motion had decreased with rising temperature to less than twenty-five seconds, the fractions had to be estimated. Suppose that in the case assumed above, where $P = 1.005600$ and $a = 20$, that an error even as large as 0.5 of a flash interval occurred in estimating the fractional part of a . Then if $a = 20$, N would be 64.5883; while if a were 20.5, N would be 64.5895, an error of but 0.001 of a vibration. However, in no case was the possible error in a as large as 0.5. Still, when the temperature was near the minimum for duration of motion the conditions for accurate work would then be the worst possible, and the error might be ± 0.5 of a flash interval if the duration of motion was very small. This would make an error of but one in the third place in decimals. This latter source of error is by far the most serious of the errors of observation. Still, only when the duration of motion becomes inconveniently small does the possible error become of much importance. Ordinarily the error due to this cause would be ± 0.0005 for ut_1 .

Recapitulating the errors due to the various causes:

(1) Irregular rate of clock	± 0.00020
(2) Timing pendulum	± 0.00005
(3) Temperature of fork	± 0.00060
(4) Error due to cycle	± 0.00050

These are probable errors when working above 100° . Between 50° and 100° error (4) might be larger. Below 50° errors (3) and (4) could be made much smaller. The total probable error then, under ordinary conditions, would be given by $\epsilon = \{ (20^2 + 5^2 + 60^2 + 50^2) \times 10^{-6} \}^{\frac{1}{2}} = \pm 0.0008$. If necessary all the errors could be limited to the fifth place in decimals for determinations near the temperature of the room.

When short-cycle lengths were used the sensitiveness of the method was sufficient to show distinct variations in the frequency in different parts of even the limited range of fork amplitude used. Also as the pendulum ran down in amplitude, the change in cycle length would show the decrease in period of the pendulum.

PART III. APPLICATION OF RESULTS.

The results in Table I. are embodied in the curves of Fig. 8. Temperatures are plotted as abscissæ and frequencies as ordinates. The curves, it will be noticed, deviate in an unmistakable way from linearity. The change in curvature per degree seems to be approximately constant. Hence $\frac{d^2N}{dt^2} = \text{a constant within the limits of observation.}$ König, working between 20° and 50° , de-

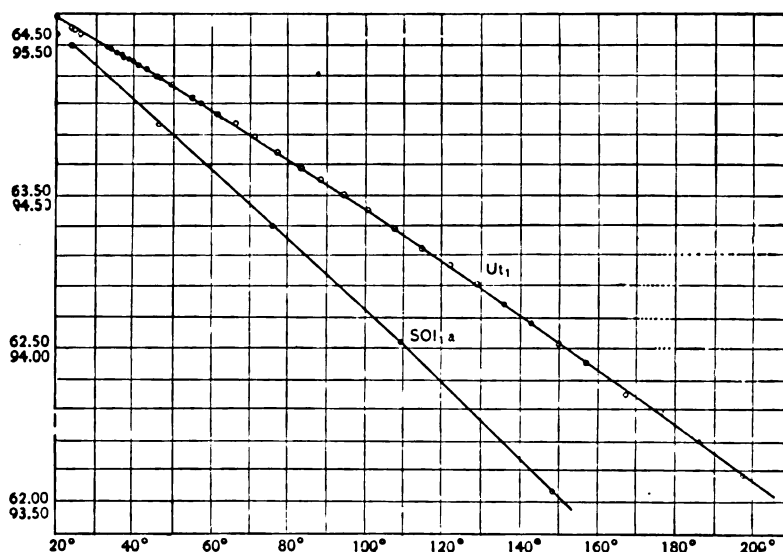


Fig. 8.

cided that the decrease in frequency decreased with higher temperatures. The above curves, however, seem to indicate that this decrease in frequency constantly *increases* with rising temperature.

Another important characteristic of the curves is that the one for *sol*_{1a} is much flatter than that for *ut*₁. The reason for this will appear when the fundamental formulæ involving relative frequencies at different temperatures are considered.

Kayser (also Mercadier) gives the formula for the frequency of a fork as follows :

$$(i) \quad N_0 = \frac{c_0 k}{l_0^2 \sqrt{D_0 \gamma_0}},$$

where k is a constant, N_0 the frequency, e_0 the thickness and l_0 the length of the prongs, D_0 the density, and γ_0 the coefficient of elasticity, all at 0°C . Parenthetically it should be stated that this elastic coefficient of Kayser, and of Wertheim, is the reciprocal of the coefficient of Mercadier, Miss Noyes and others. Therefore this coefficient will increase with rising temperature where they find their's to decrease.

The frequency at any other temperature will be given by :

$$(ii) \quad N_t = \frac{e_0(1 + \alpha t)k(1 + \alpha t)^{\frac{1}{2}}}{l_0^2(1 + \alpha t)^2 \sqrt{D_0 \gamma_t}} = \left\{ k \frac{e_0}{l_0^2 \sqrt{D_0}} \right\} \left(\frac{1 + \alpha t}{\gamma_t} \right)^{\frac{1}{2}},$$

where α is the coefficient of expansion of steel, and γ_t is the elastic coefficient at t° .

Combining (i) and (ii) we obtain

$$(iii) \quad \frac{N_t}{N_0} = \left\{ \frac{\gamma_0}{\gamma_t} (1 + \alpha t) \right\}^{\frac{1}{2}} \text{ or } N_t = N_0 \left\{ (1 + \alpha t) \frac{\gamma_0}{\gamma_t} \right\}^{\frac{1}{2}}$$

and

$$(iv) \quad \frac{\gamma_t}{\gamma_0} = \left(\frac{N_0}{N_t} \right)^2 (1 + \alpha t) \text{ or } \gamma_t = \gamma_0 (1 + \alpha t) \left(\frac{N_0}{N_t} \right)^{\frac{1}{2}}.$$

Now it will be seen from (ii), (iii) and (iv), that

1. If γ were constant N_t would be greater than N_0 . Therefore
2. The effect of temperature on the dimensions of the fork opposes that on the elasticity, as far as concerns the frequency ; and
3. The change in frequency due to the effect of temperature on the elasticity is the greater of the two, since $N_0 > N_t$.
4. Neither effect is linear.
5. The total effect is not linear but increases with the temperature.
6. The effect of temperature on γ increases with the temperature.
7. The total effect is proportional to the first power of N .
8. Since the dimensions of the fork appear as constants in (ii), the coefficient of the total effect is independent of the ratio of e to l . i. e., independent of the dimensions of the fork.

Kayser says that the total effect is linear, from 0° to 30° , and that the effect is proportional to the square root of the frequency.

König finds the total effect to be non-linear, and that it is proportional to the frequency but that the coefficient decreases with rising temperature. Pierpaoli reports results in these respects similar to those of König. The above ten conclusions from the formulæ universally accepted agree with similar conclusions from the data accumulated in this research.

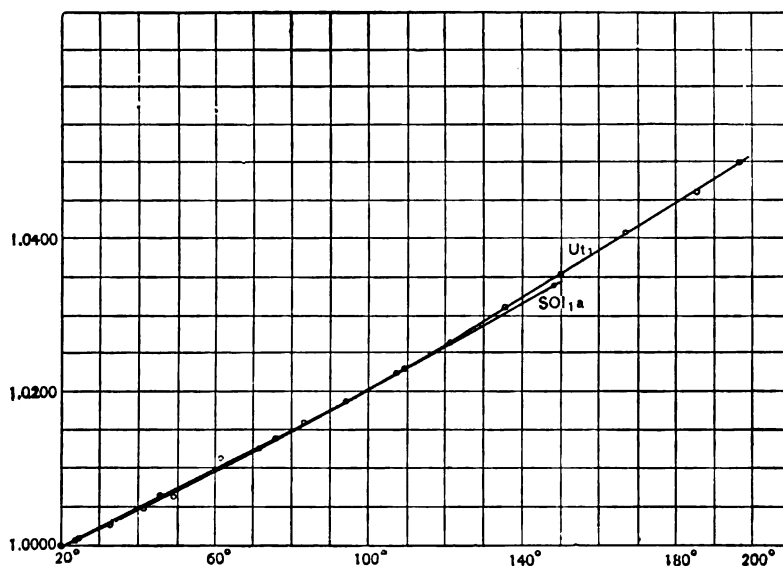


Fig. 9.

The curves in Fig. 9 represent the variations of γ with temperature. In each case γ_{20} was taken as unity and the ratio $\frac{\gamma_t}{\gamma_{20}}$ calculated by means of formula (iv) and the data in Table I. Table III. gives the calculated ratios.

The curves differ slightly for the two forks. Both were probably from the same steel but had doubtless passed through experiences sufficiently different to have changed γ permanently by a small amount. Both however show decided convexity towards the axis of temperatures, upholding conclusion (6). Still the departure from linearity is small, and especially since γ_t enters equation (iii) under the radical, the error in N_t made by assuming γ_t linear would

TABLE III.

Temperature.	$ut_1, \gamma_t + \gamma_{20}$	Temperature.	$20t_{12}, \gamma_t + \gamma_{20}$
20.00	1.0000	20.00	1.0000
24.38	1.0010	23.60	1.0008
32.40	1.0028	45.50	1.0066
41.20	1.0048	75.75	1.0139
49.60	1.0065	109.35	1.0227
61.55	1.0101	148.10	1.0337
71.55	1.0125		
83.30	1.0158		
94.05	1.0188		
107.25	1.0224		
121.40	1.0264		
135.90	1.0307		
150.00	1.0351		
167.30	1.0404		
185.80	1.0458		
197.00	1.0498		

be very small for a range of 40° . Accordingly formula (iii) may be written :

$$(v) \quad N_t = N_{20} \left\{ \frac{1 + \alpha(t - 20)}{1 + \beta(t - 20)} \right\}^{\frac{1}{2}}, \text{ where } \beta = \frac{\gamma_t - \gamma_{20}}{t - 20}$$

This expression differs in form from those usually given ; such as :

$$N_t = N_0(1 - \alpha t) \text{ or } N_t = N_0(1 - \alpha t + \beta t^2);$$

and is applicable not only to the forks experimented with, but is equally true for all forks, given the two coefficients of the material of the fork. The only assumption made is that the two coefficients, α and β , are constant over a desired small range of temperature. However for larger ranges and more accurate results, the fact that γ_t is not linear must be taken into account. Then the formula becomes

$$(vi) \quad N_t = N_{20} \left\{ \frac{1 + \alpha(t - 20)}{1 + \beta(t - 20) + \delta(t - 20)^2} \right\}^{\frac{1}{2}}$$

The values calculated for ut_1 were :

$$\alpha = 0.0000112 \text{ (Kayser)}$$

$$\beta = 0.00023$$

$$\delta = 0.0000003$$

and for sol_{1a} .

$$\begin{aligned}\beta &= 0.000238 \\ \delta &= 0.00000019\end{aligned}$$

Table IV. gives the data from which these results were obtained and also the computed values of N_t for sol_{1a} , using (vi). The computed values for ut_1 were obtained by using the conventional formula :

$$N_t = N_{20} \{1 - \alpha (t - 20) - \beta (t - 20)^2\}$$

α and β were found to be :

$$\begin{aligned}\alpha &= 0.000108 \\ \beta &= 0.00000014\end{aligned}$$

This expression of the second degree in t is not sufficient for the range of temperature covered. The computed values at the higher temperatures show a too rapid decrease. If greater accuracy over the whole range of temperature were desired, terms in values of higher powers of t would be needed. Therefore the accuracy of the observed values in Table IV. is not to be judged by the magnitude of the values in the column of differences, but rather by the exactness with which the differences themselves approximate a law of increase and decrease, changing sign at 50° and 150° and varying with more or less regularity.

As before mentioned the coefficient of elasticity as used above is the reciprocal of that of Mercadier. Taking the reciprocals of the above ratio values at 20° and 120° , we see that the coefficient, as used by Mercadier, decreases for the 100° from 20° to 120° by 2.53 per cent. Mayer¹ found that heating from 0° to 100° lowered the modulus of cast steel 2.21 per cent., and Bessemer steel 3.08 per cent. His values were obtained by observing the frequency of vibrating bars supported at the nodes. Miss Noyes, obtaining Young's modulus of a wire by stretching, found an average decrease of 4.6 per cent. per 100° . Wertheim for 100° found about 2.8 per cent., also using a stretched wire, while Kohlrausch and Loomis, working with a torsionally vibrating wire of long period, found an average decrease of 4.6 per cent. per 100° . Thus the per cent. change for 100° of the temperature coefficient

¹ Rep. of Brit. Assoc., 573, 1894.

TABLE IV.

Frequency of ν_{11} .				Frequency of ν_{12} .			
Temperature.	Observed.	Computed.	Excess of Observed.	Temperature.	Observed.	Computed.	Diff.
20.00	64.5760			20.00	95.0300		
24.38	64.5438	64.5452	- 14	23.60	94.9929	94.9910	+ 19
25.10	64.5394	64.5401	- 7	45.50	94.7305	94.7493	-188
26.30	64.5282	64.5317	- 35	75.75	94.4049	94.4075	- 26
32.40	64.4900	64.4880	+ 20	109.35	94.0146	94.0116	+ 30
33.50	64.4835	64.4801	+ 33	148.10	93.5334	93.5393	- 59
35.35	64.4708	64.4668	+ 40				
36.35	64.4633	64.4595	+ 38				
37.20	64.4544	64.4533	+ 11				
38.65	64.4509	64.4427	+ 82				
39.84	64.4377	64.4340	+ 36				
41.20	64.4263	64.4240	+ 23				
43.19	64.4151	64.4093	+ 57				
46.00	64.3939	64.3885	+ 54				
47.00	64.3859	64.3810	+ 49				
49.60	64.3676	64.3616	+ 60				
54.75	64.3199	64.3227	- 28				
57.75	64.2963	64.2998	- 35				
61.55	64.2655	64.2706	- 51				
66.25	64.2382	64.2341	+ 41				
71.55	64.1943	64.1924	+ 19				
77.35	64.1422	64.1463	- 41				
83.30	64.0926	64.0982	- 56				
88.35	64.0541	64.0571	- 30				
94.05	64.0048	64.0099	- 52				
100.41	63.9548	63.9567	- 19				
107.25	63.8945	63.8987	- 42				
114.95	63.8292	63.8323	- 31				
121.40	63.7749	63.7758	- 7				
128.70	63.7104	63.7110	- 6				
135.90	63.6460	63.6462	- 1				
142.70	63.5841	63.5841	0				
150.00	63.5170	63.5165	+ 5				
157.00	63.4577	63.4508	+ 69				
167.30	63.3595	63.3525	+ 70				
176.30	63.3009	63.2650	+359				
185.80	63.2034	63.1710	+324				
197.00	63.0881	63.0583	+298				

of elasticity seems to vary within quite wide limits with variations

in obtaining the different forms, and in the after experimenting practically amount to changing the quality of the material. For this reason, work with a wire would hardly be a criterion for bars, and *vice versa*, even if originally made from the same quality of steel. Perhaps experiments with bars and forks would have the advantage of greater permanence of the qualities of the material to be investigated, during the experimenting.

The fact remains, however, that the change of the coefficient of elasticity with temperature for several typical kinds and forms of steel, varied only from 2.5 per cent. to 4.5 per cent. for a change of temperature of 100° .

To sum up the chief results of this research :

1. The temperature coefficient for the frequency of a fork is not linear.

(a) The change in frequency is proportional to the first power of the frequency.

(b) The coefficient is independent of the dimensions of the fork.

(c) The coefficient depends only on the coefficient of expansion, and the variation of the coefficient of elasticity with temperature.

2. The coefficient of elasticity of steel has neither a maximum nor a minimum between 100° and 200° , but varies continuously and nearly linearly.

3. A change in temperature affects the logarithmic decrement of a tuning fork in a marked degree. The decrement has a maximum at about 80° C. with minima at about 20° C. and 140° C.

In conclusion the author desires to express his thanks to Professor John O. Reed, for his interest and assistance throughout the progress of this investigation.

PHYSICAL LABORATORY, UNIVERSITY OF MICHIGAN,
ANN ARBOR, December, 1902.

GENERAL LAW FOR VAPOR PRESSURES.

BY SANFORD A. MOSS.

RAMSAY and Young announced in 1886 a general law connecting the vapor pressures for all substances.¹ This has recently been verified for the vapors of liquid krypton, xenon, argon,² etc., and may be taken as very closely representing all of the available data concerning vapor pressure. The law, as now given³ is as follows:

$$\frac{T_a}{T_b} = \frac{T'_a}{T'_b} + k(T_a - T'_a)$$

T'_a and T'_b are the absolute temperatures corresponding to one vapor pressure for two different substances and T_a and T_b are the temperatures corresponding to any other vapor pressure for the same pair of substances. k is a small number, constant for each pair of substances.

Suppose that we know the vapor pressures corresponding to two temperatures for a certain substance. Then by comparing this substance with water or some other substance whose vapor pressures are known, the entire range of vapor pressures may be computed by means of the formula. A method for doing this is given by Travers.⁴

We shall proceed to express the formula in a slightly different form and give a graphical representation of the law which it states.

We may write the formula in the form

$$\frac{1}{T_b} = \frac{1}{T_a} \frac{T'_a}{T'_b} - k \frac{T'_a}{T_a} + k$$

or

¹ Phil. Mag., Vol. XXI., 1886, p. 33.

² Phil. Trans., Vol. CXCII., A, 1901, p. 67.

³ Travers, Experimental Study of Gases, p. 236.

⁴ Experimental Study of Gases, p. 236.

$$\frac{1}{T_b} = \frac{1}{T_a} \left(\frac{T_a'}{T_b'} - kT_a' \right) + k.$$

Suppose we let T_a' and T_b' be the temperatures corresponding to some particular vapor pressure. Then the coefficient of $\frac{1}{T_a}$ will be a constant, say c . Then we have

$$\frac{1}{T_b} = c \frac{1}{T_a} + k.$$

Here T_a and T_b are any absolute temperatures of any two substances, which correspond to the same vapor pressure, and c and k are constants for each pair of substances. This is a remarkably simple form of the general vapor pressure formula.

The formula may be used as follows: Suppose that we have two values of saturation temperatures corresponding to known vapor pressures for a given substance. Let us take as the other substance water vapor or some substance whose vapor pressures are known. From the two known temperatures, the constants c and k may be determined. The temperatures for all vapor pressures for the substances in question may then be readily computed.

As an example, the relation between the saturation temperatures for water and alcohol (methyl) is

$$\frac{1}{T_w} = .9198 \frac{1}{T_a} - .0000307.$$

The constants in this formula were computed from the tables of vapor pressures given by Travers, as observed by Ramsay and Young.¹

The constant k is the same in this formula as in Ramsay and Young's original formula, and is stated to be always small.

Let us consider the formula for obtaining the saturation temperatures of any substance x from those of water vapor;

$$\frac{1}{T_x} = c \frac{1}{T_w} + k.$$

Let us put

$$- \frac{1}{T} = y \text{ and } - \frac{1}{T} = x.$$

Then

$$-x = -cy + k.$$

Since y is a function of the water vapor temperature corresponding to pressure p , y is also a function of p . The value of y for a given p may be found by making use of the ordinary tables of water vapor pressure. x is a function of the temperature of the substance z corresponding to this same vapor pressure p . The curve giving x as a function of y is evidently a straight line.

Since y is a function of the pressure p and x a function of the corresponding temperature, we may label the values of x and y with the respective values of the pressure and temperature which they represent. Then we may read off directly from the diagram the corresponding values of saturation temperature and vapor pressure.

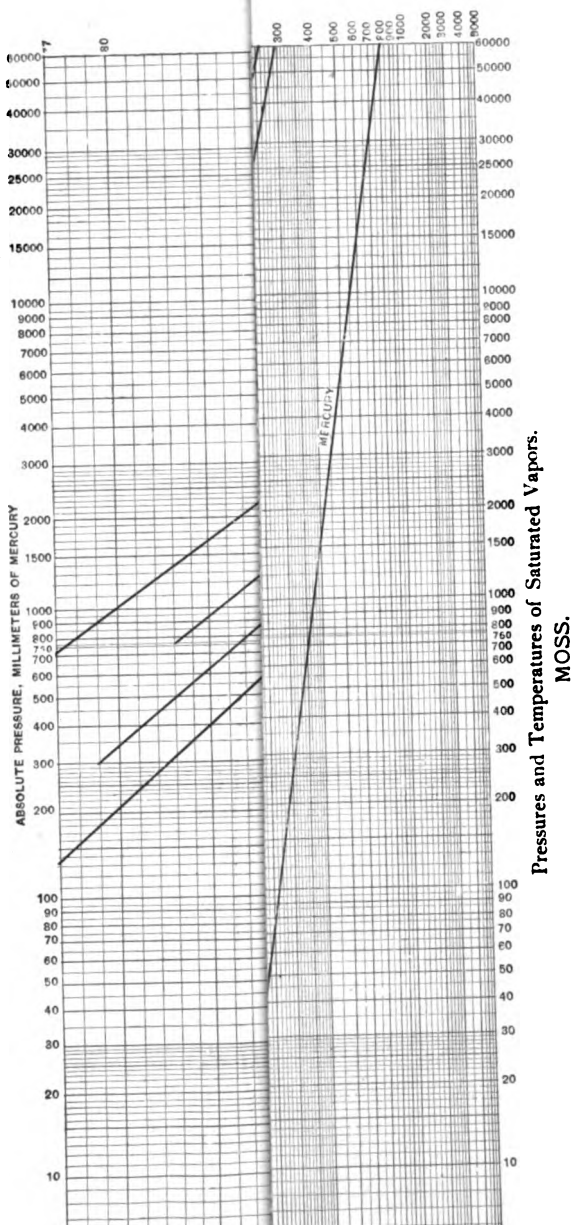
Since the same formula with different constants applies to any vapor, we may draw a straight line on the diagram for the original vapor z , to represent the pressure and temperature of any other vapor. In fact we may in the first place label the ordinates and abscissæ of the diagram without regard to any particular vapor.

Figs. 1 and 2 are diagrams constructed in this manner. The units and scales differ in the two figures as stated. These diagrams were drawn as follows.

Points on the x axis were labeled, not with the values of x which they represent, but with the values of the temperature to which the values of x correspond, according to the relation $-\frac{1}{T} = x$. Actually, integral values of the temperature were assumed and the corresponding x 's computed. These x 's were then laid off to a convenient scale and each labeled with the proper temperature.

Points on the y axis were labeled, not with the values of y but with the values of the water vapor pressure for the temperatures to which the values of y correspond according to the relation $-\frac{1}{T} = y$. Actually, integral values of the pressure were assumed, the corresponding values of the temperature found from steam tables, and the y 's corresponding to these temperatures computed. These y 's were then laid off to a convenient scale and labeled with the proper pressures.

PLATE I.
(To face page 358.)



Pressures and Temperatures of Saturated Vapors.
MOSS.

The following tables show the calculations for a few of the values of Fig. 1.

Abcissa.

Centigrade Temperature.	Absolute Temperature.	Reciprocal of Absolute Temperature.	Corresponding Value of x in Inches = $-2000 \times 1/T$.
5,000	5,273	.000190	— 0.380
1,000	1,273	.000786	— 1.572
0	273	.003663	— 7.326
	200	.005000	— 10.000
	150	.006667	— 13.334
	100	.010000	— 20.000
	77	.012987	— 25.974

Ordinates.

Pressure, mm. of Hg.	Corresponding Saturation Temperature of Water Vapor Centigrade.	Absolute Temperature.	Reciprocal of Absolute Temperature.	Corresponding Value of y in Inches referred to the Point — 17 as an Origin = $-(10,000 \times 1/T - 17)$.
60,000	295.2	568.2	.0017600	— 0.600
10,000	192.7	465.7	.0021473	— 4.473
1,000	107.9	380.9	.0026255	— 9.255
100	51.7	324.7	.0030798	— 13.798
4.5	— 0.3	272.7	.0036670	— 19.670

It will be noticed that the origin of Fig. 1 is considerably above the portion of the diagram shown. The intervening space could not be filled owing to lack of data for the pressure of water above 295.2° C.

Fig. 2 was drawn in exactly the same way except that a smaller range was covered to a larger scale.

The water vapor pressures were found as follows : From 4.5 mm. to 17,000 mm. interpolation was made in Peabody's computations from Regnault's data, which give the pressures for each degree (Tables of the Properties of Saturated Steam). From 17,000 mm. to 40,000 mm. the experimental results of Ramsay and Young as given by Traverse were used (Experimental Study of Gases, page 238). From 40,000 to 60,000 mm. the values were computed from the table of vapor pressures for methyl alcohol as given by Travers, and the formula given above connecting the temperatures for water and alcohol. The results of this computation give values

for which experimental data are lacking. However, the formula undoubtedly holds good for this range. As these values may be useful they are appended.

Saturated Water Vapor.

Pressure mm. Hg.	Temp. C.
45,000	275.9
50,000	282.9
55,000	289.3
60,000	295.2

As has been shown, the vapor pressures for any substance whatever are given by a straight line on a diagram as thus constructed. It is, of course, necessary to locate two points before the line can be drawn.

The lines shown were located from data given by Travers (Exp. Study of Gases, Chap. XVII.), and Peabody (Tables of the Properties of Saturated Steam and Other Vapors). In each case all of the values given were plotted and a straight line drawn to best represent them. In most cases the line passed almost exactly through the plotted points, thus verifying the law. In Fig. 2 the plotted points for ammonia and sulphur dioxide are indicated. It will be seen that the straight line almost exactly gives the experimentally observed data.

Some of the values from Travers did not give such close agreement. As these values were already supposed to be smoothed by Ramsay and Young's original formula they should exactly give a straight line.

The lines for hydrogen, neon and helium lie considerably to the left of the ground covered by Fig. 1. All other lines will probably fall on the figure.

Except for water vapor the lines have only been carried as far in each direction as observations have been made. Where the critical point has been determined it is indicated by a circle. There seems to be no meaning to the extension of the line beyond the critical point.

Diagrams such as those shown may be very useful in connection with observations of vapor pressure. The pressure and temperature lines may be drawn from a table of water vapor pressures, as indicated. The observed values may then be plotted and a straight

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PLATE II.
(To face page 360.)

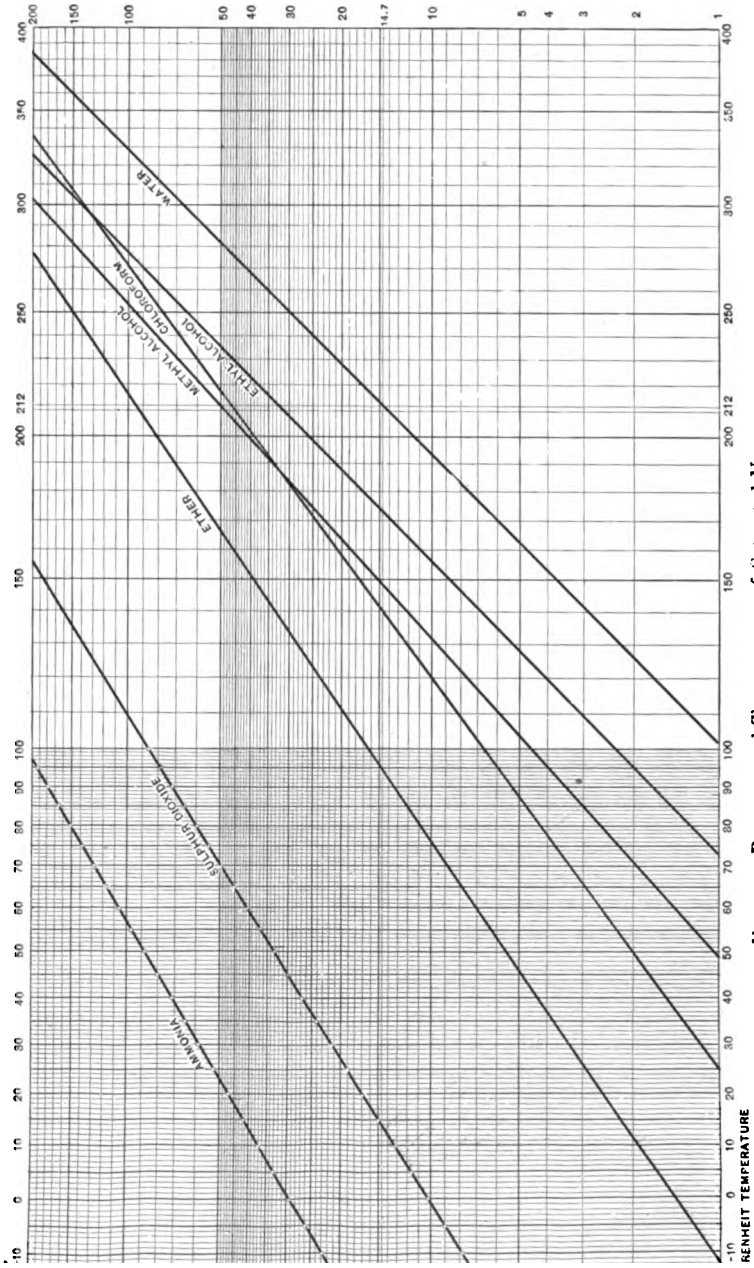


FIG. 2. Pressures and Temperatures of Saturated Vapors.

line drawn to represent them as nearly as possible. This line will then give a better approximation to the exact values than the original observations, and also enable interpolation to be made with ease. Of course two points accurately located will enable any line to be drawn and the entire range of pressures determined.

Our general formula gives the pressures of all vapors in terms of those of water vapor. Having a general formula for water vapor pressure, we could obtain a general formula for the pressures of any vapor whatever. Of the many water vapor pressure formulæ, that given by Rankine is best adapted for this purpose. Rankine's formula is applicable over the whole range of possible pressures with a single set of constants, and the values given by it agree very closely although not exactly with experimentally observed results. The formula is

$$\log p = A - \frac{B}{T_w} - \frac{C}{T_w^2}$$

where T_w is the absolute temperature of water vapor corresponding to pressure p . We may evidently write this in the form

$$\log p = l - \left(e + \frac{f}{T_w} \right)^2.$$

As we have already shown

$$\frac{1}{T_i} = \frac{c}{T_w} + k$$

where T_i is the absolute temperature of any other vapor corresponding to the pressure p . Let us find m and n such that this expression assumes the form

$$m + \frac{n}{T_i} = e + \frac{f}{T_w}.$$

Then we may substitute in the general formula for $(e + f/T_w)$ giving

$$\log p = l - \left(m + \frac{n}{T_i} \right)^2.$$

This is therefore a general formula giving the relation between the absolute saturation temperature and the corresponding vapor pressure for any vapor whatever. It is to be noticed that the constant l is the same for all substances, while m and n have particular values for every substance.

The values of the constants for water vapor have been computed by using the following three vapor pressures, taken from the final values given by Regnault (*Mémoires de l'Institut de France*, Vol. XXI.).

Centigrade Temp.	Pressure mm. Hg.
0	4.6
100	760
190	9426

Absolute zero has been taken as -273° . The constants computed on this basis are slightly different from those given by Rankine originally. The water vapor formula becomes

$$\log p = 13.7560 - \left(2.42223 + \frac{326.57}{T_w} \right)^2,$$

for pressure in mm. of Hg and temperature in degrees Centigrade.

The following table exhibits the agreement between the values as given by this formula, and the entire set of final values given by Regnault and alluded to above.

Centigrade Temp.	Pressure in Millimeters of Mercury.	
	Computed from Formula.	Given by Regnault.
-32	.31	.32
-20	.93	.91
-16	1.31	1.28
0	4.60	4.60
25	23.93	23.55
40	55.69	54.91
50	93.07	91.98
75	290.00	288.5
100	760.00	760.0
130	2028.00	2030.0
160	4613.00	4651.6
190	9426.00	9426.0
220	17418.00	17390.0
295.2	59600.00	60000.0 ¹

For engineering units these formulæ become

$$\log p = 12.04243 - \left(2.42223 + \frac{587.83}{T} \right)^2$$

for water vapor, and

$$\log p = 12.04243 - \left(m + \frac{n}{T} \right)^2$$

for any vapor whatever, where p is the vapor pressure in pounds per square inch and T is the corresponding saturation temperature, absolute Fahrenheit, which is about 459.5° greater than ordinary Fahrenheit.

SIBLEY COLLEGE,
CORNELL UNIVERSITY.

A COMPREHENSIVE BOYLE'S LAW APPARATUS.

BY W. J. HUMPHREYS.

IT is often desirable in the laboratory to attack an experimental problem in two or more ways. The most direct and obvious method has the advantage of logical simplicity and, frequently, ease of manipulation, while an indirect means that tests some consequence is, in many cases, far more accurate and instructive.

The piece of apparatus and its manipulations described below enable one to study the relation between the pressure and volume of a gas, at constant temperature, in several different ways. One of these is the usual direct and simple, but unreliable, method that assumes the bore of a tube to be perfectly constant, while certain of the others are less obvious, but instructive, free from assumptions and very accurate.

Referring to the figure, V is a tube of about one centimeter diameter, and seventy to eighty centimeters long, closed at each end with a two-way stopcock. One of the openings at the lower stopcock connects with a tube S that descends in line with V some seventy-five centimeters, where it turns back parallel to itself and rises vertically to any desired height, say for laboratory work from one to two meters above the upper stopcock A . To avoid errors due to capillary action the diameter of this branch should be the same as that of V . A third cock C is placed at the bend and arranged to empty simultaneously both sides of the tube.

CROSS-SECTION OF V CONSTANT.

If the cross-section of V is constant then the volumes of different portions of it are to each other as their respective lengths, and the manipulation is as follows: Close C , turn B till E and V are in communication and leave A open to F or G . Then fill T with clean mercury till it rises to any desired height in V . Next close A and measure the length of V that is still empty. If now the

object is to test the relation for pressures greater than one atmosphere, add more mercury to T and at convenient intervals note the lengths of the unfilled portions of V and the corresponding differences in level of the mercury in V and T . If, however, pressures less than one atmosphere are to be studied, then run some of the mercury out at C and as before measure the corresponding lengths of V and differences in level of the mercury in the two tubes. This difference is always to be taken as positive or negative according as the mercury stands highest in T or in V . Of course it is seen at once that as the pressure increases the volume decreases, and the law is easily discovered. Let a be the height of the barometer, that is, the length of mercury column that would have to be added to T , if it was in a vacuum, to produce the same pressure as actually obtains there, at the time of the experiment, due to the atmosphere above it. Also let l_v , l_t and h_v , h_t be any two corresponding lengths of V and differences in level of mercury in V and T , then it will be found that approximately

$$l_i: l_k = a + h_k: a + h_i.$$

But $a + h_k : a + h_i = p_k : p_i$ where p is the actual pressure on the gas in V , and if the cross-section of V is constant $l_i : l_k = v_i : v_k$ where v_i and v_k are the corresponding volumes. Therefore $v_i : v_k = p_k : p_i$ or p_v = a constant.

If a good barometer is not at hand a slightly different method may be adopted, that is Boyle's law, $p v = \text{a constant}$ (very nearly as we know, at ordinary temperatures and pressures) may be assumed and the assumption experimentally tested by finding the barometric pressure a from the proportion $p_i : p_k = v_k : v_i$, or $a + h_i : a + h_k = l_k : l_i$ which gives

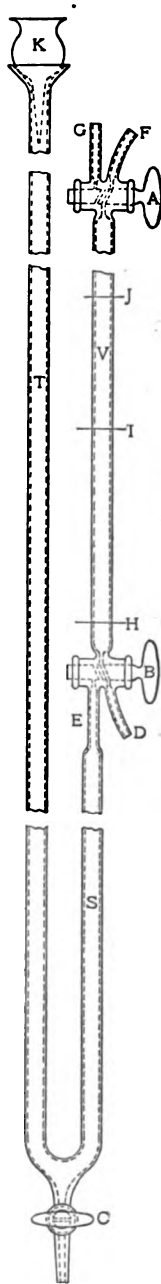


Fig. 1.

$$a = \frac{h_i l_i - h_k l_k}{l_k - l_i},$$

and the proof of the law consists in showing that a , as thus found, is, during the experiment, approximately a constant for different values of h and l .

Both the above slightly different methods are unsatisfactory, because they involve the assumption that the bore of V is uniform, which is never exactly true and often far from it, so that in the end the experiment is rather a qualitative calibration of the tube than a proof of Boyle's law.

ACCURATE RATIOS OF VOLUMES.

An exceedingly exact method that involves no assumptions is — open A to F or G and run mercury from T to V till it finally passes into or through A , then close B and turn A till it communicates with the other and still empty opening, and finally turn B sufficiently to allow the mercury that fills V to run out D where it is caught in any convenient vessel and weighed. For pressures greater than one atmosphere close A and again run mercury through B into V , this time compressing the gas, and when it has come to rest close B and measure the difference in level of the mercury in V and T , then open A and B in the order named, collect and weigh the mercury as before. Repeat for as many different pressures as desired.

Let a = height of the barometer,

W = weight of mercury to fill V ,

w = weight of mercury run in to compress the gas,

h = difference in levels of the mercury,

V = entire volume of tube V ,

v = volume of compressed gas.

The observations will show that $W : W - w = a + h : a$. But $W : W - w = V : v$. Therefore $V : v = a + h : a$ and $pv = a$ constant. Or the law may be assumed, giving at once

and the proof of the law is the constant value of a as thus determined.

This form of the experiment is identical with that given by me in the *PHYSICAL REVIEW* for February, 1900, and is repeated here only for the sake of completeness.

For pressures below one atmosphere leave A open and let the mercury into V till it rises to any common level as J , then collect this and weigh it; again fill to the same level at atmospheric pressure, close A and open C till some lower pressure is obtained, close B , take the difference in level of the mercury in V and T , open A and B in the order named and weigh the mercury that runs out. Repeat for other pressures.

The results give $W - w : W - w_1 = v : v_1 = a - h_1 : a$ from which as before, $pv = a$ constant.

Or, assuming the law,

$$a = \frac{h_1(W - w_1)}{w - w_1} = \frac{h_2(W - w_2)}{w - w_2} = \text{etc.},$$

and, as for higher pressures, the law is established through the constant value found for a .

ARBITRARY FIXED DIFFERENCES IN VOLUME.

Put V in permanent communication with T and by the aid of A and C , but finally with A closed, bring the mercury to any known point in V , to H say, then note the difference in level, now add more mercury to T till it rises to some other point as I (to get the the exact place it may be convenient to go above it and finish with C), again take the difference in level, and similarly find at least a third point J . Repeat the whole process with a different amount of gas in V , which is easily obtained by the proper use of the cocks A and C . The mercury must be brought under the resulting new pressures to the same places H, I, J in V as before and the corresponding levels taken.

Let the differences in level for one set of observations be h_H, h_I, h_J , and for another, with a different quantity of gas in V , let the corresponding differences in level be h'_H, h'_I and h'_J .

On comparing these values it will be found that within narrow limits $h_I - h_H : h_J - h_H = h'_I - h'_H : h'_J - h'_H$. That is, whatever

the pressure to begin with, increasing it any number fold decreases the volume to about the same fraction of itself. In other words the relation, whatever it is, between any given pressure and volume is approximately the same for the other pressures and volumes; and to find it for any one it is sufficient to weigh the amounts of mercury necessary to fill V up to the respective heights H , I and J . Let these weights be W_H , W_I and W_J and let the corresponding but unknown pressures be p , np , mp . In this case the differences in level of the mercury columns are to each other as $n - 1 : m - 1$, while the volumes of compressed gas of course are v , v/n , v/m , and the decrements in volume are to each other as $(n - 1)/n : (m - 1)/m$, or as $W_I - W_H : W_J - W_H$. Therefore the final part of the proof that pv is a constant consists in showing that, when the ratio of the changes in pressure is c/d , that of the corresponding differences of mercury in V is $(cd + c)/(cd + d)$.

CALIBRATION.

It is quite practicable for the average beginner to make a partial calibration by means of which accurate results may be obtained. This method is to take the weights of the mercury necessary to fill V to any two levels as H and I , and entirely full. Let V_H and V_I be the volumes of the tube above H and I , and let M_H and M_I be the weights of mercury required to fill these volumes. Let h_H and h_I be the differences in the level of the mercury when it stands at H and I , then the proof of the law consists in showing that for various initial pressures of gas in V , $a + h_H : a + h_I = M_I : M_H$. Or, by assuming the law, in showing that $\frac{h_I M_I - h_H M_H}{M_H - M_I} = \text{a constant}$ (the height of the barometer) whatever the amount of gas and pressure on it in V .

Of course instead of any of the above methods one might first carefully calibrate V through its entire length, and then proceed according to a method too obvious to justify description. But, on account of the time and skill required, this form of the experiment is scarcely adapted to the general laboratory.

The stopcocks should be kept tight with a suitable stiff lubricant — paraffin with a little vaseline works well. It is advisable to introduce the mercury into T by means of a small funnel or thistle top K whose stem is drawn down to a fine opening. In this way dirt and dust are kept out and only clean mercury admitted.

The differences in level may be read with cathetometers or otherwise as one may desire. For general work a fixed scale back of the instrument is convenient, by means of which the levels can easily be gotten with the aid of paper strips held around the tubes. The levels H, I, J should be arbitrarily chosen points on the scale, or better, temporarily fixed marks on the tube.

If it is desired to have the air in V dry, it may be accomplished by connecting F and G to a suitable drying tube, and of course the whole may be surrounded, though there is but little necessity for it, with a constant temperature jacket. Finally, if the length of T justifies it, high pressures may be obtained in V by means of a compression pump attached to F or G , though the same result may more easily be gotten by adapting the volume of S to the height of T . That is, if the length of T is na , when a is the height of the barometer, the instrument may be worked to its limit without the aid of a pump by having the volume of S roughly $n/2$ times that of V , and therefore, for most laboratory practice, it is sufficient to have these two volumes approximately equal to each other, since this would provide for working with large volumes of gas, when of course changes of volume are most marked, at pressures ranging from zero to three or four atmospheres.

UNIVERSITY OF VIRGINIA,
December, 1902.

A FLEXIBLE JOINT FOR SECURING THERMOMETER AND LIKE STEMS AND TUBES IN VESSELS UNDER PRESSURE.

BY E. G. COKER.

IN experimental work in physical and engineering laboratories, it is very often necessary to insert delicate tubes, like thermometer stems, into metal pipes and vessels, where it is important to prevent any leakage and yet not to subject the inserted tube to stress in making and keeping the joint.

When there is little or no pressure in the containing vessel, cork or india-rubber stoppers are commonly used and serve their purpose fairly well. They are, however, open to objection, for it is often difficult to avoid bending stress, owing to the length of the stopper required and the want of straightness of the inserted piece, and in consequence brittle material like glass requires very careful handling to avoid fracture. Again, if several tubes of different sizes are to be inserted successively in the same aperture, it is necessary to have a close-fitting stopper for each, or leakage will inevitably occur. The provision of a number of stoppers is generally not so serious as regards expense, as is the difficulty of obtaining the sizes required, and this often leads to annoyance and waste of time. When the vessel contains fluid under pressure, cork or india-rubber stoppers are useless, and some other device must be adopted. This often takes the form of a stuffing box, which, in its ordinary form, can only be used for one size of tube or rod and is moreover liable to cause bending stress when the gland is tightened.

In order to remove these objections, the author has used a modified form of joint, which is adapted to any size of tube or cylinder within wide limits, and in which the area of contact is reduced to a minimum, thereby lessening the chance of breakage. The device has been found so useful in practice that a description of the arrangement may be of interest to those engaged in experimental work.

The figure is a section of one form of the joint, showing a thermometer inserted in a gas pipe. An india-rubber ring *A*, slipped over the thermometer or tube, is gripped by a pair of washers *B* having slightly bevelled inner edges, which tend to force the india-rubber into close contact with the tube. The lower washer rests on the end of the pipe and the upper one is pressed down by a thimble *C*, which screws over the outside of the pipe. The thimble consists of a part of a ferrule and the shoulder is formed by a short piece of tube screwed therein.

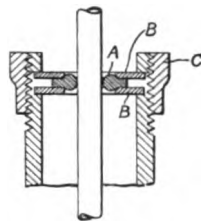


Fig. 1.

If a larger stem or tube is to be inserted, the washers may be enlarged, or a new pair made, and the joint can be renewed. It is apparent that the india-rubber ring is in line contact only, and that there can be no bending, but simply compression which can be varied to suit the circumstances of the case.

The thermometer stem is not rigidly fixed, and can be moved to some extent, depending upon the elasticity of the india-rubber and the grip of the washers.

The joint is found to be tight under considerable pressure, and it can be easily made with the ordinary appliances of a gas-fitter; should these not be available the screwed part of the thimble can be replaced by a bayonet catch.

In an arrangement used by Dr. Barnes and the author for a cylindrical jacket for heating the outer surfaces of pipes, the ends were furnished with stuffing boxes at each end, similar to the one described above, and tubes of both glass and metal from three eighths of an inch to two and one half inches were inserted. No fitting was required beyond the enlargement of the washers, and the joints were found to be perfectly tight under varied conditions of temperature and pressure. In cases where india-rubber is not suitable, an asbestos washer may be used, as for instance, in experiments with superheated steam. Joints similar to the one described have been tested for several months with water, air and steam, under pressures ranging up to eighty pounds per square inch, and have been found to work very satisfactorily.

McGILL UNIVERSITY,
February 14, 1903.

DERIVATION OF EQUATION OF DECAYING SOUND IN A ROOM AND DEFINITION OF OPEN WINDOW EQUIVALENT OF ABSORBING POWER.

BY W. S. FRANKLIN.

IT is required to find the number of units of sound energy which pass out of a room per second through a window of area a . Let V be the volume of the room and i the sound energy per unit volume at time t . It is assumed that the sound energy is the same at each point of the room and that the energy is traveling equally in all directions on the average.

The case under consideration is covered by the flow of energy in one direction through a hole in a wall built across the room so as to divide the room into halves. Describe a sphere of radius r with its center at the hole. The area of this sphere is $4\pi r^2$. We may imagine the sound energy per unit volume, i , to be divided into $4\pi r^2$ equal parts, each part traveling at the normal velocity of sound, 340 meters per second at ordinary room temperature, towards a separate unit of area of the sphere. Consider a diameter D of the sphere drawn through the hole perpendicularly to the wall. Consider the zone of the spherical surface which lies between θ and $\theta + d\theta$, where θ is an angle measured from the diameter D . The area of this zone is $2\pi r^2 \sin \theta \cdot d\theta$. Therefore the fractional part $\frac{2\pi r^2 \sin \theta \cdot d\theta}{4\pi r^2}$ of the total sound energy per unit volume is streaming towards the various elements of this zone at a velocity of $340 \cos \theta$ normal to the wall and $\frac{2\pi r^2 \sin \theta \cdot d\theta}{4\pi r^2} \times i \times 340 \cos \theta \times a$ is the rate at which sound energy is streaming through the hole towards the various elements of the zone. So that the total rate at which sound energy is streaming through the hole from right to left is

$$E = 170ai \int_0^{\pi/2} \sin \theta \cos \theta \cdot d\theta \quad (a)$$

or

$$E = 85ai. \quad (b)$$

This rate of flow of sound energy out of a room through a window of area a is equal to the rate, $\frac{d(Vi)}{dt}$, at which the total sound energy in the room is decreasing, the walls and objects in the room being assumed for the purposes of the immediate discussion to absorb no sound energy, therefore :

$$\frac{d(Vi)}{dt} = -85ai \quad (c)$$

whence

$$i = Ie^{-\frac{85a}{V} \cdot t} \quad (d)$$

in which e is the naperian base, and I is the value of i when t equals zero.

In an actual room the loss of sound energy due to absorption by the walls and objects in the room is sensibly proportional at each instant to the intensity i of the sound at that instant. That is equation (c) expresses the rate of loss of total sound energy at each instant in case of an actual room, a being the open window area which is equivalent to the walls and objects in the room, and equation (d) expresses the value of the decaying sound intensity i at time t .

Sabine has adopted as the measure of the degree of reverberation in a room the time t_1 that is required for a sound to decrease from a given initial intensity I to an intensity one millionth as great. Therefore, substituting $10^{-6} I$ for i and t_1 for t in equation (d) we have :

$$t_1 = 0.1625 \frac{V}{a}. \quad (e)$$

This result is interesting inasmuch as Sabine finds experimentally the value 0.164 for the factor in this formula.

It seems not to be correct to assume that the absorption of sound in a room is due to incomplete successive reflections between which the sound travels over a certain mean path. Such an assumption

does not take account of dissipation of sound energy due to the friction which occurs when the air particles near a wall move to and fro parallel to the wall. In general the dissipation of energy is due to this action, to friction which occurs when the air particles move to and fro perpendicularly to a porous wall, and to imperfect elasticity of walls and objects as they partake more or less of the motion of the contiguous air. All these losses would be sensibly proportional to the sound intensity i for given frequency.

PROCEEDINGS
OF THE
AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE TWENTIETH MEETING.

A REGULAR meeting of the Physical Society was held in Fayerweather Hall, Columbia University, New York City, on Saturday, April 25, 1903, President Webster being in the chair.

The Treasurer presented his report for the year 1902, showing a balance on hand of \$1,406.49. On motion the report was accepted.¹

The Secretary reported a recommendation from the Council as follows: To amend Article III. of the by-laws of the Society by the addition of the following clause:

"Life members shall receive free for the period of five years any journal or journals obtained by the society for its members. At the end of five years life members may obtain such journals through the society at cost."

The proposed amendment was adopted.

The following papers were presented:

On the Measurement of Critical Velocity, together with a study of the flow of water through pipes in stream-line motion. H. T. BARNES and E. G. COKER.

On a Form of Platinum Thermometer for use with the Continuous Flow Calorimeter. H. T. BARNES and D. McINTOSH.

Architectural Acoustics: Some Experiments in Reverberation. G. W. STEWART.

The Spectral Energy Curve of a Black Body at Room Temperature. G. W. STEWART.

The Specific Heat of Certain Organic Solids. W. F. MAGIE.

The Heat of Vaporization of Liquid Air as dependent upon its Composition. J. S. SHEARER and F. R. STRAYER.

On an Attempt to Construct an Electrostatic Transformer. J. E. IVES.

Absence of Electrification in Cases of Sudden Condensation and of Evaporation. C. BARUS.

Nuclei Produced Chemically by the Mixture of Coal Gas and Air. C. BARUS.

Preliminary Note on the Selective Absorption of Organic Compounds in the Infra-red. W. W. COBLENTZ.

Note on the Bending of Rock Salt. W. W. COBLENTZ.

A Modification of of the Quadrant Electrometer without Liquid Contacts. C. BARUS.

At the close of the afternoon session Mr. H. T. Barnes showed to the society an experiment first described by Sir William Crookes,¹ in which microscopic examination shows that the luminescence of zinc sulphide produced by radium is irregularly distributed over the field in a number of starlike bright spots, each of which lasts but a short time. It had been suggested by Sir William Crookes that each luminous spot was due to the impact of a single corpuscle emitted by the radium.

The Society adjourned at 5 P. M.

ERNEST MERRITT,
Secretary.

TREASURER'S REPORT FOR 1902.

Receipts.

Balance from the year 1901.....	\$1,001.90	
Interest on balance.....	35.07	
Annual dues for 1902.....	455.00	
Arrears of dues of previous years, paid in 1902..	75.00	
Life membership fees of three members.....	150.00	
Total receipts.....	\$1,716.97	\$1,716.97

Expenditures.

Printing the Bulletin, and notices, etc.....	\$ 189.00	
Clerk and stenographer hire.....	50.00	
Secretary's expenses.....	62.08	
Treasurer's expenses	9.40	
Total expenditures.....	\$ 310.48	310.48
Balance carried forward.....		1,406.49
		\$1,716.97

THE FLOW OF WATER THROUGH PIPES. EXPERIMENTS ON STREAM-LINE MOTION AND THE MEASUREMENT OF CRITICAL VELOCITY.¹

BY H. T. BARNES AND E. G. COKER.

THE experiments described in the present paper are a continuation of the work described by the authors in a previous paper published in the *PHYSICAL REVIEW*.² It has been possible to apply the thermal method to the measurement of critical velocity on a larger scale than was previously attempted. In the classical experiments of Osborne Reynolds³ it was shown that two critical velocities could be observed depending on the condition of the water before it entered the pipe. For undisturbed water entering a perfectly smooth pipe the critical velocity represented the flow at which the birth of eddies took place in the pipe itself, breaking up the steady stream-line flow. For disturbed water, in which eddies were initially present, the critical velocity represented the flow at which the stream-lines could no longer form in a long smooth pipe. Below this critical velocity the eddies were observed to disappear after passing through a sufficient length of pipe, and stream-line flow ultimately became the stable flow. These two critical velocities we have called the upper and lower limits of critical velocity, or stream-line flow, to avoid confusion. Reynolds has shown mathematically from the hydrodynamical equations of motion, that the change in the character of flow is related in some way to the viscosity, density and radius of the pipe. An experimental verification of these deductions showed that the critical velocity varied inversely as the diameter of the pipe and directly as the viscosity divided by the density. Experiments were tried for both the upper and lower limits of critical velocity. For the upper limit Reynolds found that the inverse diameter law held in the case of three pipes under similar conditions, but that the critical velocity was related in some way to the degree of steadiness of the water. For the lower limit he was able to connect his experiments with the experiments of D'Arcy for very large pipes on the one hand, and the experiments of Poiseuille for very small pipes on the other hand. These experiments were most complete, and showed that the lower limit was the true point from which to determine the criterion for the change in the law of resistance.

Reynolds' upper limit experiments were carried out by the method of color bands in a long rectangular tank. It seemed to us possible that by using a very much larger tank under a high head of water we would be

¹ Abstract of a paper presented at the meeting of the Physical Society held on April 25, 1903.

² *PHYSICAL REVIEW*, Vol. XII., 372, 1901.

³ *Phil. Trans.*, 1883 and 1895.

able to obtain a higher degree of steadiness than was obtained in the comparatively small tank used by Reynolds.

Adopting the thermal method of measuring critical velocity, which enabled us to use metal pipes in place of glass ones and to simplify our apparatus considerably, we carried out a series of experiments which series forms the subject of the present paper.

Briefly the result of our work may be summarized as follows :

1. The attainment of exceedingly high velocities of stream-line flow for certain sizes of pipes fed by perfectly quiet water under a high head.
2. The reformation of stream-lines in certain cases after eddies had formed with a subsequent breaking up of the stream-lines at a very much higher velocity.
3. A small divergence from the law of the change in viscosity with temperature for the upper limit of stream-line flow.
4. A verification of the viscosity temperature law for the lower limit of stream-line flow.

McGILL UNIVERSITY.

A NEW FORM OF PLATINUM RESISTANCE THERMOMETER, SPECIALLY ADAPTED FOR THE CONTINUOUS-FLOW CALORIMETER.¹

BY H. T. BARNES AND D. McINTOSH.

ONE of the difficulties to be overcome in constructing a sensitive platinum thermometer, is to restrict the size of the coil of wire forming the bulb. In order to have a sufficient length of wire it is often necessary to make the bulb inconveniently large, and thereby sacrifice quickness of register.

The form of thermometer which is described in the present paper overcomes this difficulty to a considerable extent.

Briefly the thermometer consists of two concentric glass tubes fused together at the end with the wire wound on the inner tube in a thread etched into the glass. The main point about the thermometer is that a core passes through the center of the bulb, through which the liquid or gas, the temperature of which is desired, can be made to flow. It is quite as easy to construct, if not more so, than the mica-frame type, and it has unquestionable advantages over the latter for the particular use for which it was designed, although it would be unsuited for certain types of temperature measurement. Full details of construction are given in the paper.

McGILL UNIVERSITY.

¹ Abstract of a paper presented at the meeting of the Physical Society held on April 25, 1903.

THE SPECTRAL ENERGY CURVE OF A BLACK BODY AT ROOM TEMPERATURE.¹

BY G. W. STEWART.

THE distribution of energy in the spectrum of a black body at the temperature of the room may be measured in the usual manner, if the temperature of the radiometer or bolometer and the spectrometer be made so low that their own radiation to the black body may be neglected. The device used in the present experiment was to keep the radiometer and spectrometer at room temperature, but to lower the temperature of the black body to that of liquid air. The deflections obtained were proportional to the radiation of the radiometer vane to the black body. The vanes were made of platinum, coated with platinum black, the thickness used being sufficient to make them approach very nearly to an ideally black body for the wave-lengths considered. A mirror spectrometer and a rock salt prism were used to produce the spectrum. The temperature of the room was 24° C. The maximum intensity in the spectrum occurs at 9.2 μ . According to the theory and previous observations on black bodies at higher temperatures, the maximum should occur at 9.8 μ . If Planck's equation be used to compute the distribution of energy, a curve is obtained which, between 5 μ and 13 μ , does not vary from the experimental one by more than ten per cent. The agreement of experiment with theory is as close as could be expected in view of the small radiometer deflection obtained.

ARCHITECTURAL ACOUSTICS: SOME EXPERIMENTS IN REVERBERATION.¹

BY G. W. STEWART.

PROFESSOR WALLACE C. SABINE has shown that it is possible to compute the reverberation in any proposed auditorium, if the dimensions of the room and the areas of the materials exposed are known. His formula, determined by experiment, is

$$t = \frac{0.164V}{a},$$

absorbing power of the room. Professor W. S. Franklin has derived the formula

$$t = \frac{0.162 V}{a},$$

upon the assumption that sound is diffracted with perfect ease into every part of the room. When the new auditorium of Sibley College of Cornell University neared completion, it was found that the excessive reverberation, with a small audience present, produced such a confusion that a speaker could not be understood. The auditorium offered an opportunity for the verification of the above equations, and for the measurement of the coefficients of absorption of any materials that might be introduced into the room. The apparatus used was a wooden organ pipe, 512 vibrations per second, and a chronograph. The organ pipe was blown by the mouth with a pressure sufficient to produce as loud a tone as possible without the overtones being noticeable. At the completion of these experiments, apparatus received through the kindness of Professor Sabine made it possible to ascertain the rate of production of intensity of the organ pipe. This was necessary in order to ascertain the initial intensity and to apply the formulæ which, in the form here given, demand that the initial intensity be 1,000,000 times that of minimum audibility. The value of V in this room was 4,680 cubic meters and the absorbing power, 122.9. Substituting these in the above equations, t becomes 6.23 and 6.16. The observed time of reverberation was 6.26. The agreement is closer than might be expected, for the absorption coefficients of some of the materials is not known to within less than 3 per cent.

Cocoa matting was laid on paper felt in the aisles and between the seats of the auditorium, and, at different stages of progress, observations were taken of the time of reverberation. From these data, the absolute coefficient of absorption of cocoa matting, 1.56 kgs. per sq. m., laid on paper felt, 0.69 kgs. per sq. m., was found to be 0.172. With different areas of matting laid, the time of reverberation varied in a manner indicated by the above equations. Observations made in different parts of the room show that t is practically independent of the observer's position, and that it could be measured to within a few hundredths of a second. All the experiments in this auditorium verify those of Professor Sabine, an account of which appears in a series of papers in the *American Architect* for 1900.

THE SPECIFIC HEATS OF CERTAIN ORGANIC SOLIDS.¹

BY WILLIAM FRANCIS MAGIE.

THE specific heat of a solid whose specific gravity is not too great may be obtained by introducing a weighed portion of it, finely powdered, into one of the cups of the Pfaundler calorimeter, the cups having been previously filled with appropriate amounts of some liquid which does not dissolve the solid, and comparing the heat capacities of the contents of the cups. The liquids used by me were aniline (specific heat, 0.5155, Griffiths) and kerosene (specific heat, 0.4753, obtained by comparison with nitrobenzol, specific heat 0.336 at 20°, Regnault). The solids examined were those whose apparent molecular heat in solution I had already determined. There were irregularities developed in the working of the calorimeter which render the results obtained uncertain by perhaps two per cent. or even more. In the following table are given under I., the molecular weights, under II., the specific heats, under III., the molecular heats, of the solid substances; under IV., for comparison, the apparent molecular heats of the same substances dissolved in water and in alcohol. Where one number is given in column IV. it is the molecular heat in water; where two numbers are given, the upper

TABLE I.

	I.	II.	III.	IV.
Cane sugar.	342	0.301	103 ¹	152.8
Hydrous milk sugar A. . .	360	0.299	107.6	147
Anhydrous milk sugar A. .	342	0.2895	99	147
Maltose A	360	0.323	116	143
Dextrose A.	180	0.314	56.5	78.8
Levulose K.	180	0.276	51.5	89.6
Mannite A	182	0.316	57.5	108
Dulcite K	182	0.283	51.5	97.5
Resorcin K.	110	0.266	29.2	63.4 56.8
Hydroquinone K.	110	0.258	28.3	63.4 56.8
Pyrocatechin K	110	0.313	34.4	75.5 56.8
Urea K.	60	0.321	19.2	21.5 28
Glycerine, liquid.	92	0.576	53 ²	54
Phenol, liquid.	94	0.561	52.7 ⁴	71.5 51.4

¹ Abstract of a paper presented at the meeting of the Physical Society held on April 25, 1903.

² Kopp.

³ Emo.

⁴ Schlamp.

one is the molecular heat in water, the lower one, that in alcohol. I have added three determinations by other observers to make the list as complete as possible. With acetamide I could get no satisfactory measurements, and it is therefore omitted. Those substances marked A were examined in aniline, those marked K, in kerosene.

An examination of these results in the light of a theory given by Staigmüller (Wied. Ann., Vol. 65, p. 655), shows that the observed molecular heats are consistent with reasonable suppositions as to the degrees of freedom of the atoms in the molecules, and exhibits interesting relations between the degrees of freedom assigned to the isomers and to substances of generally similar structure. On this theory the process of solution of non-electrolytes involves in most cases a considerable increase in the freedom of their atoms.

The heats of solution obtained by dissolving the substances in water are given in the following table. Column I. contains the heat devel-

TABLE II.

	I.	II.	III.
Cane sugar	- 3.6	-1231	19.7
Hydrated milk sugar.	-11.5	-4140	21.6
Dehydrated milk sugar	- 7.4	-2530	19.2
Erdmann's milk sugar.	- 3.6	-1231	19.3
Maltose	- 9.75	-3510	19.8
Dextrose	-13.02	-2344	18.7
"	-13.09	-2356	18.9
Levulose	-10.64	-1915	18.7
Mannite.	-28.2	-5132	18.7
Dulcite	-36.7	-6679	18.9
Resorcin.	-34.09	-3750	18.2
"	-35.3	-3883	20.0
Hydroquinone	-41.2	-4532	18.5
"	-41.8	-4598	20.0
Pyrocatechin.	-31.34	-3447	19.5
"	-31.03	-3413	19.0
Phenol	-30.6	-2876	19.9
Urea	-60.4	-3624	18.6
Glycerine	+14	+1290	20.0

oped, in gram-calories, by the solution of 1 gram of the substance in 37.5 grams of water; column II., the heat developed by the solution of 1 gram-molecule in 37.5 times its weight of water; column III., the temperature of the water into which the substance was introduced. Heat absorbed is marked negative.

A DETERMINATION OF THE HEAT OF FUSION OF ICE.¹

BY ARTHUR W. SMITH.

THE author shows that the determinations of this constant, made more than a third of a century ago, agree only to within 1 part in 100, and that no attempt has been made to determine its value since 1870. The problem which is now solved is the measurement, in joules, of the amount of heat required to transform one gram of ice at 0° C. into water at the same temperature.

The calorimeter proper consists of a half-liter brass vessel, nickel-plated and highly polished on the outside. This contains, besides the stirrer, thermometer and heating coil, about 200 cc. of refined kerosene oil, which provides a medium for the transference of heat from the wire to the ice. To reduce the loss of heat by radiation, conduction, etc., to a minimum, this calorimeter is placed within a chamber which is surrounded on all sides with broken ice, thus maintaining it at nearly the same temperature as that of the calorimeter.

The method employed may be briefly stated as follows: The sample of ice whose heat of fusion is to be determined is cut into small pieces and cooled several degrees below 0° C. While at this temperature it is weighed and transferred to the calorimeter, which contains kerosene oil already cooled to the same temperature. The calorimeter and contents are slowly warmed by an electric current until the temperature reaches the desired point for commencing an experiment, usually about -1° C. A larger current is then applied for sufficient time to melt the ice and raise the resulting water to about $+0.5^{\circ}$ C. The heat generated by the current has been used in four ways. (1) In raising the temperature of the ice and the calorimeter from about -1° C. to 0° C. (2) In melting the ice. (3) In raising the temperature of the water and calorimeter from 0° C. to about $+0^{\circ}$ C. (4) In supplying whatever heat is lost by the combined effects of radiation, conduction, convection, etc. Of these four quantities of heat, the second is thirty or forty times as great as all the others combined, which are determined as corrections, and when subtracted from the total amount of heat generated by the current, gives the heat required to melt the ice. This amount of heat divided by the mass of the ice gives the heat of fusion per gram of ice.

The heat generated by the current is determined by measuring both the current through the coil in the calorimeter, and the E.M.F. between its terminals, in terms of a standard cell. There is thus no question regarding the resistance or the temperature of the wire carrying the current.

¹ Abstract of a paper presented at the meeting of the Physical Society held on April 25, 1903.

The ice employed in this work was crystallized from a bath of the purest distilled water, and cut into small cubes. The specific heat of such ice is shown to be constant right up to 0°C. , at which temperature it melts. The standard cell and resistances used in the measurement of the electrical energy are being calibrated at the National Bureau of Standards. A calculation based upon a preliminary determination gives 334.25 joules as the mean of eight determinations of the heat of fusion of ice in each of which about 100 grams of ice was melted.

ABSENCE OF ELECTRIFICATION IN CASES OF SUDDEN CONDENSATION AND OF SUDDEN EVAPORATION.¹

BY CARL BARUS.

1. It has long been known that neither in cases of ordinary evaporation or condensation is there an accompaniment of electrification. On the other hand, when a mass of water is suddenly shattered as in jets, the electrification produced is marked. The electrification soon vanishes, however, whereas the nuclei persist.

The question thus occurs whether in ordinary slow evaporation the absence of an electric effect may not be due to the fact that the charges vanish too quickly to be noticeable. It seemed worth while, therefore, to examine the case for *sudden* condensation and the rapid evaporation of fog particles.

2. To test this question a graduated electroscope was introduced into the condensation chamber. To insulate it in the saturated atmosphere the stem was enclosed in a rubber tube open below, and only in contact with the stem at the top (sealing wax joint). The tube was thoroughly dried before insertion into the chamber and the instrument showed satisfactory insulation for a half hour or so, after which it was removed for desiccation prior to new experiments.

Condensation was produced by sudden exhaustion, as usual. The nuclei used were obtained from the air as well as from phosphorus.

The method of experiment consisted in observing the normal leakage of the charged electroscope by observations made every half minute.

Nuclei were then introduced and the experiment repeated with the alternate sudden production and sudden dissipation of fogs between each observation.

The two curves of leakage were not distinguishable even when the fogs were of the densely opaque character due to phosphorus. The error of the method was about 5 per cent., with somewhat larger uncer-

NUCLEI PRODUCED BY THE MIXTURE OF COAL GAS AND AIR.¹

BY CARL BARUS.

1. Some time ago the writer noticed that if coal gas is examined by the steam jet or color tube as described elsewhere, a faint pink flush is seen in the field of the tube. This indicates the presence of nuclei to the extent of many thousands per cm.³ in the gas. Inasmuch as such nuclei could not be retained in the gas pipes (they would soon be lost either by subsidence or diffusion) an explanation of the phenomenon was difficult to suggest.

2. Recently the writer examined the question by aid of coronas. Coal gas stored over water and suddenly cooled shows no condensation. It is therefore free from nuclei as would be anticipated. Filtered air under the same conditions behaves in the same way. If, however, coal gas and filtered air are *mixed* and then examined, nuclei are abundantly present to the extent of 2,000 to 3,000 per cm.³ showing that chemical reaction (attributable to the presence of sulphur gas as an impurity) has taken place.

If the air is introduced above the lighter coal gas, the nuclei are obtained at once in view of the mixture by convection. If coal gas is introduced above the air, nuclei are not at first in evidence, but they appear later as the result of diffusion at the surface of contact.

The result with the steam tube is now obvious, seeing that gas is here necessarily introduced in contact with air.

These nuclei are not ionized, appreciably, as special experiments with an electrical condenser showed.

PRELIMINARY NOTE ON THE SELECTIVE ABSORPTION OF ORGANIC COMPOUNDS.¹

BY WM. W. COBLENTZ.

THE historical side of this subject is rather limited. In 1882, Abney and Festing¹ by means of photography investigated the absorption spectra of a large number of compounds, to 1.2μ , which was the limit of the sensibility of the photographic plate. They found many interesting relations among the absorption bands of the ethyl (CH_3) radical

compounds, *e. g.*, benzene (C_6H_6) the crucial line is $.879 \mu$. Again in some of the compounds containing oxygen certain lines coincided with the iodides, which were free from oxygen, and they "were forced to the conclusion that there must be some connection between the one and the other, since such an agreement could not be fortuitous."

From 1889 to 1890 Ångström¹ using a bolometer and rock salt prism and lenses found the absorption of CO and CO_2 . He also found the absorption of methane, ethylene, ether, benzene and carbon bisulphide, several of the latter in a vapor also a liquid state. His work extended to about 8μ , and showed that the maximum of liquids and their vapors were often coincident. About the same time Julius investigated the absorption of some 20 organic compounds. He used a bolometer, and a prism and lenses of rock salt. His work extends to 10μ which by his straight line extrapolation from 5μ was supposed to be $16 + \mu$. He used a cell hollowed out of rock, about .2 mm. in thickness, and found that about one third of the substances (mostly alcohols) become opaque at 7μ . He found that all compounds containing the methyl (CH_3) group had an absorption band at 3.45μ . The conclusion reached from this investigation, which had been the most extensive thus far, was that the absorption of heat waves is due to intramolecular movements, *i. e.*, the internal structure of the molecule is the chief factor in absorption. This conclusion that the molecule and not the constituent atoms determines the absorption of a compound is further substantiated by the fact that of the six compounds containing chlorine investigated by Julius not one of them showed the Cl band found by Ångström² at 4.28μ .

Donath,³ in 1896 using a quartz prism and bolometer, investigated a half a dozen compounds and concluded that the absorption is intermolecular and not intramolecular as found by Julius. His work extended to 2.7μ and covered the region of "practical significance" as he expressed it. For the aromatic compounds and the fatty oils he found maxima common at 1.69 and 2.2μ . The most recent work in this field is that of Puccianti⁴ in 1899 to 1900 who, by means of a quartz prism and radiometer, explored a number of benzene derivatives. He found that all compounds containing carbon combined directly with hydrogen showed absorption maxima at 1.71μ while the benzene derivatives have two other maxima in common at 2.18 and 2.49μ . He found his results

sulphide remained transparent in the region of $7.3\ \mu$ where solid iodine has a larger absorption band. The results were negative. While exploring a plate of sulphur, ground from a crystal it was found that it had an absorption band at about $12.25\ \mu$ which coincided with that of CS_2 . This led to an investigation of organic compounds containing sulphur, which it was found showed these bands. When chloroform, CHCl_3 , was investigated this also gave these bands at $12.25\ \mu$. This coincidence of the absorption bands of sulphur, carbon bisulphide and chloroform (CHCl_3) added to the interest in the work, and the whole subject has been taken up anew, with the following results. Naturally the chemical side of this subject was unfamiliar in the beginning and the writer takes pleasure in acknowledging his indebtedness to Professor Orndorff and his assistants, Messrs. Teeple and Russel, of the Chemistry Department, for valuable advice, also for their kindness in placing their collection of chemicals at his disposal. When one considers the fact that the field is almost entirely new beyond $7\ \mu$, so that nothing is known of the transparency of the compounds to be investigated, also that it requires from six to eight hours to examine a single substance, it becomes apparent that this work, which has occupied two months, cannot be more than a preliminary survey of the whole field.

All the compounds thus far investigated show a deep absorption band at 3.3 to $3.7\ \mu$ which is generally complex, just as Adney found conditions at the end of the optical region — to $1.2\ \mu$. This region requires an extensive exploration by itself, with a dispersion greater than that of a single rock salt prism. The spectra of different substances investigated show that the new maxima can best be detected beyond 6 to $14\ \mu$, which is also the most useful part of a rock salt spectrum provided one has a strong source of radiation in this region. The heater of a Nernst lamp was found to give out a selective radiation from 4 to $8\ \mu$ and is most serviceable, since many of the compounds studied have their greatest general absorption from 7 to $10\ \mu$, and then became more transparent again. The suppression of the selective radiation at $5.1\ \mu$, by covering the surface with a film of iron oxide, is well shown in curve *b*. The emission maximum at $5.1\ \mu$ lies close to the absorption band of quartz found by E. F. Nichols at $5.3\ \mu$. A covering of mica (.3 mm. thick) or of borax was transparent to this radiation. In the present work a mirror spectrometer¹ rock salt prism and radiometer were employed. The rock cell varied from .04 to .3 mm. in thickness.

Thus far 38 compounds have been explored far into the infra-red, — 12 to $15\ \mu$. They include the acids, alcohols, esters and ethers.

In this survey substances were taken that had a small number of atoms like methyl iodide, CH_3I , and also those containing many atoms like stearic acid, $\text{C}_{18}\text{H}_{36}\text{O}_2$.

¹ Described in PHYSICAL REVIEW, Nos. 1 and 2, 1903.

RESULTS.

The results agree with those of Julius, which were that the constitution of the molecule, as well as the arrangement of the atoms in the molecule, determines the absorption of the compound.

New relations such as the coincidence of the absorption bands of different compounds, and the apparent occurrence of some bands in harmonic series await confirmation. The wave-lengths from 9 to 12 μ must be corrected before deciding on the latter questions. All compounds containing CH_2 have an absorption band at 3.5 μ just as found by Julius.

The effect of the constitution of the molecule is well illustrated in benzene and its derivatives. In mono-brom-benzene ($\text{C}_6\text{H}_5\text{Br}$) it so happens that the substitution of Br for H has little effect upon the maxima. But the transmission curve is quite changed throughout the whole spectrum. In benzaldehyde ($\text{C}_6\text{H}_5\text{CHO}$), where one H atom is replaced by CHO, the curve is still more affected while the maxima are shifted, or new ones introduced. From the present study of the curves it seems more probable that the occurrence of a new band near an old one is not a shifting of the old one (*e. g.*, some compounds show the CH_2 band at 6.95 or 7.1 μ instead of 7.0 μ), but is rather a modification of the whole curve in this region so that it is steeper or flatter, and consequently obliterates or brings out more distinctly the new band. It is not an error in setting and observation, for such a shift on a steep curve would mean an error of 5 to 10 per centum which would be impossible to escape detection.

Bromine seems to strengthen the bands at 8.0, 9.8, 10.6, 11.7 and 12.6 μ , and weaken the ones at 3.3, 5.4, 8.9, 12.2 and 13.3 μ .

The effect of the structural arrangement of the atoms in the molecule is well illustrated in compounds like limonene and prinine ($\text{C}_{10}\text{H}_{16}$), which have the same number of atoms of carbon and hydrogen but differently grouped.

Pinine has a greater general absorption from 7 to 10 μ . The maxima are also different. Other substances like ethyl sulphocyanide ($\text{C}_2\text{H}_5\text{SCN}$) and isothyl sulphocyanide ($\text{C}_2\text{H}_5\text{N} - \text{CS}$) show a very marked difference in the general as well as the selective absorption. In both cases the CH_2 bands occur at the approximate wave-lengths: 3.5, 7.0, 10.5 and 14.0 μ , while in ethyl sulphocyanide another series of bands exists, viz.: 3.3, 6.6, 9.9 and 13.2 μ . This same series of maxima occurs in the benzene (C_6H_6) and the other compounds containing CH and strengthens the belief that it is due to CH .

The study of the results already obtained is very incomplete and many new relations may be found in the future. At the present time the work agrees with that of Julius and Puccianti in showing that the absorption

of heat waves is due to the intramolecular condition of the molecule rather than intermolecular as found by Donath.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
April, 1903.

NOTE ON THE BENDING OF ROCK SALT.¹

BY WM. W. COBLENTZ.

WHILE splitting rock salt plates it was noticed that occasionally some plates were bent (not from splitting), showing that the whole mass must have been subjected to strain.

The conditions under which such a bending will occur is of interest, since at room temperature rock salt is very brittle.

It was found that plates of rock salt .5 mm. thick would bend after being held in the mouth for a few minutes. The bending is done by pushing the plate against the teeth with the tongue — temperature of the mouth is about 36° C. The rock salt plates can also be bent by holding them in a warm (50° to 70° C.), saturated salt solution by means of a forceps, and pressing lightly against the side of the vessel. The same is true when held in a bunsen flame, temperature 1400° C. The accompanying mounted specimens have been bent by the three different methods, the thicker ones by the second and third.

In all cases the rock salt remains brittle for a short time, after which bending begins, and (in the methods employed) then suddenly occurs more readily. It can then be bent back and forth like a piece of whalebone. The plates can also be twisted after they have become pliable.

A small plate 3 x .5 x .1 cm. held in a vise, with a weight of about 75 grams suspended from the free end, did not show a visible bending after leaving it at room temperature for 25 days.

Whether this is a case of slipping due to gliding planes, such as occurs in certain other minerals, or whether it is simply a case of viscosity like that of pitch, has not been determined. The thick plates show but slight double refraction.

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¹ Presented at the meeting of the Physical Society held on April 25, 1903.

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